Construction of Baugh-Wooley Array

Let $x = (x_4x_3x_2x_1x_0)$ and $y = (y_4y_3y_2y_1y_0)$ be two numbers in two's complement form. The Baugh-Wooley matrix of these two numbers is constructed as follows: We start with the values of these numbers:

$$x_v = -2^4x_4 + \sum_{i=0}^{3} x_i 2^i \quad \text{and} \quad y_v = -2^4y_4 + \sum_{i=0}^{3} y_i 2^i \ .$$

Since $p = y \cdot x$ implies $p_v = y_v \cdot x_v$, we have

$$\begin{aligned} p_v &= \left(-2^4y_4 + \sum_{i=0}^{3} y_i 2^i\right) \cdot \left(-2^4x_4 + \sum_{i=0}^{3} x_i 2^i\right) \\ &= 2^8x_4y_4 + \sum_{i=0}^{3}\sum_{j=0}^{3} x_i y_i 2^{i+j} - 2^4 \sum_{i=0}^{3} x_4 y_i 2^i - 2^4 \sum_{i=0}^{3} y_4 x_i 2^i \ . \end{aligned}$$

Note that the last 2 terms are subtracted from the partial product. To avoid subtraction, we obtain two's complement of these numbers and add them to the partial product. Since

$$2^4 \sum_{i=0}^{3} x_4 y_i 2^i = (0, 0, x_4y_3, x_4y_2, x_4y_1, x_4y_0, 0, 0, 0, 0)_2$$

we have

$$\begin{aligned} -2^4 \sum_{i=0}^{3} x_4 y_i 2^i &= (1, 1, \overline{x_4y_3}, \overline{x_4y_2}, \overline{x_4y_1}, \overline{x_4y_0}, 1, 1, 1, 1)_2 + 1 \\ &= (1, 1, \overline{x_4y_3}, \overline{x_4y_2}, \overline{x_4y_1}, \overline{x_4y_0}, 0, 0, 0, 0)_2 + (0, 0, 0, 1, 0, 0, 0, 0)_2 \ , \end{aligned}$$

and similarly,

$$-2^4 \sum_{i=0}^{3} y_4 x_i 2^i = (1, 1, \overline{y_4x_3}, \overline{y_4x_2}, \overline{y_4x_1}, \overline{y_4x_0}, 0, 0, 0, 0)_2 + (0, 0, 0, 1, 0, 0, 0, 0)_2 \ .$$

This gives us the following array:

9	8	7	6	5	4	3	2	1	0
					y_4	y_3	y_2	y_1	y_0
					x_4	x_3	x_2	x_1	x_0
						x_0y_3	x_0y_2	x_0y_1	x_0y_0
					x_1y_3	x_1y_2	x_1y_1	x_1y_0	
				x_2y_3	x_2y_2	x_2y_1	x_2y_0		
	x_4y_4		x_3y_3	x_3y_2	x_3y_1	x_3y_0			
1	1	$\overline{x_4y_3}$	$\overline{x_4y_2}$	$\overline{x_4y_1}$	$\overline{x_4y_0}$				
					1				
1	1	$\overline{y_4x_3}$	$\overline{y_4x_2}$	$\overline{y_4x_1}$	$\overline{y_4x_0}$				
					1				

The last 4 rows the array can be modified to produce simpler constructions. When $x_4 = 0$, the contribution of the rows

9	8	7	6	5	4	3	2	1	0
1	1	$\overline{x_4y_3}$	$\overline{x_4y_2}$	$\overline{x_4y_1}$	$\overline{x_4y_0}$				
					1				

should be equal to zero. This is easily verified by plugging a zero for x_4 in the above. The resulting 10-bit number have all zeros:

9	8	7	6	5	4	3	2	1	0
1	1	1	1	1	1				
					1				
0	0	0	0	0	0				

When $x_4 = 1$, we obtain:

9	8	7	6	5	4	3	2	1	0
1	1	$\overline{y_3}$	$\overline{y_2}$	$\overline{y_1}$	$\overline{y_0}$				
					1				

The following row produces the same values for both $x_4 = 0$ and $x_4 = 1$, and can be substituted:

9	8	7	6	5	4	3	2	1	0
1	$\overline{x_4}$	$x_4\overline{y_3}$	$x_4\overline{y_2}$	$x_4\overline{y_1}$	$x_4\overline{y_0}$				
	1				x_4				

We apply the same reasoning to the row with the terms $\overline{y_4x_i}$ and reproduce the array as follows:

9	8	7	6	5	4	3	2	1	0
						x_0y_3	x_0y_2	x_0y_1	x_0y_0
					x_1y_3	x_1y_2	x_1y_1	x_1y_0	
				x_2y_3	x_2y_2	x_2y_1	x_2y_0		
	x_4y_4		x_3y_3	x_3y_2	x_3y_1	x_3y_0			
1	$\overline{x_4}$	$x_4\overline{y_3}$	$x_4\overline{y_2}$	$x_4\overline{y_1}$	$x_4\overline{y_0}$				
	1				x_4				
1	$\overline{y_4}$	$y_4\overline{x_3}$	$y_4\overline{x_2}$	$y_4\overline{x_1}$	$y_4\overline{x_0}$				
	1				y_4				

We can also remove the 1s in the 8th column by adding. This will produce a 1 in the 9th column, which in turn will produce a 1 in the 9th and a 1 in the 10th column. Omitting the 10th column (since our numbers need at most 10 bits: column 0 to column 9), we obtain the Baugh-Wooley array as

9	8	7	6	5	4	3	2	1	0
						x_0y_3	x_0y_2	x_0y_1	x_0y_0
					x_1y_3	x_1y_2	x_1y_1	x_1y_0	
				x_2y_3	x_2y_2	x_2y_1	x_2y_0		
	x_4y_4		x_3y_3	x_3y_2	x_3y_1	x_3y_0			
	$\overline{x_4}$	$x_4\overline{y_3}$	$x_4\overline{y_2}$	$x_4\overline{y_1}$	$x_4\overline{y_0}$				
1	$\overline{y_4}$	$y_4\overline{x_3}$	$y_4\overline{x_2}$	$y_4\overline{x_1}$	$y_4\overline{x_0}$				
					x_4				
					y_4				

The short paper by Blankenship offers slight improvements on the construction of the values in the 8th and 9th columns. It is straightforward to show that the following three constructions are equivalent:

9	8
	x_4y_4
	$\overline{x_4}$
1	$\overline{y_4}$

9	8
	1
1	$\overline{x_4}$ $\overline{y_4}$

9	8
$x_4 + y_4$	$x_4 + y_4$