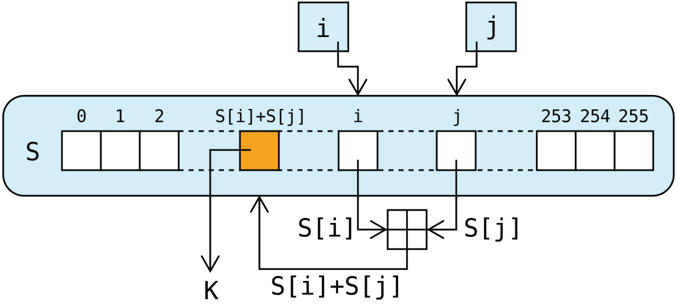
Advanced Stream Ciphers

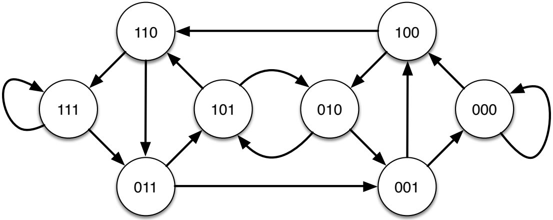


Nonlinear Feedback Shift Registers

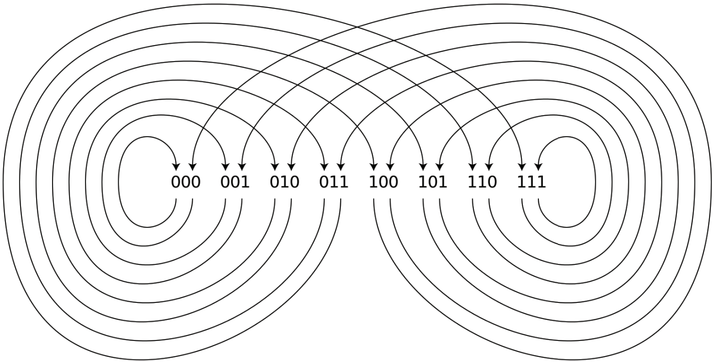
- LFSRs have very desirable statistical properties and excellent theory to build stream ciphers with large (maximal) periods
- However, due to linearity and the Berlekamp-Massey algorithm), they provide no security against known text attacks
- One way to overcome the cryptographic weakness is to use a nonlinear feedback function
- As in the case for LFSRs, the sequences of maximal period are of special interest; furthermore, the all-zero state is not excluded
- There are several mathematical tools for studying the properties of nonlinear shift register sequences

de Bruijin Graphs

- A general shift register sequence can be represented using a graph of 2^n vertices, labeled by the words of $\{0,1\}^n$
- The edges show possible transitions of a shift register, for example, we would have a transition from $(s_3, s_2, s_2, s_1, s_0)$ to $(0, s_3, s_2, s_2, s_1)$ and to $(1, s_3, s_2, s_2, s_1)$
- These graphs are named de Bruijin graphs, and have applications in other fields as well, such as genome research



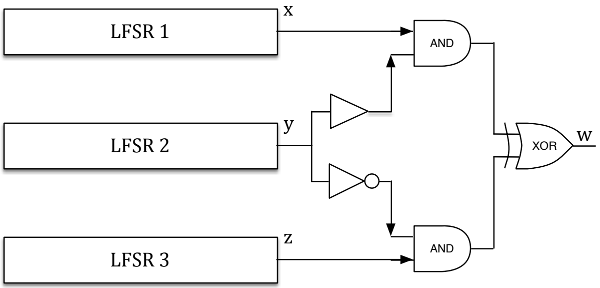
A pretty de Bruijin Graph :)



Nonlinear Combination of LFSRs

- While many mathematical properties of nonlinear shift register sequences can be studied using de Bruijin graphs (and associated de Bruijin sequences), there are no fast algorithms for generating such sequences
- An approach that makes use of the LFSRs (particularly, statistical properties, simple design, fast generation, maximality) but removing the cryptographic weakness is the *nonlinear combination* of LFSRs
- A very simple generator is proposed by Geffe: 3 LFSRs are used, and the output of one them decides which of the output of the other 2 LFSRs is to be used

Geffe Generator



- If x, y, z are the output bits of the LFSRs at the ith step, the combined generator is

$$w = (x \wedge y) \oplus (z \wedge \overline{y})$$

Properties of Geffe Generator

- The LFSRs are not identical, each one having a length n_i, a particular connection polynomial $c_i(x)$ for $i = 1, 2, 3$
- Also, the lengths n_i are selected to be pairwise relative prime: $\gcd(n_1, n_2) = \gcd(n_1, n_3) = \gcd(n_2, n_3) = 1$
- When the connection polynomials are primitive, each LFSR will be maximal, having the period of $2^{n_i} - 1$ for $i = 1, 2, 3$
- The Geffe generator has the period of

$$(2^{n_1} - 1)(2^{n_2} - 1)(2^{n_3} - 1)$$

- Unfortunately, this simple generator, while it has a long period, is still not secure due to correlation properties

Properties of Geffe Generator

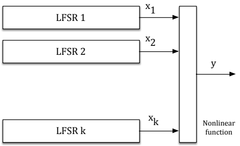
- The truth table of the Geffe generator output is given as

x	y	z	w	
0	0	0	0	$\leftarrow$
0	0	1	1	$\leftarrow$
1	0	0	0	$\leftarrow$
1	0	1	1	$\leftarrow$
0	1	0	0	$\leftarrow$
0	1	1	0	
1	1	0	1	
1	1	1	1	$\leftarrow$

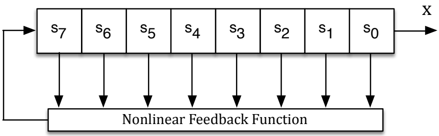
- The output the Geffe generator (w) matches the output of the LFSR3 (z) in 6 out of 8 times, correlating 75%
- If the known text attack gives us a set of w bits, we can apply the Berlekamp-Massey and exhaustive search to reconstruct LFSR3

General Nonlinear Generators

- The Geffe generator can be generalized to include other nonlinear functions that combine several LFSRs



- Or, more generally, we can use nonlinear feedback shift registers



- There are a myriad of other design options

RC4

- RC4 is a software-suitable stream cipher, designed by Ron Rivest at RSA Labs (research arm of RSA Inc.)
- RC stands for Ron's Cipher, since he designed several ciphers for the company RSA Inc. (where I also worked between 1990-1995)
- RC4 became very important because of its use in TLS suites, providing confidentiality for e-commerce communication
- RC4 was designed for 8-bit processors, and requires a small state table, but it has very large period
- The key (seed) size is flexible, and can be any multiple of 8 bits
- The mathematics of RC4 is difficult, making it a hard cipher to cryptanalyze

RC4 Key Table

- The RC4 Key Table is a linear array of k cells, where each cell is a byte

$T[0]$	$T[1]$	$T[2]$	$\cdots$	$T[k-2]$	$T[k-1]$

- The number of bits in an RC4 key is a multiple of 8, and the value of k is between 1 and 256
- In other words, the minimum size key is 8 bits, while the maximum key size 2048 bits!
- During the times of export restrictions (until late 90s), the most common RC4 key was 48 bits ($k = 6$)
- Many US-based implementations used 128-bit ($k = 16$) keys

RC4 State Table

- The RC4 State Table is a linear array of 256 cells, where each cell is a byte

$S[0]$	$S[1]$	$S[2]$	$\cdots$	$S[254]$	$S[255]$

- RC4 algorithm has 3 phases: Initialization, key scheduling, and pseudorandom byte generation
- First the State Table is initialized as

```
for i in range(256):
    S[i] = i
```

- Thus, it becomes

0	1	2	$\cdots$	254	255

which is a permutation of 256 integer values from 0 to 255

RC4 Key Scheduling

- The k-byte Key Table T is used to permute the elements of the State Table using the following algorithm

```
j = 0
for i in range(256):
    j = j + S[i] + T[i % k] % 256
    temp = S[i]
    S[i] = S[j]
    S[j] = temp
```

- As the index i progresses from 0 to 255, a new value of j is computed, and the State Table cells $S[i]$ and $S[j]$ are swapped
- At the end of the Key Scheduling process, the State Table is still a a permutation of 256 integer values from 0 to 255

RC4 Pseudorandom Byte Generation

- The Key Scheduling algorithm runs once, and the Key Table is not needed again
- The cipher starts producing the running key bytes, however, at each step, two cells in the the State Table is swapped, mixing up the State Table as it proceeds

```
i = 0
j = 0
while GenerateOutput == True
    i = i + 1 % 256
    j = j + S[i] % 256
    temp = S[i]
    S[i] = S[j]
    S[j] = temp
    R = S[ S[i] + S[j] % 256 ]
    print(R)
```

Properties of RC4

- The State Table runs through all possible permutations of 256 values, and there are 256! distinct state tables
- This is indeed a huge number, which is about 2^{1684}
- However, note that the same key will always produce the same output sequence; RC4 does not have a nonce variable alongside with the key
- If a nonce is to be used, there must be a way to incorporate with the single long-term key, and thus, an incorrect use of the nonce and key may weaken the key scheduling algorithm
- A protocol involving RC4 that does not discard the beginning part of the output stream or that uses nonrandom (or related) keys will be vulnerable to attack, such as WEP

Using Stream Cipher Modes of Block Ciphers

- An efficient way to generate a stream of deterministic random numbers is to use block ciphers, turning a block cipher box into a stream cipher
- There are 3 basic methods: OFB (output feedback), CFB (cipher feedback), and CTR (counter)
- In block cipher context, these methods are called "modes of operation"
- There are other modes of operation for block ciphers, each one of which is serving a different purpose

Setup

- For all three modes, we assume the following:
- A block cipher encryption function is available, which produces an m-bit ciphertext C from an m-bit plaintext M using a n-bit key K:

$$C = E_k(M) \text{ such that } |C| = |M| = m \text{ and } |K| = n$$

- Also assume an initial value is available, which is generally called *initializing variable* and written as IV

The Output Feedback Mode

- The OFB produces a key stream r_i of s bits at each step, for $s = 0, 1, 2, \ldots$, and computes the ciphertext $c_i = r_i \oplus m_i$
- We have $s \leq m$, and generally s is a small number, such as 1, 2, or 8
- The algorithm performs for $i = 0, 1, 2, \ldots$, starting with $S_0 = IV$

$$\begin{aligned} T_i &= E_K(S_i) \\ r_i &= TR_s(T_i) \\ c_i &= r_i \oplus m_i \\ S_{i+1} &= r_i || RS_s(S_i) \end{aligned}$$

- $TR_s(T_i)$ is the function that truncates the m-bit number T_i to s bits, either by taking the leftmost s bits
- Then, S_i is shifted s bits to right using $RS_s(S_i)$ function, and the s-bit r_i is left-appended to get the new m-bit S_{i+1}

The Cipher Feedback Mode

- The CFB produces a key stream r_i of s bits at each step, for $s = 0, 1, 2, \ldots$, and computes the ciphertext $c_i = r_i \oplus m_i$
- We have $s \leq m$, and generally s is a small number, such as 1, 2, or 8
- The algorithm performs for $i = 0, 1, 2, \ldots$, starting with $S_0 = IV$

$$
\begin{aligned}
T_i &= E_K(S_i) \\
r_i &= TR_s(T_i) \\
c_i &= r_i \oplus m_i \\
S_{i+1} &= c_i || RS_s(S_i)
\end{aligned}
$$

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The Counter Mode

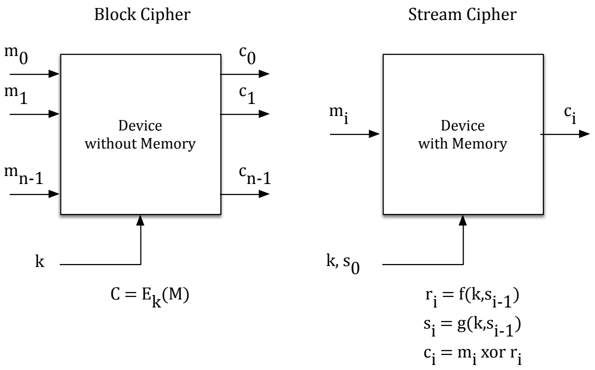
- In the counter mode the m-bit initial state S_0 consists of two parts: The u-bit count value I on the right and a $(m-u)$-bit nonce value N on the left: $S_0 = N||I$
- The initial value of $I = 1$, and u is selected appropriately
- The algorithm performs for $i = 0, 1, 2, \ldots$, starting with $S_0 = N||1$

$$
\begin{aligned}
T_i &= E_K(S_i) \\
r_i &= TR_s(T_i) \\
c_i &= r_i \oplus m_i \\
I &= I + 1 \\
S_{i+1} &= N||I
\end{aligned}
$$

- The new value S_{i+1} is obtained by incrementing the counter value I and keeping the nonce N as the same

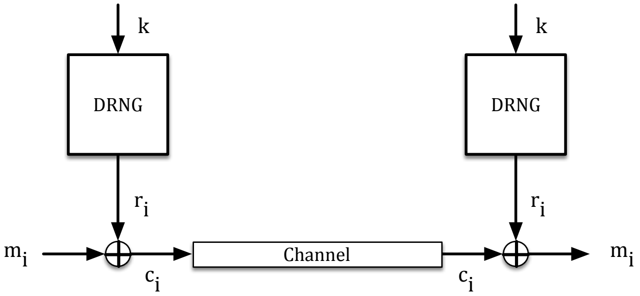
Block Ciphers vs Stream Ciphers

- The fundamental difference between block ciphers and stream ciphers is the memory



Synchronous Stream Ciphers

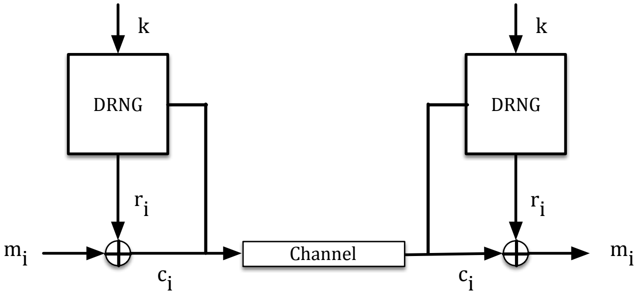
- The synchronous stream ciphers rely on the communication protocol to stay synchronous



- If the synchronization is lost, the cipher needs to be initialized, and to restart

Self-synchronizing Stream Ciphers

- The self-synchronizing stream ciphers keep the synchrony



- Whenever a correct set of r consecutive ciphertext bits are transmitted, the cipher function will produce the same r_i value

$$r_i = f(k, c_j, c_{j+1}, \ldots, c_{j+r-1})$$

When to Use Stream Ciphers?

- Block ciphers are better understood than stream ciphers: If there are no special requirements, a block cipher used in one of the stream cipher modes (OFB or CFB) is a good choice
- There are some platforms where block cipher modes may not be suitable, for example, embedded devices where we try to save chip area (code space) and energy in embedded devices
- A shift-register based stream cipher needs fewer gates by several magnitudes than even a simple CPU, and therefore, much more suitable small, mobile, low-energy embedded systems

When to Use Stream Ciphers?

- Also, stream ciphers can reach higher speeds than block ciphers; some stream ciphers can be produce 64 or 128 bits per clock cycle which is significantly higher than any block cipher
- Currently hard disk space is growing faster than CPU speed; this might imply that in future we will have a greater need for high-speed ciphers, making stream ciphers good choices
- Furthermore, stream ciphers may also be a lot more useful for RFID devices, which is an important application
- If low energy consumption is important, stream ciphers will win out over block ciphers