

## 7.4.2 DES algorithm

DES is a Feistel cipher which processes plaintext blocks of  $n = 64$  bits, producing 64-bit ciphertext blocks (Figure 7.8). The effective size of the secret key  $K$  is  $k = 56$  bits; more precisely, the input key  $K$  is specified as a 64-bit key, 8 bits of which (bits 8, 16, . . . , 64) may be used as parity bits. The  $2^{56}$  keys implement (at most)  $2^{56}$  of the  $2^{64}$  possible bijections on 64-bit blocks. A widely held belief is that the parity bits were introduced to reduce the effective key size from 64 to 56 bits, to intentionally reduce the cost of exhaustive key search by a factor of 256.

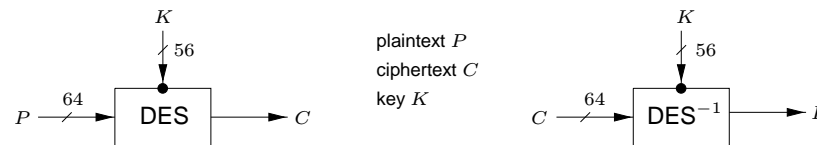


Figure 7.8: DES input-output.

Full details of DES are given in Algorithm 7.82 and Figures 7.9 and 7.10. An overview follows. Encryption proceeds in 16 stages or *rounds*. From the input key  $K$ , sixteen 48-bit subkeys  $K_i$  are generated, one for each round. Within each round, 8 fixed, carefully selected 6-to-4 bit substitution mappings (*S-boxes*)  $S_i$ , collectively denoted  $S$ , are used. The 64-bit plaintext is divided into 32-bit halves  $L_0$  and  $R_0$ . Each round is functionally equivalent, taking 32-bit inputs  $L_{i-1}$  and  $R_{i-1}$  from the previous round and producing 32-bit outputs  $L_i$  and  $R_i$  for  $1 \leq i \leq 16$ , as follows:

$$L_i = R_{i-1}; \quad (7.4)$$

$$R_i = L_{i-1} \oplus f(R_{i-1}, K_i), \text{ where } f(R_{i-1}, K_i) = P(S(E(R_{i-1}) \oplus K_i)) \quad (7.5)$$

Here  $E$  is a fixed expansion permutation mapping  $R_{i-1}$  from 32 to 48 bits (all bits are used once; some are used twice).  $P$  is another fixed permutation on 32 bits. An initial bit permutation (IP) precedes the first round; following the last round, the left and right halves are exchanged and, finally, the resulting string is bit-permuted by the inverse of IP. Decryption involves the same key and algorithm, but with subkeys applied to the internal rounds in the reverse order (Note 7.84).

A simplified view is that the right half of each round (after expanding the 32-bit input to 8 characters of 6 bits each) carries out a key-dependent substitution on each of 8 characters, then uses a fixed bit transposition to redistribute the bits of the resulting characters to produce 32 output bits.

Algorithm 7.83 specifies how to compute the DES round keys  $K_i$ , each of which contains 48 bits of  $K$ . These operations make use of tables PC1 and PC2 of Table 7.4, which are called *permuted choice 1* and *permuted choice 2*. To begin, 8 bits ( $k_8, k_{16}, \dots, k_{64}$ ) of  $K$  are discarded (by PC1). The remaining 56 bits are permuted and assigned to two 28-bit variables  $C$  and  $D$ ; and then for 16 iterations, both  $C$  and  $D$  are rotated either 1 or 2 bits, and 48 bits ( $K_i$ ) are selected from the concatenated result.

**7.82 Algorithm** Data Encryption Standard (DES)

INPUT: plaintext  $m_1 \dots m_{64}$ ; 64-bit key  $K = k_1 \dots k_{64}$  (includes 8 parity bits).

OUTPUT: 64-bit ciphertext block  $C = c_1 \dots c_{64}$ . (For decryption, see Note 7.84.)

1. (key schedule) Compute sixteen 48-bit round keys  $K_i$  from  $K$  using Algorithm 7.83.
2.  $(L_0, R_0) \leftarrow \text{IP}(m_1 m_2 \dots m_{64})$ . (Use IP from Table 7.2 to permute bits; split the result into left and right 32-bit halves  $L_0 = m_{58} m_{50} \dots m_8$ ,  $R_0 = m_{57} m_{49} \dots m_7$ .)
3. (16 rounds) for  $i$  from 1 to 16, compute  $L_i$  and  $R_i$  using Equations (7.4) and (7.5) above, computing  $f(R_{i-1}, K_i) = P(S(E(R_{i-1}) \oplus K_i))$  as follows:
  - (a) Expand  $R_{i-1} = r_1 r_2 \dots r_{32}$  from 32 to 48 bits using  $E$  per Table 7.3:  
 $T \leftarrow E(R_{i-1})$ . (Thus  $T = r_{32} r_1 r_2 \dots r_{32} r_1$ .)
  - (b)  $T' \leftarrow T \oplus K_i$ . Represent  $T'$  as eight 6-bit character strings:  $(B_1, \dots, B_8) = T'$ .
  - (c)  $T'' \leftarrow (S_1(B_1), S_2(B_2), \dots, S_8(B_8))$ . (Here  $S_i(B_i)$  maps  $B_i = b_1 b_2 \dots b_6$  to the 4-bit entry in row  $r$  and column  $c$  of  $S_i$  in Table 7.8, page 260 where  $r = 2 \cdot b_1 + b_6$ , and  $b_2 b_3 b_4 b_5$  is the radix-2 representation of  $0 \leq c \leq 15$ . Thus  $S_1(011011)$  yields  $r = 1$ ,  $c = 13$ , and output 5, i.e., binary 0101.)
  - (d)  $T''' \leftarrow P(T'')$ . (Use  $P$  per Table 7.3 to permute the 32 bits of  $T''' = t_1 t_2 \dots t_{32}$ , yielding  $t_{16} t_7 \dots t_{25}$ .)
4.  $b_1 b_2 \dots b_{64} \leftarrow (R_{16}, L_{16})$ . (Exchange final blocks  $L_{16}, R_{16}$ .)
5.  $C \leftarrow \text{IP}^{-1}(b_1 b_2 \dots b_{64})$ . (Transpose using  $\text{IP}^{-1}$  from Table 7.2;  $C = b_{40} b_8 \dots b_{25}$ .)

IP							
58	50	42	34	26	18	10	2
60	52	44	36	28	20	12	4
62	54	46	38	30	22	14	6
64	56	48	40	32	24	16	8
57	49	41	33	25	17	9	1
59	51	43	35	27	19	11	3
61	53	45	37	29	21	13	5
63	55	47	39	31	23	15	7

IP <sup>-1</sup>							
40	8	48	16	56	24	64	32
39	7	47	15	55	23	63	31
38	6	46	14	54	22	62	30
37	5	45	13	53	21	61	29
36	4	44	12	52	20	60	28
35	3	43	11	51	19	59	27
34	2	42	10	50	18	58	26
33	1	41	9	49	17	57	25

**Table 7.2:** DES initial permutation and inverse (IP and IP<sup>-1</sup>).

E					
32	1	2	3	4	5
4	5	6	7	8	9
8	9	10	11	12	13
12	13	14	15	16	17
16	17	18	19	20	21
20	21	22	23	24	25
24	25	26	27	28	29
28	29	30	31	32	1

P			
16	7	20	21
29	12	28	17
1	15	23	26
5	18	31	10
2	8	24	14
32	27	3	9
19	13	30	6
22	11	4	25

**Table 7.3:** DES per-round functions: expansion  $E$  and permutation  $P$ .

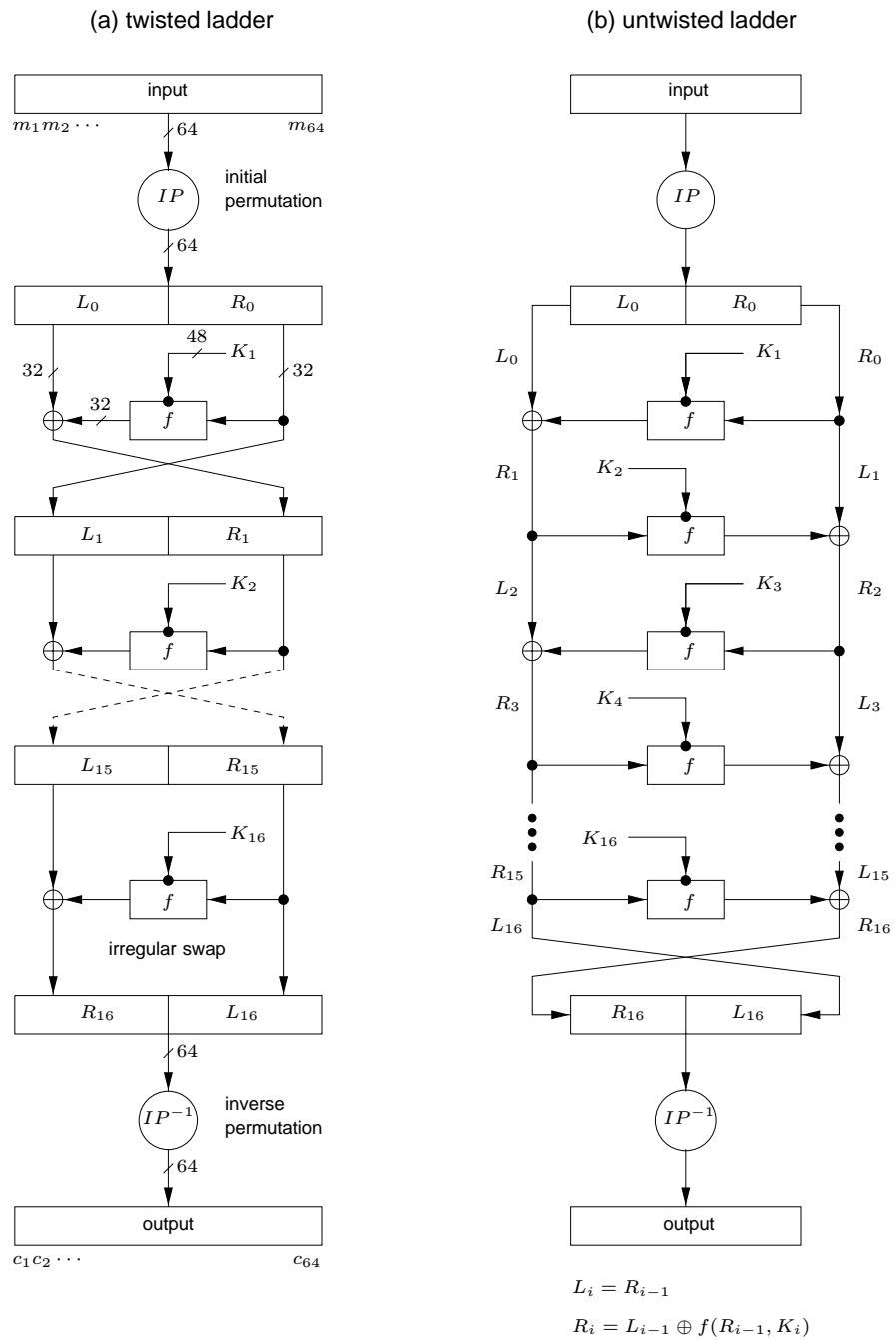
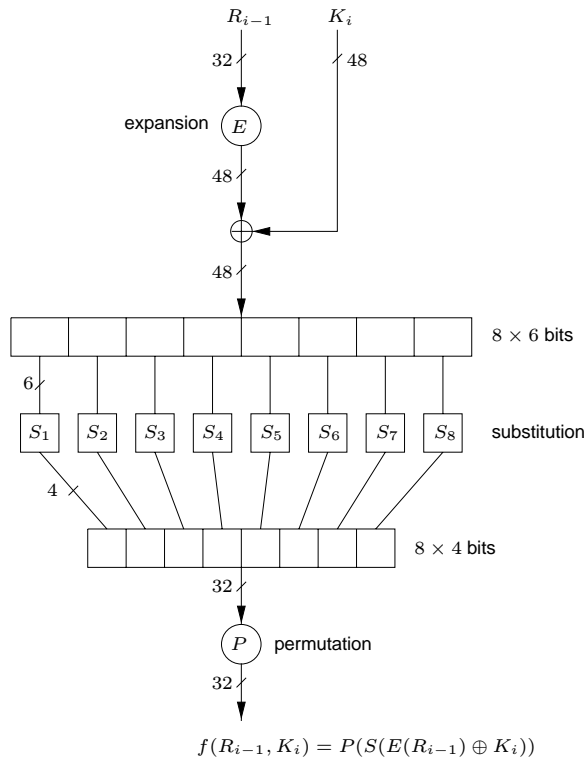


Figure 7.9: DES computation path.



**Figure 7.10:** DES inner function  $f$ .

**7.83 Algorithm** DES key schedule

INPUT: 64-bit key  $K = k_1 \dots k_{64}$  (including 8 odd-parity bits).

OUTPUT: sixteen 48-bit keys  $K_i, 1 \leq i \leq 16$ .

1. Define  $v_i, 1 \leq i \leq 16$  as follows:  $v_i = 1$  for  $i \in \{1, 2, 9, 16\}$ ;  $v_i = 2$  otherwise. (These are left-shift values for 28-bit circular rotations below.)
2.  $T \leftarrow \text{PC1}(K)$ ; represent  $T$  as 28-bit halves  $(C_0, D_0)$ . (Use PC1 in Table 7.4 to select bits from  $K$ :  $C_0 = k_{57}k_{49} \dots k_{36}, D_0 = k_{63}k_{55} \dots k_4$ .)
3. For  $i$  from 1 to 16, compute  $K_i$  as follows:  $C_i \leftarrow (C_{i-1} \leftarrow v_i), D_i \leftarrow (D_{i-1} \leftarrow v_i), K_i \leftarrow \text{PC2}(C_i, D_i)$ . (Use PC2 in Table 7.4 to select 48 bits from the concatenation  $b_1b_2 \dots b_{56}$  of  $C_i$  and  $D_i$ :  $K_i = b_{14}b_{17} \dots b_{32}$ . ' $\leftarrow$ ' denotes left circular shift.)

If decryption is designed as a simple variation of the encryption function, savings result in hardware or software code size. DES achieves this as outlined in Note 7.84.

**7.84 Note** (*DES decryption*) DES decryption consists of the encryption algorithm with the same key but reversed key schedule, using in order  $K_{16}, K_{15}, \dots, K_1$  (see Note 7.85). This works as follows (refer to Figure 7.9). The effect of  $\text{IP}^{-1}$  is cancelled by IP in decryption, leaving  $(R_{16}, L_{16})$ ; consider applying round 1 to this input. The operation on the left half yields, rather than  $L_0 \oplus f(R_0, K_1)$ , now  $R_{16} \oplus f(L_{16}, K_{16})$  which, since  $L_{16} = R_{15}$  and  $R_{16} = L_{15} \oplus f(R_{15}, K_{16})$ , is equal to  $L_{15} \oplus f(R_{15}, K_{16}) \oplus f(R_{15}, K_{16}) = L_{15}$ . Thus round 1 decryption yields  $(R_{15}, L_{15})$ , i.e., inverting round 16. Note that the cancellation

PC1						
57	49	41	33	25	17	9
1	58	50	42	34	26	18
10	2	59	51	43	35	27
19	11	3	60	52	44	36
above for $C_i$ ; below for $D_i$						
63	55	47	39	31	23	15
7	62	54	46	38	30	22
14	6	61	53	45	37	29
21	13	5	28	20	12	4

PC2					
14	17	11	24	1	5
3	28	15	6	21	10
23	19	12	4	26	8
16	7	27	20	13	2
41	52	31	37	47	55
30	40	51	45	33	48
44	49	39	56	34	53
46	42	50	36	29	32

**Table 7.4:** DES key schedule bit selections (PC1 and PC2).

of each round is independent of the definition of  $f$  and the specific value of  $K_i$ ; the swapping of halves combined with the XOR process is inverted by the second application. The remaining 15 rounds are likewise cancelled one by one in reverse order of application, due to the reversed key schedule.

**7.85 Note** (*DES decryption key schedule*) Subkeys  $K_1, \dots, K_{16}$  may be generated by Algorithm 7.83 and used in reverse order, or generated in reverse order directly as follows. Note that after  $K_{16}$  is generated, the original values of the 28-bit registers  $C$  and  $D$  are restored (each has rotated 28 bits). Consequently, and due to the choice of shift-values, modifying Algorithm 7.83 as follows generates subkeys in order  $K_{16}, \dots, K_1$ : replace the left-shifts by right-shift rotates; change the shift value  $v_1$  to 0.

**7.86 Example** (*DES test vectors*) The plaintext “Now is the time for all ”, represented as a string of 8-bit hex characters (7-bit ASCII characters plus leading 0-bit), and encrypted using the DES key specified by the hex string  $K = 0123456789ABCDEF$  results in the following plaintext/ciphertext:

$P = 4E6F772069732074 \ 68652074696D6520 \ 666F7220616C6C20$   
 $C = 3FA40E8A984D4815 \ 6A271787AB8883F9 \ 893D51EC4B563B53.$  □

### 7.4.3 DES properties and strength

There are many desirable characteristics for block ciphers. These include: each bit of the ciphertext should depend on all bits of the key and all bits of the plaintext; there should be no statistical relationship evident between plaintext and ciphertext; altering any single plaintext or key bit should alter each ciphertext bit with probability  $\frac{1}{2}$ ; and altering a ciphertext bit should result in an unpredictable change to the recovered plaintext block. Empirically, DES satisfies these basic objectives. Some known properties and anomalies of DES are given below.

#### (i) Complementation property

**7.87 Fact** Let  $E$  denote DES, and  $\bar{x}$  the bitwise complement of  $x$ . Then  $y = E_K(x)$  implies  $\bar{y} = E_{\bar{K}}(\bar{x})$ . That is, bitwise complementing both the key  $K$  and the plaintext  $x$  results in complemented DES ciphertext.

*Justification:* Compare the first round output (see Figure 7.10) to  $(L_0, R_0)$  for the uncomplemented case. The combined effect of the plaintext and key being complemented results

in the inputs to the XOR preceding the S-boxes (the expanded  $R_{i-1}$  and subkey  $K_i$ ) both being complemented; this double complementation cancels out in the XOR operation, resulting in S-box inputs, and thus an overall result  $f(R_0, K_1)$ , as before. This quantity is then XORed (Figure 7.9) to  $\bar{L}_0$  (previously  $L_0$ ), resulting in  $\bar{L}_1$  (rather than  $L_1$ ). The same effect follows in the remaining rounds.

The complementation property is normally of no help to a cryptanalyst in known-plaintext exhaustive key search. If an adversary has, for a fixed unknown key  $K$ , a chosen-plaintext set of  $(x, y)$  data  $(P_1, C_1), (P_1, C_2)$ , then  $C_2 = E_K(\bar{P}_1)$  implies  $\bar{C}_2 = E_{\bar{K}}(P_1)$ . Checking if the key  $K$  with plaintext  $P_1$  yields either  $C_1$  or  $\bar{C}_2$  now rules out two keys with one encryption operation, thus reducing the expected number of keys required before success from  $2^{55}$  to  $2^{54}$ . This is not a practical concern.

### (ii) Weak keys, semi-weak keys, and fixed points

If subkeys  $K_1$  to  $K_{16}$  are equal, then the reversed and original schedules create identical subkeys:  $K_1 = K_{16}, K_2 = K_{15}$ , and so on. Consequently, the encryption and decryption functions coincide. These are called weak keys (and also: *palindromic keys*).

**7.88 Definition** A DES *weak key* is a key  $K$  such that  $E_K(E_K(x)) = x$  for all  $x$ , i.e., defining an involution. A pair of DES *semi-weak keys* is a pair  $(K_1, K_2)$  with  $E_{K_1}(E_{K_2}(x)) = x$ .

Encryption with one key of a semi-weak pair operates as does decryption with the other.

**7.89 Fact** DES has four weak keys and six pairs of semi-weak keys.

The four DES weak keys are listed in Table 7.5, along with corresponding 28-bit variables  $C_0$  and  $D_0$  of Algorithm 7.83; here  $\{0\}^j$  represents  $j$  repetitions of bit 0. Since  $C_0$  and  $D_0$  are all-zero or all-one bit vectors, and rotation of these has no effect, it follows that all subkeys  $K_i$  are equal and an involution results as noted above.

The six pairs of DES semi-weak keys are listed in Table 7.6. Note their defining property (Definition 7.88) occurs when subkeys  $K_1$  through  $K_{16}$  of the first key, respectively, equal subkeys  $K_{16}$  through  $K_1$  of the second. This requires that a 1-bit circular left-shift of each of  $C_0$  and  $D_0$  for the first 56-bit key results in the  $(C_0, D_0)$  pair for the second 56-bit key (see Note 7.84), and thereafter left-rotating  $C_i$  and  $D_i$  one or two bits for the first results in the same value as right-rotating those for the second the same number of positions. The values in Table 7.6 satisfy these conditions. Given any one 64-bit semi-weak key, its paired semi-weak key may be obtained by splitting it into two halves and rotating each half through 8 bits.

**7.90 Fact** Let  $E$  denote DES. For each of the four DES weak keys  $K$ , there exist  $2^{32}$  *fixed points* of  $E_K$ , i.e., plaintexts  $x$  such that  $E_K(x) = x$ . Similarly, four of the twelve semi-weak keys  $K$  each have  $2^{32}$  *anti-fixed points*, i.e.,  $x$  such that  $E_K(x) = \bar{x}$ .

The four semi-weak keys of Fact 7.90 are in the upper portion of Table 7.6. These are called *anti-palindromic keys*, since for these  $K_1 = \bar{K}_{16}, K_2 = \bar{K}_{15}$ , and so on.

### (iii) DES is not a group

For a fixed DES key  $K$ , DES defines a permutation from  $\{0, 1\}^{64}$  to  $\{0, 1\}^{64}$ . The set of DES keys defines  $2^{56}$  such (potentially different) permutations. If this set of permutations was closed under composition (i.e., given any two keys  $K_1, K_2$ , there exists a third key  $K_3$  such that  $E_{K_3}(x) = E_{K_2}(E_{K_1}(x))$  for all  $x$ ) then multiple encryption would be equivalent to single encryption. Fact 7.91 states that this is not the case for DES.

weak key (hexadecimal)	$C_0$	$D_0$
0101 0101 0101 0101	$\{0\}^{28}$	$\{0\}^{28}$
FEFE FEFE FEFE FEFE	$\{1\}^{28}$	$\{1\}^{28}$
1F1F 1F1F 0E0E 0E0E	$\{0\}^{28}$	$\{1\}^{28}$
E0E0 E0E0 F1F1 F1F1	$\{1\}^{28}$	$\{0\}^{28}$

**Table 7.5:** Four DES weak keys.

$C_0$	$D_0$	semi-weak key pair (hexadecimal)	$C_0$	$D_0$
$\{01\}^{14}$	$\{01\}^{14}$	01FE 01FE 01FE 01FE, FE01 FE01 FE01 FE01	$\{10\}^{14}$	$\{10\}^{14}$
$\{01\}^{14}$	$\{10\}^{14}$	1FEO 1FEO 0EF1 0EF1, E01F E01F F10E F10E	$\{10\}^{14}$	$\{01\}^{14}$
$\{01\}^{14}$	$\{0\}^{28}$	01EO 01EO 01F1 01F1, E001 E001 F101 F101	$\{10\}^{14}$	$\{0\}^{28}$
$\{01\}^{14}$	$\{1\}^{28}$	1FFE 1FFE 0EFE 0EFE, FE1F FE1F FE0E FE0E	$\{10\}^{14}$	$\{1\}^{28}$
$\{0\}^{28}$	$\{01\}^{14}$	011F 011F 010E 010E, 1F01 1F01 0E01 0E01	$\{0\}^{28}$	$\{10\}^{14}$
$\{1\}^{28}$	$\{01\}^{14}$	E0FE E0FE F1FE F1FE, FEE0 FEE0 FEF1 FEF1	$\{1\}^{28}$	$\{10\}^{14}$

**Table 7.6:** Six pairs of DES semi-weak keys (one pair per line).

**7.91 Fact** The set of  $2^{56}$  permutations defined by the  $2^{56}$  DES keys is not closed under functional composition. Moreover, a lower bound on the size of the group generated by composing this set of permutations is  $10^{2499}$ .

The lower bound in Fact 7.91 is important with respect to using DES for multiple encryption. If the group generated by functional composition was too small, then multiple encryption would be less secure than otherwise believed.

#### (iv) Linear and differential cryptanalysis of DES

Assuming that obtaining enormous numbers of known-plaintext pairs is feasible, linear cryptanalysis provides the most powerful attack on DES to date; it is not, however, considered a threat to DES in practical environments. Linear cryptanalysis is also possible in a ciphertext-only environment if some underlying plaintext redundancy is known (e.g., parity bits or high-order 0-bits in ASCII characters).

Differential cryptanalysis is one of the most general cryptanalytic tools to date against modern iterated block ciphers, including DES, Lucifer, and FEAL among many others. It is, however, primarily a chosen-plaintext attack. Further information on linear and differential cryptanalysis is given in §7.8.

**7.92 Note** (*strength of DES*) The complexity (see §7.2.1) of the best attacks currently known against DES is given in Table 7.7; percentages indicate success rate for specified attack parameters. The ‘processing complexity’ column provides only an estimate of the expected cost (operation costs differ across the various attacks); for exhaustive search, the cost is in DES operations. Regarding storage complexity, both linear and differential cryptanalysis require only negligible storage in the sense that known or chosen texts can be processed individually and discarded, but in a practical attack, storage for accumulated texts would be required if ciphertext was acquired prior to commencing the attack.

attack method	data complexity		storage complexity	processing complexity
	known	chosen		
exhaustive precomputation	—	1	$2^{56}$	1 (table lookup)
exhaustive search	1	—	negligible	$2^{55}$
linear cryptanalysis	$2^{43}$ (85%)	—	for texts	$2^{43}$
	$2^{38}$ (10%)	—	for texts	$2^{50}$
differential cryptanalysis	—	$2^{47}$	for texts	$2^{47}$
	$2^{55}$	—	for texts	$2^{55}$

**Table 7.7:** DES strength against various attacks.

**7.93 Remark** (*practicality of attack models*) To be meaningful, attack comparisons based on different models (e.g., Table 7.7) must appropriately weigh the feasibility of extracting (acquiring) enormous amounts of chosen (known) plaintexts, which is considerably more difficult to arrange than a comparable number of computing cycles on an adversary's own machine. Exhaustive search with one known plaintext-ciphertext pair (for ciphertext-only, see Example 7.28) and  $2^{55}$  DES operations is significantly more feasible in practice (e.g., using highly parallelized custom hardware) than linear cryptanalysis (LC) requiring  $2^{43}$  known pairs.

While exhaustive search, linear, and differential cryptanalysis allow recovery of a DES key and, therefore, the entire plaintext, the attacks of Note 7.8, which become feasible once about  $2^{32}$  ciphertexts are available, may be more efficient if the goal is to recover only part of the text.

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## 7.5 FEAL

The Fast Data Encipherment Algorithm (FEAL) is a family of algorithms which has played a critical role in the development and refinement of various advanced cryptanalytic techniques, including linear and differential cryptanalysis. FEAL- $N$  maps 64-bit plaintext to 64-bit ciphertext blocks under a 64-bit secret key. It is an  $N$ -round Feistel cipher similar to DES (cf. Equations (7.4), (7.5)), but with a far simpler  $f$ -function, and augmented by initial and final stages which XOR the two data halves as well as XOR subkeys directly onto the data halves.

FEAL was designed for speed and simplicity, especially for software on 8-bit microprocessors (e.g., chipcards). It uses byte-oriented operations (8-bit addition mod 256, 2-bit left rotation, and XOR), avoids bit-permutations and table look-ups, and offers small code size. The initial commercially proposed version with 4 rounds (FEAL-4), positioned as a fast alternative to DES, was found to be considerably less secure than expected (see Table 7.10). FEAL-8 was similarly found to offer less security than planned. FEAL-16 or FEAL-32 may yet offer security comparable to DES, but throughput decreases as the number of rounds rises. Moreover, whereas the speed of DES implementations can be improved through very large lookup tables, this appears more difficult for FEAL.

Algorithm 7.94 specifies FEAL-8. The  $f$ -function  $f(A, Y)$  maps an input pair of  $32 \times 16$  bits to a 32-bit output. Within the  $f$  function, two byte-oriented data substitutions (S-boxes)  $S_0$  and  $S_1$  are each used twice; each maps a pair of 8-bit inputs to an 8-bit output

*Handbook of Applied Cryptography* by A. Menezes, P. van Oorschot and S. Vanstone.



row	column number															
	[0]	[1]	[2]	[3]	[4]	[5]	[6]	[7]	[8]	[9]	[10]	[11]	[12]	[13]	[14]	[15]
$S_1$																
[0]	14	4	13	1	2	15	11	8	3	10	6	12	5	9	0	7
[1]	0	15	7	4	14	2	13	1	10	6	12	11	9	5	3	8
[2]	4	1	14	8	13	6	2	11	15	12	9	7	3	10	5	0
[3]	15	12	8	2	4	9	1	7	5	11	3	14	10	0	6	13
$S_2$																
[0]	15	1	8	14	6	11	3	4	9	7	2	13	12	0	5	10
[1]	3	13	4	7	15	2	8	14	12	0	1	10	6	9	11	5
[2]	0	14	7	11	10	4	13	1	5	8	12	6	9	3	2	15
[3]	13	8	10	1	3	15	4	2	11	6	7	12	0	5	14	9
$S_3$																
[0]	10	0	9	14	6	3	15	5	1	13	12	7	11	4	2	8
[1]	13	7	0	9	3	4	6	10	2	8	5	14	12	11	15	1
[2]	13	6	4	9	8	15	3	0	11	1	2	12	5	10	14	7
[3]	1	10	13	0	6	9	8	7	4	15	14	3	11	5	2	12
$S_4$																
[0]	7	13	14	3	0	6	9	10	1	2	8	5	11	12	4	15
[1]	13	8	11	5	6	15	0	3	4	7	2	12	1	10	14	9
[2]	10	6	9	0	12	11	7	13	15	1	3	14	5	2	8	4
[3]	3	15	0	6	10	1	13	8	9	4	5	11	12	7	2	14
$S_5$																
[0]	2	12	4	1	7	10	11	6	8	5	3	15	13	0	14	9
[1]	14	11	2	12	4	7	13	1	5	0	15	10	3	9	8	6
[2]	4	2	1	11	10	13	7	8	15	9	12	5	6	3	0	14
[3]	11	8	12	7	1	14	2	13	6	15	0	9	10	4	5	3
$S_6$																
[0]	12	1	10	15	9	2	6	8	0	13	3	4	14	7	5	11
[1]	10	15	4	2	7	12	9	5	6	1	13	14	0	11	3	8
[2]	9	14	15	5	2	8	12	3	7	0	4	10	1	13	11	6
[3]	4	3	2	12	9	5	15	10	11	14	1	7	6	0	8	13
$S_7$																
[0]	4	11	2	14	15	0	8	13	3	12	9	7	5	10	6	1
[1]	13	0	11	7	4	9	1	10	14	3	5	12	2	15	8	6
[2]	1	4	11	13	12	3	7	14	10	15	6	8	0	5	9	2
[3]	6	11	13	8	1	4	10	7	9	5	0	15	14	2	3	12
$S_8$																
[0]	13	2	8	4	6	15	11	1	10	9	3	14	5	0	12	7
[1]	1	15	13	8	10	3	7	4	12	5	6	11	0	14	9	2
[2]	7	11	4	1	9	12	14	2	0	6	10	13	15	3	5	8
[3]	2	1	14	7	4	10	8	13	15	12	9	0	3	5	6	11

Table 7.8: DES S-boxes.

(see Table 7.9).  $S_0$  and  $S_1$  add a single bit  $d \in \{0, 1\}$  to 8-bit arguments  $x$  and  $y$ , ignore the carry out of the top bit, and left rotate the result 2 bits (ROT2):

$$S_d(x, y) = ROT2(x + y + d \bmod 256) \tag{7.6}$$

The key schedule uses a function  $f_K(A, B)$  similar to the  $f$ -function (see Table 7.9;  $A_i, B_i, Y_i, t_i$ , and  $U_i$  are 8-bit variables), mapping two 32-bit inputs to a 32-bit output.

	$U \leftarrow f(A, Y)$	$U \leftarrow f_K(A, B)$
$t_1 =$	$(A_0 \oplus A_1) \oplus Y_0$	$A_0 \oplus A_1$
$t_2 =$	$(A_2 \oplus A_3) \oplus Y_1$	$A_2 \oplus A_3$
$U_1 =$	$S_1(t_1, t_2)$	$S_1(t_1, t_2 \oplus B_0)$
$U_2 =$	$S_0(t_2, U_1)$	$S_0(t_2, U_1 \oplus B_1)$
$U_0 =$	$S_0(A_0, U_1)$	$S_0(A_0, U_1 \oplus B_2)$
$U_3 =$	$S_1(A_3, U_2)$	$S_1(A_3, U_2 \oplus B_3)$

**Table 7.9:** Output  $U = (U_0, U_1, U_2, U_3)$  for FEAL functions  $f, f_K$  (Algorithm 7.94).

As the operations of 2-bit rotation and XOR are both linear, the only nonlinear elementary operation in FEAL is addition mod 256.

**7.94 Algorithm** Fast Data Encipherment Algorithm (FEAL-8)

INPUT: 64-bit plaintext  $M = m_1 \dots m_{64}$ ; 64-bit key  $K = k_1 \dots k_{64}$ .

OUTPUT: 64-bit ciphertext block  $C = c_1 \dots c_{64}$ . (For decryption, see Note 7.96.)

1. (key schedule) Compute sixteen 16-bit subkeys  $K_i$  from  $K$  using Algorithm 7.95.
2. Define  $M_L = m_1 \dots m_{32}, M_R = m_{33} \dots m_{64}$ .
3.  $(L_0, R_0) \leftarrow (M_L, M_R) \oplus ((K_8, K_9), (K_{10}, K_{11}))$ . (XOR initial subkeys.)
4.  $R_0 \leftarrow R_0 \oplus L_0$ .
5. For  $i$  from 1 to 8 do:  $L_i \leftarrow R_{i-1}, R_i \leftarrow L_{i-1} \oplus f(R_{i-1}, K_{i-1})$ . (Use Table 7.9 for  $f(A, Y)$  with  $A = R_{i-1} = (A_0, A_1, A_2, A_3)$  and  $Y = K_{i-1} = (Y_0, Y_1)$ .)
6.  $L_8 \leftarrow L_8 \oplus R_8$ .
7.  $(R_8, L_8) \leftarrow (R_8, L_8) \oplus ((K_{12}, K_{13}), (K_{14}, K_{15}))$ . (XOR final subkeys.)
8.  $C \leftarrow (R_8, L_8)$ . (Note the order of the final blocks is exchanged.)

**7.95 Algorithm** FEAL-8 key schedule

INPUT: 64-bit key  $K = k_1 \dots k_{64}$ .

OUTPUT: 256-bit extended key (16-bit subkeys  $K_i, 0 \leq i \leq 15$ ).

1. (initialize)  $U^{(-2)} \leftarrow 0, U^{(-1)} \leftarrow k_1 \dots k_{32}, U^{(0)} \leftarrow k_{33} \dots k_{64}$ .
2.  $U \stackrel{\text{def}}{=} (U_0, U_1, U_2, U_3)$  for 8-bit  $U_i$ . Compute  $K_0, \dots, K_{15}$  as  $i$  runs from 1 to 8:
  - (a)  $U \leftarrow f_K(U^{(i-2)}, U^{(i-1)} \oplus U^{(i-3)})$ . ( $f_K$  is defined in Table 7.9, where  $A$  and  $B$  denote 4-byte vectors  $(A_0, A_1, A_2, A_3), (B_0, B_1, B_2, B_3)$ .)
  - (b)  $K_{2i-2} = (U_0, U_1), K_{2i-1} = (U_2, U_3), U^{(i)} \leftarrow U$ .

**7.96 Note** (FEAL decryption) Decryption may be achieved using Algorithm 7.94 with the same key  $K$  and ciphertext  $C = (R_8, L_8)$  as the plaintext input  $M$ , but with the key schedule reversed. More specifically, subkeys  $((K_{12}, K_{13}), (K_{14}, K_{15}))$  are used for the initial XOR (step 3),  $((K_8, K_9), (K_{10}, K_{11}))$  for the final XOR (step 7), and the round keys are used from  $K_7$  back to  $K_0$  (step 5). This is directly analogous to decryption for DES (Note 7.84).

**7.97 Note** (*FEAL-N*) FEAL with 64-bit key can be generalized to  $N$ -rounds,  $N$  even.  $N = 2^x$  is recommended;  $x = 3$  yields FEAL-8 (Algorithm 7.94). FEAL- $N$  uses  $N + 8$  sixteen-bit subkeys:  $K_0, \dots, K_{N-1}$ , respectively, in round  $i$ ;  $K_N, \dots, K_{N+3}$  for the initial XOR; and  $K_{N+4}, \dots, K_{N+7}$  for the final XOR. The key schedule of Algorithm 7.95 is directly generalized to compute keys  $K_0$  through  $K_{N+7}$  as  $i$  runs from 1 to  $(N/2) + 4$ .

**7.98 Note** (*FEAL-NX*) Extending FEAL- $N$  to use a 128-bit key results in FEAL- $NX$ , with altered key schedule as follows. The key is split into 64-bit halves  $(K_L, K_R)$ .  $K_R$  is partitioned into 32-bit halves  $(K_{R1}, K_{R2})$ . For  $1 \leq i \leq (N/2) + 4$ , define  $Q_i = K_{R1} \oplus K_{R2}$  for  $i \equiv 1 \pmod{3}$ ;  $Q_i = K_{R1}$  for  $i \equiv 2 \pmod{3}$ ; and  $Q_i = K_{R2}$  for  $i \equiv 0 \pmod{3}$ . The second argument  $(U^{(i-1)} \oplus U^{(i-3)})$  to  $f_K$  in step 2a of Algorithm 7.95 is replaced by  $U^{(i-1)} \oplus U^{(i-3)} \oplus Q_i$ . For  $K_R = 0$ , FEAL- $NX$  matches FEAL- $N$  with  $K_L$  as the 64-bit FEAL- $N$  key  $K$ .

**7.99 Example** (*FEAL test vectors*) For hex plaintext  $M = 00000000\ 00000000$  and hex key  $K = 01234567\ 89ABCDEF$ , Algorithm 7.95 generates subkeys  $(K_0, \dots, K_7) = DF3BCA36\ F17C1AEC\ 45A5B9C7\ 26EBAD25$ ,  $(K_8, \dots, K_{15}) = 8B2AECB7\ AC509D4C\ 22CD479B\ A8D50CB5$ . Algorithm 7.94 generates FEAL-8 ciphertext  $C = CEEF2C86\ F2490752$ . For FEAL-16, the corresponding ciphertext is  $C' = 3ADE0D2A\ D84D0B6F$ ; for FEAL-32,  $C'' = 69B0FAE6\ DDED6B0B$ . For 128-bit key  $(K_L, K_R)$  with  $K_L = K_R = K$  as above,  $M$  has corresponding FEAL-8X ciphertext  $C''' = 92BEB65D\ 0E9382FB$ .  $\square$

**7.100 Note** (*strength of FEAL*) Table 7.10 gives various published attacks on FEAL; LC and DC denote linear and differential cryptanalysis, and times are on common personal computers or workstations.

attack method	data complexity		storage complexity	processing complexity
	known	chosen		
FEAL-4 – LC	5	—	30K bytes	6 minutes
FEAL-6 – LC	100	—	100K bytes	40 minutes
FEAL-8 – LC	$2^{24}$	—	—	10 minutes
FEAL-8 – DC	—	$2^7$ pairs	280K bytes	2 minutes
FEAL-16 – DC	—	$2^{29}$ pairs	—	$2^{30}$ operations
FEAL-24 – DC	—	$2^{45}$ pairs	—	$2^{46}$ operations
FEAL-32 – DC	—	$2^{66}$ pairs	—	$2^{67}$ operations

**Table 7.10:** FEAL strength against various attacks.

## 7.6 IDEA

The cipher named IDEA (International Data Encryption Algorithm) encrypts 64-bit plaintext to 64-bit ciphertext blocks, using a 128-bit input key  $K$ . Based in part on a novel generalization of the Feistel structure, it consists of 8 computationally identical rounds followed by an output transformation (see Figure 7.11). Round  $r$  uses six 16-bit subkeys  $K_i^{(r)}$ ,  $1 \leq i \leq 6$ , to transform a 64-bit input  $X$  into an output of four 16-bit blocks, which are input to the next round. The round 8 output enters the output transformation, employing four additional subkeys  $K_i^{(9)}$ ,  $1 \leq i \leq 4$  to produce the final ciphertext  $Y = (Y_1, Y_2, Y_3, Y_4)$ . All subkeys are derived from  $K$ .

A dominant design concept in IDEA is mixing operations from three different algebraic groups of  $2^n$  elements. The corresponding group operations on sub-blocks  $a$  and  $b$  of bitlength  $n = 16$  are bitwise XOR:  $a \oplus b$ ; addition mod  $2^n$ :  $(a + b)$  AND  $0\text{xFFFF}$ , denoted  $a \boxplus b$ ; and (modified) multiplication mod  $2^n + 1$ , with  $0 \in \mathbb{Z}_{2^n}$  associated with  $2^n \in \mathbb{Z}_{2^n + 1}$ :  $a \odot b$  (see Note 7.104).

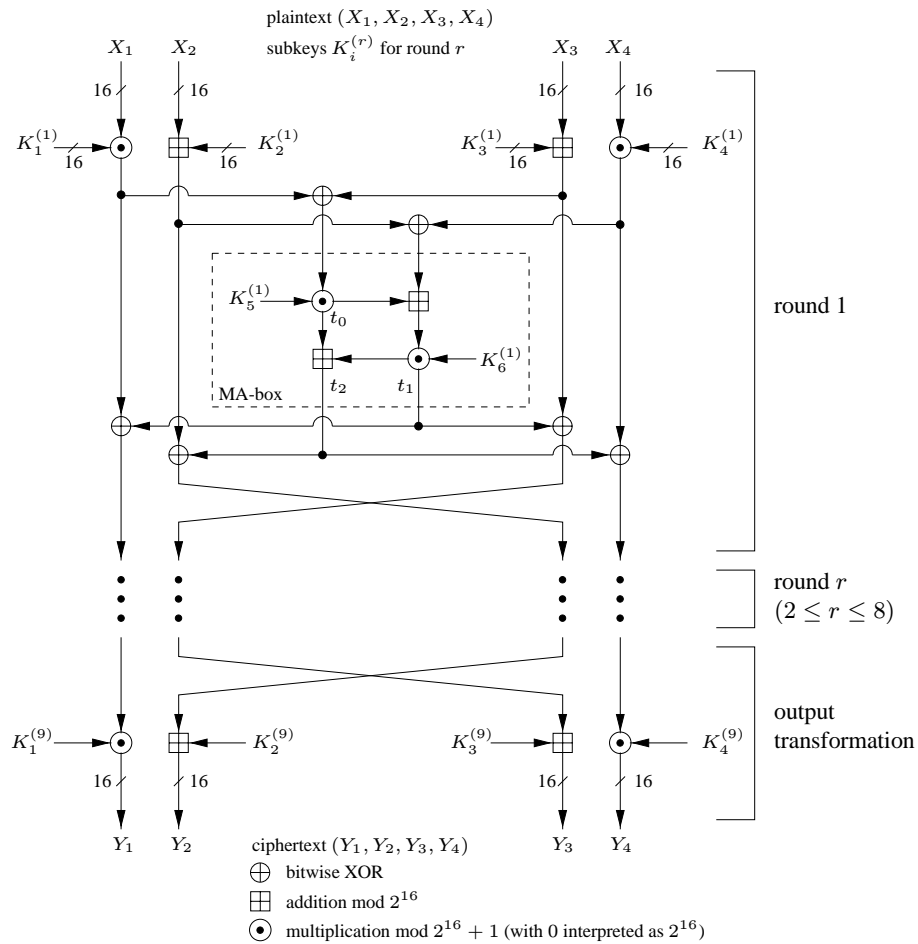


Figure 7.11: IDEA computation path.

**7.101 Algorithm** IDEA encryption

INPUT: 64-bit plaintext  $M = m_1 \dots m_{64}$ ; 128-bit key  $K = k_1 \dots k_{128}$ .

OUTPUT: 64-bit ciphertext block  $Y = (Y_1, Y_2, Y_3, Y_4)$ . (For decryption, see Note 7.103.)

1. (key schedule) Compute 16-bit subkeys  $K_1^{(r)}, \dots, K_6^{(r)}$  for rounds  $1 \leq r \leq 8$ , and  $K_1^{(9)}, \dots, K_4^{(9)}$  for the output transformation, using Algorithm 7.102.
2.  $(X_1, X_2, X_3, X_4) \leftarrow (m_1 \dots m_{16}, m_{17} \dots m_{32}, m_{33} \dots m_{48}, m_{49} \dots m_{64})$ , where  $X_i$  is a 16-bit data store.
3. For round  $r$  from 1 to 8 do:
  - (a)  $X_1 \leftarrow X_1 \odot K_1^{(r)}, X_4 \leftarrow X_4 \odot K_4^{(r)}, X_2 \leftarrow X_2 \boxplus K_2^{(r)}, X_3 \leftarrow X_3 \boxplus K_3^{(r)}$ .
  - (b)  $t_0 \leftarrow K_5^{(r)} \odot (X_1 \boxplus X_3), t_1 \leftarrow K_6^{(r)} \odot (t_0 \boxplus (X_2 \boxplus X_4)), t_2 \leftarrow t_0 \boxplus t_1$ .
  - (c)  $X_1 \leftarrow X_1 \boxplus t_1, X_4 \leftarrow X_4 \boxplus t_2, a \leftarrow X_2 \boxplus t_2, X_2 \leftarrow X_3 \boxplus t_1, X_3 \leftarrow a$ .
4. (output transformation)  $Y_1 \leftarrow X_1 \odot K_1^{(9)}, Y_4 \leftarrow X_4 \odot K_4^{(9)}, Y_2 \leftarrow X_3 \boxplus K_2^{(9)}, Y_3 \leftarrow X_2 \boxplus K_3^{(9)}$ .

**7.102 Algorithm** IDEA key schedule (encryption)

INPUT: 128-bit key  $K = k_1 \dots k_{128}$ .

OUTPUT: 52 16-bit key sub-blocks  $K_i^{(r)}$  for 8 rounds  $r$  and the output transformation.

1. Order the subkeys  $K_1^{(1)} \dots K_6^{(1)}, K_1^{(2)} \dots K_6^{(2)}, \dots, K_1^{(8)} \dots K_6^{(8)}, K_1^{(9)} \dots K_4^{(9)}$ .
2. Partition  $K$  into eight 16-bit blocks; assign these directly to the first 8 subkeys.
3. Do the following until all 52 subkeys are assigned: cyclic shift  $K$  left 25 bits; partition the result into 8 blocks; assign these blocks to the next 8 subkeys.

The key schedule of Algorithm 7.102 may be converted into a table which lists, for each of the 52 keys blocks, which 16 (consecutive) bits of the input key  $K$  form it.

**7.103 Note** (*IDEA decryption*) Decryption is achieved using Algorithm 7.101 with the ciphertext  $Y$  provided as input  $M$ , and the same encryption key  $K$ , but the following change to the key schedule. First use  $K$  to derive all encryption subkeys  $K_i^{(r)}$ ; from these compute the decryption subkeys  $K_i'^{(r)}$  per Table 7.11; then use  $K_i'^{(r)}$  in place of  $K_i^{(r)}$  in Algorithm 7.101. In Table 7.11,  $-K_i$  denotes the additive inverse (mod  $2^{16}$ ) of  $K_i$ ; the integer  $u = (2^{16} - K_i) \text{ AND } 0\text{x}\text{FFFF}$ ,  $0 \leq u \leq 2^{16} - 1$ .  $K_i^{-1}$  denotes the multiplicative inverse (mod  $2^{16} + 1$ ) of  $K_i$ , also in  $\{0, 1, \dots, 2^{16} - 1\}$ , derivable by the Extended Euclidean algorithm (Algorithm 2.107), which on inputs  $a \geq b \geq 0$  returns integers  $x$  and  $y$  such that  $ax + by = \text{gcd}(a, b)$ . Using  $a = 2^{16} + 1$  and  $b = K_i$ , the gcd is always 1 (except for  $K_i = 0$ , addressed separately) and thus  $K_i^{-1} = y$ , or  $2^{16} + 1 + y$  if  $y < 0$ . When  $K_i = 0$ , this input is mapped to  $2^{16}$  (since the inverse is defined by  $K_i \odot K_i^{-1} = 1$ ; see Note 7.104) and  $(2^{16})^{-1} = 2^{16}$  is then defined to give  $K_i^{-1} = 0$ .

**7.104 Note** (*definition of  $\odot$* ) In IDEA,  $a \odot b$  corresponds to a (modified) multiplication, modulo  $2^{16} + 1$ , of unsigned 16-bit integers  $a$  and  $b$ , where  $0 \in \mathbb{Z}_{2^{16}}$  is associated with  $2^{16} \in \mathbb{Z}_{2^{16}+1}^*$  as follows:<sup>2</sup> if  $a = 0$  or  $b = 0$ , replace it by  $2^{16}$  (which is  $\equiv -1 \pmod{2^{16} + 1}$ ) prior to modular multiplication; and if the result is  $2^{16}$ , replace this by 0. Thus,  $\odot$  maps two 16-bit inputs to a 16-bit output. Pseudo-code for  $\odot$  is as follows (cf. Note 7.105, for ordinary

<sup>2</sup>Thus the operands of  $\odot$  are from a set of cardinality  $2^{16}$  ( $\mathbb{Z}_{2^{16}+1}^*$ ) as are those of  $\oplus$  and  $\boxplus$ .

round $r$	$K_1^{(r)}$	$K_2^{(r)}$	$K_3^{(r)}$	$K_4^{(r)}$	$K_5^{(r)}$	$K_6^{(r)}$
$r = 1$	$(K_1^{(10-r)})^{-1}$	$-K_2^{(10-r)}$	$-K_3^{(10-r)}$	$(K_4^{(10-r)})^{-1}$	$K_5^{(9-r)}$	$K_6^{(9-r)}$
$2 \leq r \leq 8$	$(K_1^{(10-r)})^{-1}$	$-K_3^{(10-r)}$	$-K_2^{(10-r)}$	$(K_4^{(10-r)})^{-1}$	$K_5^{(9-r)}$	$K_6^{(9-r)}$
$r = 9$	$(K_1^{(10-r)})^{-1}$	$-K_2^{(10-r)}$	$-K_3^{(10-r)}$	$(K_4^{(10-r)})^{-1}$	—	—

**Table 7.11:** IDEA decryption subkeys  $K_i^{(r)}$  derived from encryption subkeys  $K_i^{(r)}$ .

multiplication mod  $2^{16} + 1$ , for  $c$  a 32-bit unsigned integer: if  $(a = 0) r \leftarrow (0 \times 10001 - b)$  (since  $2^{16}b \equiv -b$ ), elseif  $(b = 0) r \leftarrow (0 \times 10001 - a)$  (by similar reasoning), else  $\{c \leftarrow ab; r \leftarrow ((c \text{ AND } 0 \times \text{FFFF}) - (c >> 16))\}$ ; if  $(r < 0) r \leftarrow (0 \times 10001 + r)$ , with return value  $(r \text{ AND } 0 \times \text{FFFF})$  in all 3 cases.

**7.105 Note** (implementing  $ab \bmod 2^n + 1$ ) Multiplication mod  $2^{16} + 1$  may be efficiently implemented as follows, for  $0 \leq a, b \leq 2^{16}$  (cf. §14.3.4). Let  $c = ab = c_0 \cdot 2^{32} + c_H \cdot 2^{16} + c_L$ , where  $c_0 \in \{0, 1\}$  and  $0 \leq c_L, c_H < 2^{16}$ . To compute  $c' = c \bmod (2^{16} + 1)$ , first obtain  $c_L$  and  $c_H$  by standard multiplication. For  $a = b = 2^{16}$ , note that  $c_0 = 1, c_L = c_H = 0$ , and  $c' = (-1)(-1) = 1$ , since  $2^{16} \equiv -1 \pmod{2^{16} + 1}$ ; otherwise,  $c_0 = 0$ . Consequently,  $c' = c_L - c_H + c_0$  if  $c_L \geq c_H$ , while  $c' = c_L - c_H + (2^{16} + 1)$  if  $c_L < c_H$  (since then  $-2^{16} < c_L - c_H < 0$ ).

**7.106 Example** (IDEA test vectors) Sample data for IDEA encryption of 64-bit plaintext  $M$  using 128-bit key  $K$  is given in Table 7.12. All entries are 16-bit values displayed in hexadecimal. Table 7.13 details the corresponding decryption of the resulting 64-bit ciphertext  $C$  under the same key  $K$ . □

128-bit key $K = (1, 2, 3, 4, 5, 6, 7, 8)$							64-bit plaintext $M = (0, 1, 2, 3)$			
$r$	$K_1^{(r)}$	$K_2^{(r)}$	$K_3^{(r)}$	$K_4^{(r)}$	$K_5^{(r)}$	$K_6^{(r)}$	$X_1$	$X_2$	$X_3$	$X_4$
1	0001	0002	0003	0004	0005	0006	00f0	00f5	010a	0105
2	0007	0008	0400	0600	0800	0a00	222f	21b5	f45e	e959
3	0c00	0e00	1000	0200	0010	0014	0f86	39be	8ee8	1173
4	0018	001c	0020	0004	0008	000c	57df	ac58	c65b	ba4d
5	2800	3000	3800	4000	0800	1000	8e81	ba9c	f77f	3a4a
6	1800	2000	0070	0080	0010	0020	6942	9409	e21b	1c64
7	0030	0040	0050	0060	0000	2000	99d0	c7f6	5331	620e
8	4000	6000	8000	a000	c000	e001	0a24	0098	ec6b	4925
9	0080	00c0	0100	0140	—	—	11fb	ed2b	0198	6de5

**Table 7.12:** IDEA encryption sample: round subkeys and ciphertext  $(X_1, X_2, X_3, X_4)$ .

**7.107 Note** (security of IDEA) For the full 8-round IDEA, other than attacks on weak keys (see page 279), no published attack is better than exhaustive search on the 128-bit key space. The security of IDEA currently appears bounded only by the weaknesses arising from the relatively small (compared to its keylength) blocklength of 64 bits.

$K = (1, 2, 3, 4, 5, 6, 7, 8)$							$C = (11fb,ed2b,0198,6de5)$			
$r$	$K_1^{(r)}$	$K_2^{(r)}$	$K_3^{(r)}$	$K_4^{(r)}$	$K_5^{(r)}$	$K_6^{(r)}$	$X_1$	$X_2$	$X_3$	$X_4$
1	fe01	ff40	ff00	659a	c000	e001	d98d	d331	27f6	82b8
2	fffd	8000	a000	cccc	0000	2000	bc4d	e26b	9449	a576
3	a556	ffb0	ffc0	52ab	0010	0020	0aa4	f7ef	da9c	24e3
4	554b	ff90	e000	fe01	0800	1000	ca46	fe5b	dc58	116d
5	332d	c800	d000	fffd	0008	000c	748f	8f08	39da	45cc
6	4aab	ffe0	ffe4	c001	0010	0014	3266	045e	2fb5	b02e
7	aa96	f000	f200	ff81	0800	0a00	0690	050a	00fd	1dfa
8	4925	fc00	fff8	552b	0005	0006	0000	0005	0003	000c
9	0001	fffe	fffd	c001	—	—	0000	0001	0002	0003

**Table 7.13:** IDEA decryption sample: round subkeys and variables ( $X_1, X_2, X_3, X_4$ ).

## 7.7 SAFER, RC5, and other block ciphers

### 7.7.1 SAFER

SAFER K-64 (Secure And Fast Encryption Routine, with 64-bit key) is an iterated block cipher with 64-bit plaintext and ciphertext blocks. It consists of  $r$  identical rounds followed by an output transformation. The original recommendation of 6 rounds was followed by a recommendation to adopt a slightly modified key schedule (yielding SAFER SK-64, which should be used rather than SAFER K-64 – see Note 7.110) and to use 8 rounds (maximum  $r = 10$ ). Both key schedules expand the 64-bit external key into  $2r + 1$  subkeys each of 64-bits (two for each round plus one for the output transformation). SAFER consists entirely of simple byte operations, aside from byte-rotations in the key schedule; it is thus suitable for processors with small word size such as chipcards (cf. FEAL).

Details of SAFER K-64 are given in Algorithm 7.108 and Figure 7.12 (see also page 280 regarding SAFER K-128 and SAFER SK-128). The XOR-addition stage beginning each round (identical to the output transformation) XORs bytes 1, 4, 5, and 8 of the (first) round subkey with the respective round input bytes, and respectively adds (mod 256) the remaining 4 subkey bytes to the others. The XOR and addition (mod 256) operations are interchanged in the subsequent addition-XOR stage. The S-boxes are an invertible byte-to-byte substitution using one fixed 8-bit bijection (see Note 7.111). A linear transformation  $f$  (the *Pseudo-Hadamard Transform*) used in the 3-level linear layer was specially constructed for rapid diffusion. The introduction of additive key biases in the key schedule eliminates weak keys (cf. DES, IDEA). In contrast to Feistel-like and many other ciphers, in SAFER the operations used for encryption differ from those for decryption (see Note 7.113). SAFER may be viewed as an SP network (Definition 7.79).

Algorithm 7.108 uses the following definitions ( $L, R$  denote left, right 8-bit inputs):

1.  $f(L, R) = (2L + R, L + R)$ . Addition here is mod 256 (also denoted by  $\boxplus$ );
2. tables  $S$  and  $S_{\text{inv}}$ , and the constant table for *key biases*  $B_i[j]$  as per Note 7.111.

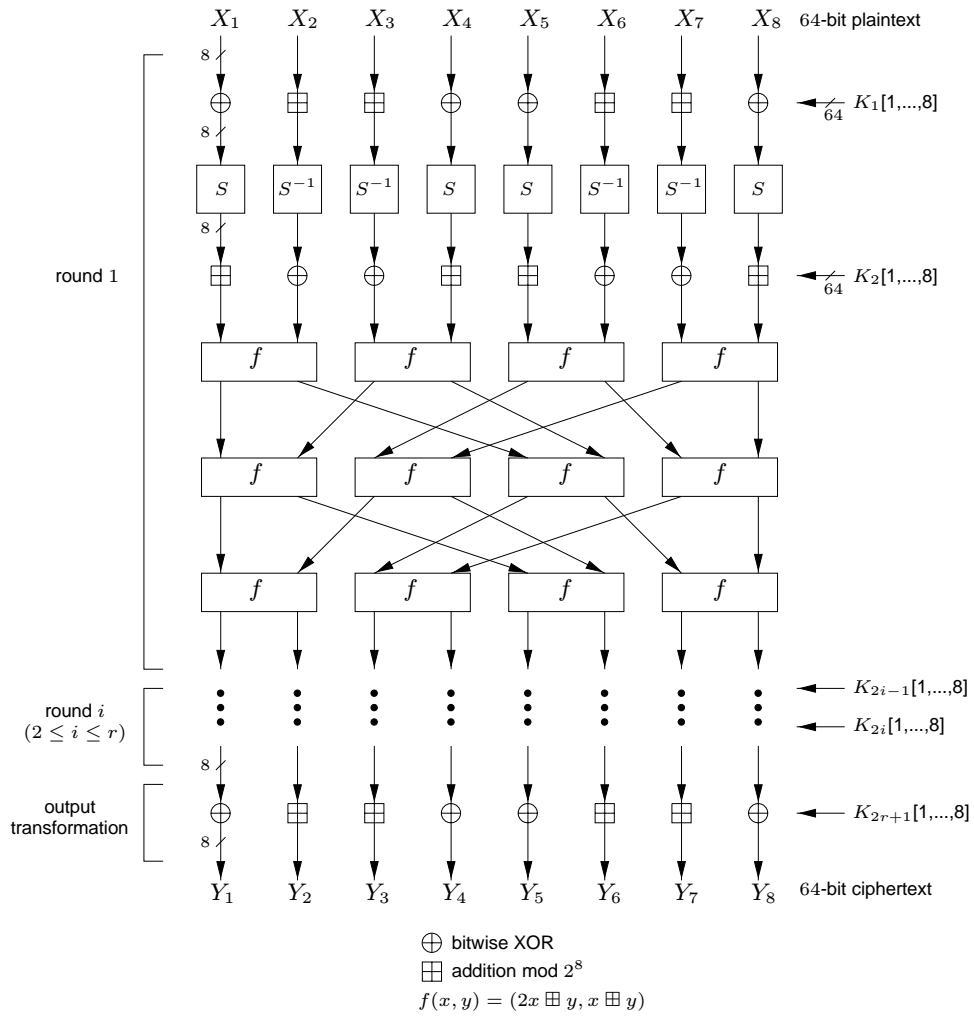


Figure 7.12: SAFER K-64 computation path ( $r$  rounds).



**7.108 Algorithm SAFER K-64 encryption** ( $r$  rounds)

INPUT:  $r$ ,  $6 \leq r \leq 10$ ; 64-bit plaintext  $M = m_1 \cdots m_{64}$  and key  $K = k_1 \cdots k_{64}$ .

OUTPUT: 64-bit ciphertext block  $Y = (Y_1, \dots, Y_8)$ . (For decryption, see Note 7.113.)

1. Compute 64-bit subkeys  $K_1, \dots, K_{2r+1}$  by Algorithm 7.109 with inputs  $K$  and  $r$ .
2.  $(X_1, X_2, \dots, X_8) \leftarrow (m_1 \cdots m_8, m_9 \cdots m_{16}, \dots, m_{57} \cdots m_{64})$ .
3. For  $i$  from 1 to  $r$  do: (XOR-addition, S-box, addition-XOR, and 3 linear layers)
  - (a) For  $j = 1, 4, 5, 8$ :  $X_j \leftarrow X_j \oplus K_{2i-1}[j]$ .  
For  $j = 2, 3, 6, 7$ :  $X_j \leftarrow X_j \boxplus K_{2i-1}[j]$ .
  - (b) For  $j = 1, 4, 5, 8$ :  $X_j \leftarrow S[X_j]$ . For  $j = 2, 3, 6, 7$ :  $X_j \leftarrow S_{\text{inv}}[X_j]$ .
  - (c) For  $j = 1, 4, 5, 8$ :  $X_j \leftarrow X_j \boxplus K_{2i}[j]$ . For  $j = 2, 3, 6, 7$ :  $X_j \leftarrow X_j \oplus K_{2i}[j]$ .
  - (d) For  $j = 1, 3, 5, 7$ :  $(X_j, X_{j+1}) \leftarrow f(X_j, X_{j+1})$ .
  - (e)  $(Y_1, Y_2) \leftarrow f(X_1, X_3)$ ,  $(Y_3, Y_4) \leftarrow f(X_5, X_7)$ ,  
 $(Y_5, Y_6) \leftarrow f(X_2, X_4)$ ,  $(Y_7, Y_8) \leftarrow f(X_6, X_8)$ .  
For  $j$  from 1 to 8 do:  $X_j \leftarrow Y_j$ .
  - (f)  $(Y_1, Y_2) \leftarrow f(X_1, X_3)$ ,  $(Y_3, Y_4) \leftarrow f(X_5, X_7)$ ,  
 $(Y_5, Y_6) \leftarrow f(X_2, X_4)$ ,  $(Y_7, Y_8) \leftarrow f(X_6, X_8)$ .  
For  $j$  from 1 to 8 do:  $X_j \leftarrow Y_j$ . (This mimics the previous step.)
4. (output transformation):  
For  $j = 1, 4, 5, 8$ :  $Y_j \leftarrow X_j \oplus K_{2r+1}[j]$ . For  $j = 2, 3, 6, 7$ :  $Y_j \leftarrow X_j \boxplus K_{2r+1}[j]$ .

**7.109 Algorithm SAFER K-64 key schedule**

INPUT: 64-bit key  $K = k_1 \cdots k_{64}$ ; number of rounds  $r$ .

OUTPUT: 64-bit subkeys  $K_1, \dots, K_{2r+1}$ .  $K_i[j]$  is byte  $j$  of  $K_i$  (numbered left to right).

1. Let  $R[i]$  denote an 8-bit data store and let  $B_i[j]$  denote byte  $j$  of  $B_i$  (Note 7.111).
2.  $(R[1], R[2], \dots, R[8]) \leftarrow (k_1 \cdots k_8, k_9 \cdots k_{16}, \dots, k_{57} \cdots k_{64})$ .
3.  $(K_1[1], K_1[2], \dots, K_1[8]) \leftarrow (R[1], R[2], \dots, R[8])$ .
4. For  $i$  from 2 to  $2r + 1$  do: (rotate key bytes left 3 bits, then add in the bias)
  - (a) For  $j$  from 1 to 8 do:  $R[j] \leftarrow (R[j] \leftarrow 3)$ .
  - (b) For  $j$  from 1 to 8 do:  $K_i[j] \leftarrow R[j] \boxplus B_i[j]$ . (See Note 7.110.)

**7.110 Note** (*SAFER SK-64 – strengthened key schedule*) An improved key schedule for Algorithm 7.108, resulting in SAFER SK-64, involves three changes as follows. (i) After initializing the  $R[i]$  in step 1 of Algorithm 7.109, set  $R[9] \leftarrow R[1] \oplus R[2] \oplus \cdots \oplus R[8]$ . (ii) Change the upper bound on the loop index in step 4a from 8 to 9. (iii) Replace the iterated line in step 4b by:  $K_i[j] \leftarrow R[(i+j-2) \bmod 9 + 1] \boxplus B_i[j]$ . Thus, key bytes 1,  $\dots$ , 8 of  $R[\cdot]$  are used for  $K_1$ ; bytes 2,  $\dots$ , 9 for  $K_2$ ; bytes 3,  $\dots$ , 9, 1 for  $K_3$ , etc. Here and originally,  $\boxplus$  denotes addition mod 256. No attack against SAFER SK-64 better than exhaustive key search is known.

**7.111 Note** (*S-boxes and key biases in SAFER*) The S-box, inverse S-box, and key biases for Algorithm 7.108 are constant tables as follows.  $g \leftarrow 45$ .  $S[0] \leftarrow 1$ ,  $S_{\text{inv}}[1] \leftarrow 0$ . for  $i$  from 1 to 255 do:  $t \leftarrow g \cdot S[i-1] \bmod 257$ ,  $S[i] \leftarrow t$ ,  $S_{\text{inv}}[t] \leftarrow i$ . Finally,  $S[128] \leftarrow 0$ ,  $S_{\text{inv}}[0] \leftarrow 128$ . (Since  $g$  generates  $\mathbb{Z}_{257}^*$ ,  $S[i]$  is a bijection on  $\{0, 1, \dots, 255\}$ . (Note that  $g^{128} \equiv 256 \pmod{257}$ , and associating 256 with 0 makes  $S$  a mapping with 8-bit input and output.) The *additive key biases* are 8-bit constants used in the key schedule (Algorithm 7.109), intended to behave as random numbers, and defined  $B_i[j] = S[S[9i+j]]$  for  $i$  from 2 to  $2r+1$  and  $j$  from 1 to 8. For example:  $B_2 = (22, 115, 59, 30, 142, 112, 189, 134)$  and  $B_{13} = (143, 41, 221, 4, 128, 222, 231, 49)$ .

**7.112 Remark** (*S-box mapping*) The S-box of Note 7.111 is based on the function  $S(x) = g^x \pmod{257}$  using a primitive element  $g = 45 \in \mathbb{Z}_{257}$ . This mapping is nonlinear with respect to both  $\mathbb{Z}_{257}$  arithmetic and the vector space of 8-tuples over  $\mathbb{F}_2$  under the XOR operation. The inverse S-box is based on the base- $g$  logarithm function.

**7.113 Note** (*SAFER K-64 decryption*) For decryption of Algorithm 7.108, the same key  $K$  and subkeys  $K_i$  are used as for encryption. Each encryption step is undone in reverse order, from last to first. Begin with an input transformation (XOR-subtraction stage) with key  $K_{2r+1}$  to undo the output transformation, replacing modular addition with subtraction. Follow with  $r$  decryption rounds using keys  $K_{2r}$  through  $K_1$  (two-per-round), inverting each round in turn. Each starts with a 3-stage inverse linear layer using  $f_{\text{inv}}(L, R) = (L - R, 2R - L)$ , with subtraction here mod 256, in a 3-step sequence defined as follows (to invert the byte-permutations between encryption stages):

Level 1 (for  $j = 1, 3, 5, 7$ ):  $(X_j, X_{j+1}) \leftarrow f_{\text{inv}}(X_j, X_{j+1})$ .

Levels 2 and 3 (each):  $(Y_1, Y_2) \leftarrow f_{\text{inv}}(X_1, X_5)$ ,  $(Y_3, Y_4) \leftarrow f_{\text{inv}}(X_2, X_6)$ ,

$(Y_5, Y_6) \leftarrow f_{\text{inv}}(X_3, X_7)$ ,  $(Y_7, Y_8) \leftarrow f_{\text{inv}}(X_4, X_8)$ ; for  $j$  from 1 to 8 do:  $X_j \leftarrow Y_j$ .

A subtraction-XOR stage follows (replace modular addition with subtraction), then an inverse substitution stage (exchange  $S$  and  $S^{-1}$ ), and an XOR-subtraction stage.

**7.114 Example** (*SAFER test vectors*) Using 6-round SAFER K-64 (Algorithm 7.108) on the 64-bit plaintext  $M = (1, 2, 3, 4, 5, 6, 7, 8)$  with the key  $K = (8, 7, 6, 5, 4, 3, 2, 1)$  results in the ciphertext  $C = (200, 242, 156, 221, 135, 120, 62, 217)$ , written as 8 bytes in decimal. Using 6-round SAFER SK-64 (Note 7.110) on the plaintext  $M$  above with the key  $K = (1, 2, 3, 4, 5, 6, 7, 8)$  results in the ciphertext  $C = (95, 206, 155, 162, 5, 132, 56, 199)$ .  $\square$

## 7.7.2 RC5

The RC5 block cipher has a word-oriented architecture for variable word sizes  $w = 16, 32$ , or 64 bits. It has an extremely compact description, and is suitable for hardware or software. The number of rounds  $r$  and the key byte-length  $b$  are also variable. It is successively more completely identified as RC5- $w$ , RC5- $w/r$ , and RC5- $w/r/b$ . RC5-32/12/16 is considered a common choice of parameters;  $r = 12$  rounds are recommended for RC5-32, and  $r = 16$  for RC5-64.

Algorithm 7.115 specifies RC5. Plaintext and ciphertext are blocks of bitlength  $2w$ . Each of  $r$  rounds updates both  $w$ -bit data halves, using 2 subkeys in an input transformation and 2 more for each round. The only operations used, all on  $w$ -bit words, are addition mod  $2^w$  ( $\boxplus$ ), XOR ( $\oplus$ ), and rotations (left  $\leftarrow$  and right  $\rightarrow$ ). The XOR operation is linear, while the addition may be considered nonlinear depending on the metric for linearity. The data-dependent rotations featured in RC5 are the main nonlinear operation used:  $x \leftarrow y$  denotes cyclically shifting a  $w$ -bit word left  $y$  bits; the rotation-count  $y$  may be reduced mod  $w$  (the low-order  $\lg(w)$  bits of  $y$  suffice). The key schedule expands a key of  $b$  bytes into  $2r + 2$  subkeys  $K_i$  of  $w$  bits each. Regarding packing/unpacking bytes into words, the byte-order is *little-endian*: for  $w = 32$ , the first plaintext byte goes in the low-order end of  $A$ , the fourth in  $A$ 's high-order end, the fifth in  $B$ 's low order end, and so on.

**7.115 Algorithm RC5 encryption** ( $w$ -bit wordsize,  $r$  rounds,  $b$ -byte key)

INPUT:  $2w$ -bit plaintext  $M = (A, B)$ ;  $r$ ; key  $K = K[0] \dots K[b-1]$ .  
 OUTPUT:  $2w$ -bit ciphertext  $C$ . (For decryption, see Note 7.117.)

1. Compute  $2r + 2$  subkeys  $K_0, \dots, K_{2r+1}$  by Algorithm 7.116 from inputs  $K$  and  $r$ .
2.  $A \leftarrow A \boxplus K_0$ ,  $B \leftarrow B \boxplus K_1$ . (Use addition modulo  $2^w$ .)
3. For  $i$  from 1 to  $r$  do:  $A \leftarrow ((A \oplus B) \leftrightarrow B) \boxplus K_{2i}$ ,  $B \leftarrow ((B \oplus A) \leftrightarrow A) \boxplus K_{2i+1}$ .
4. The output is  $C \leftarrow (A, B)$ .

**7.116 Algorithm RC5 key schedule**

INPUT: word bitsize  $w$ ; number of rounds  $r$ ;  $b$ -byte key  $K[0] \dots K[b-1]$ .  
 OUTPUT: subkeys  $K_0, \dots, K_{2r+1}$  (where  $K_i$  is  $w$  bits).

1. Let  $u = w/8$  (number of bytes per word) and  $c = \lceil b/u \rceil$  (number of words  $K$  fills). Pad  $K$  on the right with zero-bytes if necessary to achieve a byte-count divisible by  $u$  (i.e.,  $K[j] \leftarrow 0$  for  $b \leq j \leq c \cdot u - 1$ ). For  $i$  from 0 to  $c - 1$  do:  $L_i \leftarrow \sum_{j=0}^{u-1} 2^{8j} K[i \cdot u + j]$  (i.e., fill  $L_i$  low-order to high-order byte using each byte of  $K[\cdot]$  once).
2.  $K_0 \leftarrow P_w$ ; for  $i$  from 1 to  $2r + 1$  do:  $K_i \leftarrow K_{i-1} \boxplus Q_w$ . (Use Table 7.14.)
3.  $i \leftarrow 0, j \leftarrow 0, A \leftarrow 0, B \leftarrow 0, t \leftarrow \max(c, 2r + 2)$ . For  $s$  from 1 to  $3t$  do:
  - (a)  $K_i \leftarrow (K_i \boxplus A \boxplus B) \leftrightarrow 3$ ,  $A \leftarrow K_i$ ,  $i \leftarrow i + 1 \bmod (2r + 2)$ .
  - (b)  $L_j \leftarrow (L_j \boxplus A \boxplus B) \leftrightarrow (A \boxplus B)$ ,  $B \leftarrow L_j$ ,  $j \leftarrow j + 1 \bmod c$ .
4. The output is  $K_0, K_1, \dots, K_{2r+1}$ . (The  $L_i$  are not used.)

**7.117 Note** (*RC5 decryption*) Decryption uses the Algorithm 7.115 subkeys, operating on ciphertext  $C = (A, B)$  as follows (subtraction is mod  $2^w$ , denoted  $\boxminus$ ). For  $i$  from  $r$  down to 1 do:  $B \leftarrow ((B \boxminus K_{2i+1}) \leftrightarrow A) \oplus A$ ,  $A \leftarrow ((A \boxminus K_{2i}) \leftrightarrow B) \oplus B$ . Finally  $M \leftarrow (A \boxminus K_0, B \boxminus K_1)$ .

$w$ :	16	32	64
$P_w$ :	B7E1	B7E15163	B7E15162 8AED2A6B
$Q_w$ :	9E37	9E3779B9	9E3779B9 7F4A7C15

**Table 7.14:** RC5 magic constants (given as hex strings).

**7.118 Example** (*RC5-32/12/16 test vectors*) For the hexadecimal plaintext  $M = 65C178B2$   $84D197CC$  and key  $K = 5269F149$   $D41BA015$   $2497574D$   $7F153125$ , RC5 with  $w = 32$ ,  $r = 12$ , and  $b = 16$  generates ciphertext  $C = EB44E415$   $DA319824$ .  $\square$

**7.7.3 Other block ciphers**

LOKI'91 (and earlier, LOKI'89) was proposed as a DES alternative with a larger 64-bit key, a matching 64-bit blocksize, and 16 rounds. It differs from DES mainly in key-scheduling and the  $f$ -function. The  $f$ -function of each round uses four identical 12-to-8 bit S-boxes,

4 input bits of which select one of 16 functions, each of which implements exponentiation with a fixed exponent in a different representation of  $GF(2^8)$ . While no significant exploitable weaknesses have been found in LOKI'91 when used for encryption, related-key attacks (see page 281) are viewed as a certification weakness.

Khufu and Khafre are DES-like ciphers which were proposed as fast software-oriented alternatives to DES. They have 64-bit blocks,  $8 \times 32$  bit S-boxes, and a variable number of rounds (typically 16, 24, or 32). Khufu keys may be up to 512 bits. Khafre keys have bitlength that is a multiple of 64 (64 and 128-bit keys are typical); 64 key bits are XORed onto the data block before the first and thereafter following every 8 rounds. Whereas a DES round involves eight 6-to-4 bit S-boxes, one round of Khufu involves a single 8-to-32 bit table look-up, with a different S-box for every 8 rounds. The S-boxes are generated pseudorandomly from the user key. Khafre uses fixed S-boxes generated pseudorandomly from an initial S-box constructed from random numbers published by the RAND corporation in 1955. Under the best currently known attacks, 16-round Khufu and 24-round Khafre are each more difficult to break than DES.

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## 7.8 Notes and further references

### §7.1

The extensive and particularly readable survey by Diffie and Hellman [347], providing a broad introduction to cryptography especially noteworthy for its treatment of Hagelin and rotor machines and the valuable annotated bibliography circa 1979, is a source for much of the material in §7.2, §7.3, and §7.4 herein. Aside from the appearance of DES [396] in the mid 1970s and FEAL [884] later in the 1980s, prior to 1990 few fully-specified serious symmetric block cipher proposals were widely available or discussed. (See Chapter 15 for Pohlig and Hellman's 1978 discrete exponentiation cipher.) With the increasing feasibility of exhaustive search on 56-bit DES keys, the period 1990-1995 resulted in a large number of proposals, beginning with PES [728], the preliminary version of IDEA [730]. The *Fast Software Encryption* workshops (Cambridge, U.K., Dec. 1993; Leuven, Belgium, Dec. 1994; and again Cambridge, Feb. 1996) were a major stimulus and forum for new proposals.

The most significant cryptanalytic advances over the 1990-1995 period were Matsui's linear cryptanalysis [796, 795], and the differential cryptanalysis of Biham and Shamir [138] (see also [134, 139]). Extensions of these included the differential-linear analysis by Langford and Hellman [741], and the truncated differential analysis of Knudsen [686]. For additional background on linear cryptanalysis, see Biham [132]; see also Matsui and Yamagishi [798] for a preliminary version of the method. Additional background on differential cryptanalysis is provided by many authors including Lai [726], Lai, Massey, and Murphy [730], and Coppersmith [271]; although more efficient 6-round attacks are known, Stinson [1178] provides detailed examples of attacks on 3-round and 6-round DES. Regarding both linear and differential cryptanalysis, see also Knudsen [684] and Kaliski and Yin [656].

### §7.2

Lai [726, Chapter 2] provides an excellent concise introduction to block ciphers, including a lucid discussion of design principles (recommended for all block cipher designers). Regarding text dictionary and matching ciphertext attacks (Note 7.8), see Coppersmith, Johnson, and Matyas [278]. Rivest and Sherman [1061] provide a unified framework for randomized encryption (Definition 7.3); a common example is the use of random "salt" appended

*Handbook of Applied Cryptography* by A. Menezes, P. van Oorschot and S. Vanstone.

to passwords prior to password encryption in some operating systems (§10.2.3). Fact 7.9 is due to Shannon [1121], whose contributions are many (see below).

The four basic modes of operation (including  $k$ -bit OFB feedback) were originally defined specifically for DES in 1980 by FIPS 81 [398] and in 1983 by ANSI X3.106 [34], while ISO 8732 [578] and ISO/IEC 10116 [604], respectively, defined these modes for general 64-bit and general  $n$ -bit block ciphers, mandating  $n$ -bit OFB feedback (see also Chapter 15). Brassard [192] gives a concise summary of modes of operation; Davies and Price [308] provide a comprehensive discussion, including OFB cycling (Note 7.24; see also Jueneman [643] and Davies and Parkin [307]), and a method for encrypting incomplete CBC final blocks without data expansion, which is important if plaintext must be encrypted and returned into its original store. See Voydock and Kent [1225] for additional requirements on  $IV$ s. Recommending  $r = s$  for maximum strength, ISO/IEC 10116 [604] specifies the CFB variation of Example 7.19, and provides extensive discussion of properties of the various modes. The counter mode (Example 7.23) was suggested by Diffie and Hellman [347].

The 1977 exhaustive DES key search machine (Example 7.27) proposed by Diffie and Hellman [346] contained  $10^6$  DES chips, with estimated cost US\$20 million (1977 technology) and 12-hour expected search time; Diffie later revised the estimate upwards one order of magnitude in a BNR Inc. report (US\$50 million machine, 2-day expected search time, 1980 technology). Diffie and Hellman noted the feasibility of a ciphertext-only attack (Example 7.28), and that attempting to preclude exhaustive search by changing DES keys more frequently, at best, doubles the expected search time before success.

Subsequently Wiener [1241] provided a gate-level design for a US\$1 million machine (1993 technology) using 57 600 DES chips with expected success in 3.5 hours. Each chip contains 16 pipelined stages, each stage completing in one clock tick at 50 MHz; a chip with full pipeline completes a key test every 20 nanoseconds, providing a machine  $57\,600 \times 50$  times faster than the 1142 years noted in FIPS 74 [397] as the time required to check  $2^{55}$  keys if one key can be tested each microsecond. Comparable key search machines of equivalent cost by Eberle [362] and Wayner [1231] are, respectively, 55 and 200 times slower, although the former does not require a chip design, and the latter uses a general-purpose machine. Wiener also noted adaptations of the ECB known-plaintext attack to other 64-bit modes (CBC, OFB, CFB) and 1-bit and 8-bit CFB.

Even and Goldreich [376] discuss the unicity distance of cascade ciphers under known-plaintext attack (Fact 7.35), present a generalized time-memory meet-in-the-middle trade-off (Note 7.38), and give several other concise results on cascades, including that under reasonable assumptions, the number of permutations realizable by a cascade of  $L$  random cipher stages is, with high probability,  $2^{Lk}$ .

Diffie and Hellman [346] noted the meet-in-the-middle attack on double encryption (Fact 7.33), motivating their recommendation that multiple encipherment, if used, should be at least three-fold; Hoffman [558] credits them with suggesting E-E-E triple encryption with three independent keys. Merkle's June 1979 thesis [850] explains the attack on two-key triple-encryption of Fact 7.39 (see also Merkle and Hellman [858]), and after noting Tuchman's proposal of two-key E-D-E triple encryption in a June 1978 conference talk (*National Computer Conference*, Anaheim, CA; see also [1199]), recommended that E-D-E be used with three independent keys:  $E_{K_3}(E_{K_2}^{-1}(E_{K_1}(x)))$ . The two-key E-D-E idea, adopted in ANSI X9.17 [37] and ISO 8732 [578], was reportedly conceived circa April 1977 by Tuchman's colleagues, Matyas and Meyer. The attack of Fact 7.40 is due to van Oorschot and Wiener [1206]. See Coppersmith, Johnson, and Matyas [278] for a proposed construction for a triple-DES algorithm. Other techniques intended to extend the strength of DES in-

clude the *DESX* proposal of Rivest as analyzed by Kilian and Rogaway [672], and the work of Biham and Biryukov [133].

Hellman [549] proposes a time-memory tradeoff for exhaustive key search on a cipher with  $N = 2^m$  ciphertexts requiring a chosen-plaintext attack,  $O(N^{2/3})$  time and  $O(N^{2/3})$  space after an  $O(N)$  precomputation; search time can be reduced somewhat by use of Rivest's suggestion of distinguished points (see Denning [326, p.100]). Kusuda and Matsumoto [722] recently extended this analysis. Fiat and Naor [393] pursue time-memory tradeoffs for more general functions. Amirazizi and Hellman [25] note that time-memory tradeoff with constant time-memory product offers no asymptotic cost advantage over exhaustive search; they examine tradeoffs between time, memory, and parallel processing, and using standard parallelization techniques, propose under a simplified model a search machine architecture for which doubling the machine budget (cost) increases the solution rate four-fold. This approach may be applied to exhaustive key search on double-encryption, as can the parallel collision search technique of van Oorschot and Wiener [1207, 1208]; see also Quisquater and Delescaille [1017, 1018].

Regarding Note 7.41, see Biham [131] (and earlier [130]) as well as Coppersmith, Johnson, and Matyas [278]. Biham's analysis on DES and FEAL shows that, in many cases, the use of intermediate data as feedback into an intermediate stage reduces security. 15 years earlier, reflecting on his chosen-plaintext attack on two-key triple-encryption, Merkle [850, p.149] noted "multiple encryption with any cryptographic system is liable to be much less secure than a system designed originally for the longer key".

Maurer and Massey [822] formalize Fact 7.42, where "break" means recovering plaintext from ciphertext (under a known-plaintext attack) or recovering the key; the results hold also for chosen-plaintext and chosen-ciphertext attack. They illustrate, however, that the earlier result and commonly-held belief proven by Even and Goldreich [376] – that a cascade is as strong as any of its component ciphers – requires the important qualifying (and non-practical) assumption that an adversary will not exploit statistics of the underlying plaintext; thus, the intuitive result is untrue for most practical ciphertext-only attacks.

### §7.3

Kahn [648] is the definitive historical reference for classical ciphers and machines up to 1967, including much of §7.3 and the notes below. The selection of classical ciphers presented largely follows Shannon's lucid 1949 paper [1121]. Standard references for classical cryptanalysis include Friedman [423], Gaines [436], and Sinkov [1152]. More recent books providing expository material on classical ciphers, machines, and cryptanalytic examples include Beker and Piper [84], Meyer and Matyas [859], Denning [326], and Davies and Price [308].

Polyalphabetic ciphers were invented circa 1467 by the Florentine architect Alberti, who devised a cipher disk with a larger outer and smaller inner wheel, respectively indexed by plaintext and ciphertext characters. Letter alignments defined a simple substitution, modified by rotating the disk after enciphering a few words. The first printed book on cryptography, *Polygraphia*, written in 1508 by the German monk Trithemius and published in 1518, contains the first *tableau* – a square table on 24 characters listing all shift substitutions for a fixed ordering of plaintext alphabet characters. Tableau rows were used sequentially to substitute one plaintext character each for 24 letters, where-after the same tableau or one based on a different alphabet ordering was used. In 1553 Belaso (from Lombardy) suggested using an easily changed key (and key-phrases as memory aids) to define the fixed alphabetic (shift) substitutions in a polyalphabetic substitution. The 1563 book of Porta (from Naples) noted the ordering of tableau letters may define arbitrary substitutions (vs. simply shifted

alphabets).

Various polyalphabetic auto-key ciphers, wherein the key changes with each message (the alteration depending on the message), were explored in the 16th century, most significantly by the Frenchman B. de Vigenère. His 1586 book *Traicté des Chiffres* proposed the combined use of a mixed tableau (mixed alphabet on both the tableau top and side) and an auto-keying technique (cf. Example 7.61). A single character served as a priming key to select the tableau row for the first character substitution, where-after the  $i$ th plaintext character determined the alphabet (tableau row) for substituting the next. The far less secure simple Vigenère cipher (Definition 7.53) is incorrectly attributed to Vigenère.

The Playfair cipher (Example 7.51), popularized by L. Playfair in England circa 1854 and invented by the British scientist C. Wheatstone, was used as a British field cipher [648, p.6]. J. Mauborgne (see also the Vernam and PURPLE ciphers below) is credited in 1914 with the first known solution of this digram cipher.

The Jefferson cylinder was designed by American statesman T. Jefferson, circa 1790-1800. In 1817, fellow American D. Wadsworth introduced the principle of plaintext and ciphertext alphabets of different lengths. His disk (cf. Alberti above) implemented a cipher similar to Trithemius' polyalphabetic substitution, but wherein the various alphabets were brought into play irregularly in a plaintext-dependent manner, foreshadowing both the polyalphabetic ciphers of later 20th century rotor machines, and the concept of chaining. The inner disk had 26 letters while the outer had an additional 7 digits; one full revolution of the larger caused the smaller to advance 7 characters into its second revolution. The driving disk was always turned in the same clockwise sense; when the character revealed through an aperture in the plaintext disk matched the next plaintext character, that visible through a corresponding ciphertext aperture indicated the resulting ciphertext. In 1867, Wheatstone displayed an independently devised similar device thereafter called the *Wheatstone disc*, receiving greater attention although less secure (having disks of respectively 26 and 27 characters, the extra character a plaintext space).

Vernam [1222] recorded his idea for telegraph encryption in 1917; a patent filed in September 1918 was issued July 1919. Vernam's device combined a stream of plaintext (5-bit Baudot coded) characters, via XOR, with a keystream of 5-bit (key) values, resulting in the *Vernam cipher* (a term often used for related techniques). This, the first polyalphabetic substitution automated using electrical impulses, had period equal to the length of the key stream; each 5-bit key value determined one of 32 fixed mono-alphabetic substitutions. Credit for the actual *one-time system* goes to J. Mauborgne (U.S. Army) who, after seeing Vernam's device with a repeated tape, realized that use of a random, non-repeated key improved security. While Vernam's device was a commercial failure, a related German system engineered by W. Kunze, R. Schauffler, and E. Langlotz was put into practice circa 1921-1923 for German diplomatic communications; their encryption system, which involved manually adding a key string to decimal-coded plaintext, was secured by using as the numerical key a random non-repeating decimal digit stream – the original *one-time pad*. Pads of 50 numbered sheets were used, each with 48 five-digit groups; no pads were repeated aside for one identical pad for a communicating partner, and no sheet was to be used twice; sheets were destroyed once used. The Vernam cipher proper, when used as a one-time system, involves only 32 alphabets, but provides more security than rotor machines with a far greater number of alphabets because the latter eventually repeat, whereas there is total randomness (for each plaintext character) in selecting among the 32 Vernam alphabets.

The matrix cipher of Example 7.52 was proposed in 1929 by Hill [557], providing a practical method for polygraphic substitution, albeit a linear transformation susceptible to known-

plaintext attack. Hill also recognized that using an involution as the encryption mapping allowed the same function to provide decryption. Recent contributions on homophonic substitution include Günther [529] and Jendal, Kuhn, and Massey [636].

Among the unrivalled cryptanalytic contributions of the Russian-born American Friedman is his 1920 Riverbank Publication no.22 [426] on cryptanalysis using the index of coincidence. Friedman coined the term *cryptanalysis* in 1920, using it in his 1923 book *Elements of Cryptanalysis* [425], a 1944 expansion of which, *Military Cryptanalysis* [423], remains highly recommended. The method of Kasiski (from West Prussia) was originally published in 1863; see Kahn [648, pp.208-213] for a detailed example. The discussion on IC and MR follows that of Denning [326], itself based on Sinkov [1152]. Fact 7.75 follows from a standard expectation computation weighted by  $\kappa_p$  or  $\kappa_r$ , depending on whether the second of a pair of randomly selected ciphertext characters is from the same ciphertext alphabet or one of the  $t - 1$  remaining alphabets. The values in Table 7.1 are from Kahn [648], and vary somewhat over time as languages evolve.

Friedman teaches how to cryptanalyze running-key ciphers in his (circa 1918) Riverbank Publication no.16, *Methods for the Solution of Running-Key Ciphers*; the two basic techniques are outlined by Diffie and Hellman [347]. The first is a *probable word* attack wherein an attacker guesses an (e.g., 10 character) word hopefully present in underlying text, and subtracts that word (mod 26) from all possible starting locations in the ciphertext in hopes of finding a recognizable 10-character result, where-after the guessed word (as either partial running-key or plaintext) might be extended using context. Probable-word attacks also apply to polyalphabetic substitution. The second technique is based on the fact that each ciphertext letter  $c$  results from a pair of plaintext/running-key letters  $(m_i, m'_i)$ , and is most likely to result from such pairs wherein both  $m_i$  and  $m'_i$  are high-frequency characters; one isolates the highest-probability pairs for each such ciphertext character value  $c$ , makes trial assumptions, and attempts to extend apparently successful guesses by similarly decrypting adjacent ciphertext characters; see Denning [326, p.83] for a partial example. Diffie and Hellman [347] note Fact 7.59 as an obvious method that is little-used (modern ciphers being more convenient); their suggestion that use of four iterative running keys is unbreakable follows from English being 75% redundant. They also briefly summarize various *scrambling* techniques (encryption via analog rather than digital methods), noting that analog scramblers are sometimes used in practice due to lower bandwidth and cost requirements, although such known techniques appear relatively insecure (possibly an inherent characteristic) and their use is waning as digital networks become prevalent.

Denning [326] tabulates digrams into high, medium, low, and rare classes. Konheim [705, p.24] provides transition probabilities  $p(t|s)$ , the probability that the next letter is  $t$  given that the current character is  $s$  in English text, in a table also presented by H. van Tilborg [1210]. Single-letter distributions in plaintext languages other than English are given by Davies and Price [308]. The letter frequencies in Figure 7.5, which should be interpreted only as an estimate, were derived by Meyer and Matyas [859] using excerpts totaling 4 million characters from the 1964 publication: W. Francis, *A Standard Sample of Present-Day Edited American English for Use with Digital Computers*, Linguistics Dept., Brown University, Providence, Rhode Island, USA. Figure 7.6 is based on data from Konheim [705, p.19] giving an estimated probability distribution of 2-grams in English, derived from a sample of size 67 320 digrams.

See Shannon [1122] and Cover and King [285] regarding redundancy and Fact 7.67. While not proven in any concrete manner, Fact 7.68 is noted by Friedman [424] and generally accepted. Unicity distance was defined by Shannon [1121]. Related issues are discussed in detail in various appendices of Meyer and Matyas [859]. Fact 7.71 and the random cipher

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model are due to Shannon [1121]; see also Hellman [548].

Diffie and Hellman [347] give an instructive overview of rotor machines (see also Denning [326]), and note their use in World War II by the Americans in their highest level system, the British, and the Germans (Enigma); they also give Fact 7.63 and the number of characters required under ciphertext-only and known-plaintext attacks (Note 7.66). Beker and Piper [84] provide technical details of the Hagelin M-209, as does Kahn [648, pp.427-431] who notes its remarkable compactness and weight: 3.25 x 5.5 x 7 inches and 6 lb. (including case); see also Barker [74], Morris [906], and Rivest [1053]. Davies and Price [308] briefly discuss the Enigma, noting it was cryptanalyzed during World War II in Poland, France, and then in the U.K. (Bletchley Park); see also Konheim [705].

The Japanese PURPLE cipher, used during World War II, was a polyalphabetic cipher cryptanalyzed August 1940 [648, p.18-23] by Friedman's team in the U.S. Signal Intelligence Service, under (Chief Signal Officer) Mauborgne. The earlier RED cipher used two rotor arrays; preceding it, the ORANGE system implemented a vowels-to-vowels, consonants-to-consonants cipher using sets of rotors.

#### §7.4

The concept of *fractionation*, related to product ciphers, is noted by Feistel [387], Shannon [1121], and Kahn [648, p.344] who identifies this idea in an early product cipher, the WWI German *ADFGVX* field cipher. As an example, an encryption function might operate on a block of  $t = 8$  plaintext characters in three stages as follows: the first substitutes two symbols for each individual character; the second transposes (mixes) the substituted symbols among themselves; the third re-groups adjacent resulting symbols and maps them back to the plaintext alphabet. The action of the transposition on partial (rather than complete) characters contributes to the strength of the principle.

Shannon [1121, §5 and §23-26] explored the idea of the product of two ciphers, noted the principles of confusion and diffusion (Remark 1.36), and introduced the idea of a *mixing transformation*  $F$  (suggesting a preliminary transposition followed by a sequence of alternating substitution and simple linear operations), and combining ciphers in a product using an intervening transformation  $F$ . Transposition and substitution, respectively, rest on the principles of diffusion and confusion. Harpes, Kramer, and Massey [541] discuss a general model for iterated block ciphers (cf. Definition 7.80).

The name *Lucifer* is associated with two very different algorithms. The first is an SP network described by Feistel [387], which employs (bitwise nonlinear)  $4 \times 4$  invertible S-boxes; the second, closely related to DES (albeit significantly weaker), is described by Smith [1160] (see also Sorkin [1165]). Principles related to both are discussed by Feistel, Notz, and Smith [388]; both are analyzed by Biham and Shamir [138], and the latter in greater detail by Ben-Aroya and Biham [108] whose extension of differential cryptanalysis allows, using  $2^{36}$  chosen plaintexts and complexity, attack on 55% of the key space in Smith's Lucifer – still infeasible in practice, but illustrating inferiority to DES despite the longer 128-bit key.

Feistel's product cipher Lucifer [387], instantiated by a blocksize  $n = 128$ , consists of an unspecified number of alternating substitution and permutation (transposition) stages, using a fixed (unpublished)  $n$ -bit permutation  $P$  and 32 parallel identical S-boxes each effecting a mapping  $S_0$  or  $S_1$  (fixed but unpublished bijections on  $\{0, 1\}^4$ ), depending on the value of one key bit; the unpublished key schedule requires 32-bits per S-box stage. Each stage operates on all  $n$  bits; decryption is by stage-wise inversion of  $P$  and  $S_i$ .

The structure of so-called Feistel ciphers (Definition 7.81) was first introduced in the Lucifer algorithm of Smith [1160], the direct predecessor of DES. This 16-round algorithm

with 128-bit key operates on alternating half-blocks of a 128-bit message block with a simplified  $f$  function based on two published invertible  $4 \times 4$  bit S-boxes  $S_0$  and  $S_1$  (cf. above). Feistel, Notz, and Smith [388] discuss both the abstract Feistel cipher structure (suggesting its use with non-invertible S-boxes) and SP networks based on invertible (distinct) S-boxes. Suggestions for SP networks include the use of single key bits to select one of two mappings (a fixed bijection or its inverse) from both S-boxes and permutation boxes; decryption then uses a reversed key schedule with complemented key. They also noted the multi-round *avalanche effect* of changing a single input bit, subsequently pursued by Kam and Davida [659] in relation to SP networks and S-boxes having a *completeness* property: for every pair of bit positions  $i, j$ , there must exist at least two input blocks  $x, y$  which differ only in bit  $i$  and whose outputs differ in at least bit  $j$ . More simply, a function is *complete* if each output bit depends on all input bits. Webster and Tavares [1233] proposed the more stringent *strict avalanche criterion*: whenever one input bit is changed, every output bit must change with probability  $1/2$ .

DES resulted from IBM's submission to the 1974 U.S. National Bureau of Standards (NBS) solicitation for encryption algorithms for the protection of computer data. The original specification is the 1977 U.S. Federal Information Processing Standards Publication 46 [396], reprinted in its entirety as Appendix A in Meyer and Matyas [859]. DES is now specified in FIPS 46-2, which succeeded FIPS 46-1; the same cipher is defined in the American standard ANSI X3.92 [33] and referred to as the Data Encryption Algorithm (DEA). Differences between FIPS 46/46-1 and ANSI X3.92 included the following: these earlier FIPS required that DES be implemented in hardware and that the parity bits be used for parity; ANSI X3.92 specifies that the parity bits *may* be used for parity. Although no purpose was stated by the DES designers for the permutations  $IP$  and  $IP^{-1}$ , Preneel et al. [1008] provided some evidence of their cryptographic value in the CFB mode.

FIPS 81 [398] specifies the common modes of operation. Davies and Price [308] provide a comprehensive discussion of both DES and modes of operation; see also Diffie and Hellman [347], and the extensive treatment of Meyer and Matyas [859]. The survey of Smid and Branstad [1156] discusses DES, its history, and its use in the U.S. government. Test vectors for various modes of DES, including the ECB vectors of Example 7.86, may be found in ANSI X3.106 [34]. Regarding exhaustive cryptanalysis of DES and related issues, see also the notes under §7.2.

The 1981 publication FIPS 74 [397] notes that DES is not (generally) commutative under two keys, and summarizes weak and semi-weak keys using the term *dual keys* to include both (weak keys being self-dual); see also Davies [303] and Davies and Price [308]. Coppersmith [268] noted Fact 7.90; Moore and Simmons [900] pursue weak and semi-weak DES keys and related phenomena more rigorously.

The 56-bit keylength of DES was criticized from the outset as being too small (e.g., see Diffie and Hellman [346], and p.272 above). Claims which have repeatedly arisen and been denied (e.g., see Tuchman [1199]) over the past 20 years regarding built-in weaknesses of DES (e.g., trap-door S-boxes) remain un-substantiated. Fact 7.91 is significant in that if the permutation group were closed under composition, DES would fall to a known-plaintext attack requiring  $2^{28}$  steps – see Kaliski, Rivest, and Sherman [654], whose cycling experiments provided strong evidence against this. Campbell and Wiener [229] prove the fact conclusively (and give the stated lower bound), through their own cycling experiments utilizing collision key search and an idea outlined earlier by Coppersmith [268] for establishing a lower bound on the group size; they attribute to Coppersmith the same result (in unpublished work), which may also be deduced from the cycle lengths published by Moore and Simmons [901].

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Countless papers have analyzed various properties of DES; Davies and Price [308, pp.73-75] provide a partial summary to 1987. Subsequent to the discovery of differential cryptanalysis (DC) by Biham and Shamir, Coppersmith [271] explains how DES was specifically designed 15 years earlier to counter DC, citing national security concerns regarding the design team publishing neither the attack nor design criteria; then gives the (relevant) design criteria – some already noted by others, e.g., see Hellman et al. [552] – for DES S-boxes and the permutation  $P$ , explaining how these preclude DC. Coppersmith notes elements of DC were present in the work of den Boer [322], followed shortly by Murphy [913]. DES was not, however, specifically designed to preclude linear cryptanalysis (LC); Matsui [797] illustrates the order of the 8 DES S-boxes, while a strong (but not optimal) choice against DC, is relatively weak against LC, and that DES can be strengthened (vs. DC and LC) by carefully re-arranging these. Despite Remark 7.93, a DES key has actually been recovered by Matsui [795] using LC under experimental conditions (using  $2^{43}$  known-plaintext pairs from randomly generated plaintexts, and  $2^{43}$  complexity running twelve 99 MHz machines over 50 days); such a result remains to be published for exhaustive search or DC.

Ben-Aroya and Biham [108] note that often suggestions to redesign DES, some based on design criteria and attempts to specifically resist DC, have resulted in (sometimes far) weaker systems, including the RDES (randomized DES) proposal of Koyama and Terada [709], which fall to variant attacks. The lesson is that in isolation, individual design principles do not guarantee security.

DES alternatives are sought not only due to the desire for a keylength exceeding 56 bits, but also because its bit-oriented operations are inconvenient in conventional software implementations, often resulting in poor performance; this makes triple-DES less attractive. Regarding fast software implementations of DES, see Shepherd [1124], Pfitzmann and Aßmann [970], and Feldmeier and Karn [391].

### §7.5

FEAL stimulated the development of a sequence of advanced cryptanalytic techniques of unparalleled richness and utility. While it appears to remain relatively secure when iterated a sufficient number of rounds (e.g., 24 or more), this defeats its original objective of speed. FEAL-4 as presented at Eurocrypt'87 (Abstracts of Eurocrypt'87, April 1987) was found to have certain vulnerabilities by den Boer (unpublished Eurocrypt'87 rump session talk), resulting in Shimizu and Miyaguchi [1126] (or see Miyaguchi, Shiraiishi, and Shimizu [887]) increasing FEAL to 8 rounds in the final proceedings. In 1988 den Boer [322] showed FEAL-4 vulnerable to an adaptive chosen plaintext attack with 100 to 10 000 plaintexts. In 1990, Gilbert and Chassé [455] devised a chosen-plaintext attack (called a statistical meet-in-the-middle attack) on FEAL-8 requiring 10 000 pairs of plaintexts, the bitwise XOR of each pair being selected to be an appropriate constant (thus another early variant of differential cryptanalysis).

FEAL- $N$  with  $N$  rounds, and its extension FEAL-NX with 128-bit key (Notes 7.97 and 7.98) were then published by Miyaguchi [884] (or see Miyaguchi et al. [885]), who nonetheless opined that chosen-plaintext attacks on FEAL-8 were not practical threats. However, improved chosen-plaintext attacks were subsequently devised, as well as known-plaintext attacks. Employing den Boer's  $G$  function expressing linearity in the FEAL  $f$ -function, Murphy [913] defeated FEAL-4 with 20 chosen plaintexts in under 4 hours (under 1 hour for most keys) on a Sun 3/60 workstation. A statistical method of Tardy-Corfdir and Gilbert [1187] then allowed a known-plaintext attack on FEAL-4 (1000 texts; or 200 in an announced improvement) and FEAL-6 ( $2 \times 10\,000$  texts), involving linear approximation of FEAL S-boxes. Thereafter, the first version of linear cryptanalysis (LC) introduced by Matsui and Yamagishi [798] allowed known-plaintext attack of FEAL-4 (5 texts, 6 minutes on

a 25MHz 68040 processor), FEAL-6 (100 texts, 40 minutes), and FEAL-8 ( $2^{28}$  texts, in time equivalent to exhaustive search on 50-bit keys); the latter betters the  $2^{38}$  texts required for FEAL-8 by Biham and Shamir [136] in their known-plaintext conversion of differential cryptanalysis (DC). Biham and Shamir [138, p.101] later implemented a DC chosen-plaintext attack recovering FEAL-8 keys in two minutes on a PC using 128 chosen pairs, the program requiring 280K bytes of storage. Biham [132] subsequently used LC to defeat FEAL-8 with  $2^{24}$  known-plaintexts in 10 minutes on a personal computer. Ohta and Aoki [943] suggest that FEAL-32 is as secure as DES against DC, while FEAL-16 is as secure as DES against certain restricted forms of LC.

*Differential-linear cryptanalysis* was introduced by Langford and Hellman [741], combining linear and differential cryptanalysis to allow a reduced 8-round version of DES to be attacked with fewer chosen-plaintexts than previous attacks. Aoki and Ohta [53] refined these ideas for FEAL-8 yielding a differential-linear attack requiring only 12 chosen texts and 35 days of computer time (cf. Table 7.10).

Test vectors for FEAL-N and FEAL-NX (Example 7.99) are given by Miyaguchi [884]. The DC attack of Biham and Shamir [137], which finds FEAL-N subkeys themselves, is equally as effective on FEAL-NX. Biham [132] notes that an LC attack on FEAL-N is possible with less than  $2^{64}$  known plaintexts (and complexity) for up to  $N = 20$ . For additional discussion of properties of FEAL, see Biham and Shamir [138, §6.3].

### §7.6

The primary reference for IDEA is Lai [726]. A preliminary version introduced by Lai and Massey [728] was named PES (Proposed Encryption Standard). Lai, Massey, and Murphy [730] showed that a generalization (see below) of differential cryptanalysis (DC) allowed recovery of PES keys, albeit requiring all  $2^{64}$  possible ciphertexts (cf. exhaustive search of  $2^{128}$  operations). Minor modifications resulted in IPES (Improved PES): in stage  $r$ ,  $1 \leq r \leq 9$ , the group operations keyed by  $K_2^{(r)}$  and  $K_4^{(r)}$  ( $\boxplus$  and  $\odot$  in Figure 7.11) were reversed from PES; the permutation on 16-bit blocks after stage  $r$ ,  $1 \leq r \leq 9$ , was altered; and necessary changes were made in the decryption (but not encryption) key schedule. IPES was commercialized under the name IDEA, and is patented (see Chapter 15).

The ingenious design of IDEA is supported by a careful analysis of the interaction and algebraic incompatibilities of operations across the groups  $(\mathbb{F}_2^n, \oplus)$ ,  $(\mathbb{Z}_{2^n}, \boxplus)$ , and  $(\mathbb{Z}_{2^n+1}^*, \odot)$ . The design of the MA structure (see Figure 7.11) results in IDEA being “complete” after a single round; for other security properties, see Lai [726]. Regarding mixing operations from different algebraic systems, see also the 1974 examination by Grossman [522] of transformations arising by alternating mod  $2^n$  and mod 2 addition ( $\oplus$ ), and the use of arithmetic modulo  $2^{32} - 1$  and  $2^{32} - 2$  in MAA (Algorithm 9.68).

Daemen [292, 289] identifies several classes of so-called *weak keys for IDEA*, and notes a small modification to the key schedule to eliminate them. The largest is a class of  $2^{51}$  keys for which membership can be tested in two encryptions plus a small number of computations, whereafter the key itself can be recovered using 16 chosen plaintext-difference encryptions, on the order of  $2^{16}$  group operations, plus  $2^{17}$  key search encryptions. The probability of a randomly chosen key being in this class is  $2^{51}/2^{128} = 2^{-77}$ . A smaller number of weak key blocks were observed earlier by Lai [726], and dismissed as inconsequential. The analysis of Meier [832] revealed no attacks feasible against full 8-round IDEA, and supports the conclusion of Lai [726] that IDEA appears to be secure against DC after 4 of its 8 rounds (cf. Note 7.107). Daemen [289] also references attacks on reduced-round variants of IDEA. While linear cryptanalysis (LC) can be applied to any iterated block cipher,

Harpes, Kramer, and Massey [541] provide a generalization thereof; IDEA and SAFER K-64 are argued to be secure against this particular generalization.

Lai, Massey, and Murphy [730] (see also Lai [726]) generalized DC to apply to *Markov ciphers* (which they introduced for this purpose; DES, FEAL, and LOKI are all examples under the assumption of independent round keys) including IDEA; broadened the notion of a *difference* from that based on  $\oplus$  to:  $\Delta X = X \otimes (X^*)^{-1}$  where  $\otimes$  is a specified group operation and  $(X^*)^{-1}$  is the group inverse of an element  $X^*$ ; and defined an *i-round differential* (as opposed to an *i-round characteristic* used by Biham and Shamir [138] on DES) to be a pair  $(\alpha, \beta)$  such that two distinct plaintexts with difference  $\Delta X = \alpha$  results in a pair of round *i* outputs with difference  $\beta$ .

Decimal values corresponding to Tables 7.12 and 7.13 may be found in Lai [726]. A table-based alternative for multiplication mod  $2^{16} + 1$  (cf. Note 7.104) is to look up the anti-log of  $\log_\alpha(a) + \log_\alpha(b) \bmod 2^{16}$ , relative to a generator  $\alpha$  of  $\mathbb{Z}_{2^{16}+1}^*$ ; the required tables, however, are quite large.

### §7.7

Massey [787] introduced SAFER K-64 with a 64-bit key and initially recommended 6 rounds, giving a reference implementation and test vectors (cf. Example 7.114). It is not patented. Massey [788] then published SAFER K-128 (with a reference implementation), differing only in its use of a non-proprietary (and backwards compatible) key schedule accommodating 128-bit keys, proposed by a Singapore group; 10 rounds were recommended (12 maximum). Massey [788] gave further justification for design components of SAFER K-64. Vaudenay [1215] showed SAFER K-64 is weakened if the S-box mapping (Remark 7.112) is replaced by a random permutation.

Knudsen [685] proposed the modified key schedule of Note 7.110 after finding a weakness in 6-round SAFER K-64 that, while not of practical concern for encryption (with  $2^{45}$  chosen plaintexts, it finds 8 bits of the key), permitted collisions when using the cipher for hashing. This and a subsequent certification attack on SAFER K-64 by S. Murphy (to be published) lead Massey (“Strengthened key schedule for the cipher SAFER”, posted to the USENET newsgroup sci.crypt, September 9 1995) to advise adoption of the new key schedule, with the resulting algorithm distinguished as SAFER SK-64 with 8 rounds recommended (minimum 6, maximum 10); an analogous change to the 128-bit key schedule yields SAFER SK-128 for which 10 rounds remain recommended (maximum 12). A new variant of DC by Knudsen and Berson [687] using *truncated differentials* (building on Knudsen [686]) yields a certification attack on 5-round SAFER K-64 with  $2^{45}$  chosen plaintexts; the attack, which does not extend to 6 rounds, indicates that security is less than argued by Massey [788], who also notes that preliminary attempts at linear cryptanalysis of SAFER were unsuccessful.

RC5 was designed by Rivest [1056], and published along with a reference implementation. The magic constants of Table 7.14 are based on the golden ratio and the base of natural logarithms. The data-dependent rotations (which vary across rounds) distinguish RC5 from iterated ciphers which have identical operations each round; Madryga [779] proposed an earlier (less elegant) cipher involving data-dependent rotations. A preliminary examination by Kaliski and Yin [656] suggested that, while variations remain to be explored, standard linear and differential cryptanalysis appear impractical for RC5–32 (64-bit blocksize) for  $r = 12$ : their differential attacks on 9 and 12 round RC5 require, respectively,  $2^{45}$ ,  $2^{62}$  chosen-plaintext pairs, while their linear attacks on 4, 5, and 6-round RC5–32 require, respectively,  $2^{37}$ ,  $2^{47}$ ,  $2^{57}$  known plaintexts. Both attacks depend on the number of rounds and the blocksize, but not the byte-length of the input key (since subkeys are recovered di-

rectly). Knudsen and Meier [689] subsequently presented differential attacks on RC5 which improved on those of Kaliski and Yin by a factor up to 512, and showed that RC5 has so-called *weak keys* (independent of the key schedule) for which these differential attacks perform even better.

LOKI was introduced by Brown, Pieprzyk, and Seberry [215] and renamed LOKI'89 after the discovery of weaknesses lead to the introduction of LOKI'91 by Brown et al. [214]. Knudsen [682] noted each LOKI'89 key fell into a class of 16 equivalent keys, and the differential cryptanalysis of Biham and Shamir [137] was shown to be effective against reduced-round versions. LOKI'91 failed to succumb to differential analysis by Knudsen [683]; Tokita et al. [1193] later confirmed the optimality of Knudsen's characteristics, suggesting that LOKI'89 and LOKI'91 were resistant to both ordinary linear and differential cryptanalysis. However, neither should be used for hashing as originally proposed (see Knudsen [682]) or in other modes (see Preneel [1003]). Moreover, both are susceptible to *related-key attacks* (Note 7.6), popularized by Biham [128, 129]; but see also the earlier ideas of Knudsen [683]. Distinct from these are *key clustering attacks* (see Diffie and Hellman [347, p.410]), wherein a cryptanalyst first finds a key "close" to the correct key, and then searches a cluster of "nearby" keys to find the correct one.

$8 \times 32$  bit S-boxes first appeared in the Snefru hash function of Merkle [854]; here such fixed S-boxes created from random numbers were used in its internal encryption mapping. Regarding large S-boxes, see also Gordon and Retkin [517], Adams and Tavares [7], and Biham [132]. Merkle [856] again used  $8 \times 32$  S-boxes in Khufu and Khafre (see also §15.2.3(viii)). In this 1990 paper, Merkle gives a chosen-plaintext differential attack defeating 8 rounds of Khufu (with secret S-box). Regarding 16-round Khafre, a DC attack by Biham and Shamir [138, 137] requires somewhat over 1500 chosen plaintexts and one hour on a personal computer, and their known-plaintext differential attack requires  $2^{37.5}$  plaintexts; for 24-round Khafre, they require  $2^{53}$  chosen plaintexts or  $2^{58.5}$  known plaintexts. Khufu with 16 rounds was examined by Gilbert and Chauvaud [456], who gave an attack using  $2^{43}$  chosen plaintexts and about  $2^{43}$  operations.

CAST is a design procedure for a family of DES-like ciphers, featuring fixed  $m \times n$  bit S-boxes ( $m < n$ ) based on bent functions. Adams and Tavares [7] examine the construction of large S-boxes resistant to differential cryptanalysis, and give a partial example (with 64-bit blocklength and  $8 \times 32$  bit S-boxes) of a CAST cipher. CAST ciphers have variable keysize and numbers of rounds. Rijmen and Preneel [1049] presented a cryptanalytic technique applicable to Feistel ciphers with non-surjective round functions (e.g., LOKI'91 and an example CAST cipher), noting cases where 6 to 8 rounds is insufficient.

Blowfish is a 16-round DES-like cipher due to Schneier [1093], with 64-bit blocks and keys of length up to 448 bits. The computationally intensive key expansion phase creates eighteen 32-bit subkeys plus four  $8 \times 32$  bit S-boxes derived from the input key (cf. Khafre above), for a total of 4168 bytes. See Vaudenay [1216] for a preliminary analysis of Blowfish.

3-WAY is a block cipher with 96-bit blocksize and keysize, due to Daemen [289] and introduced by Daemen, Govaerts, and Vandewalle [290] along with a reference C implementation and test vectors. It was designed for speed in both hardware and software, and to resist differential and linear attacks. Its core is a 3-bit nonlinear S-box and a linear mapping representable as polynomial multiplication in  $\mathbb{Z}_2^{12}$ .

SHARK is an SP-network block cipher due to Rijmen et al. [1048] (coordinates for a reference implementation are given) which may be viewed as a generalization of SAFER, employing highly nonlinear S-boxes and the idea of MDS codes (cf. Note 12.36) for diffusion

to allow a small number of rounds to suffice. The block ciphers BEAR and LION of Anderson and Biham [30] are 3-round unbalanced Feistel networks, motivated by the earlier construction of Luby and Rackoff [776] (see also Maurer [816] and Lucks [777]) which provides a provably secure (under suitable assumptions) block cipher from pseudorandom functions using a 3-round Feistel structure. SHARK, BEAR, and LION all remain to be subjected to independent analysis in order to substantiate their conjectured security levels. SKIPJACK is a classified block cipher whose specification is maintained by the U.S. National Security Agency (NSA). FIPS 185 [405] notes that its specification is available to organizations entering into a Memorandum of Agreement with the NSA, and includes interface details (e.g., it has an 80-bit secret key). A public report contains results of a preliminary security evaluation of this 64-bit block cipher (“SKIPJACK Review, Interim Report, The SKIPJACK Algorithm”, 1993 July 28, by E.F. Brickell, D.E. Denning, S.T. Kent, D.P. Maher, and W. Tuchman). See also Roe [1064, p.312] regarding curious results on the cyclic closure tests on SKIPJACK, which give evidence related to the size of the cipher keyspace.

GOST 28147-89 is a Soviet government encryption algorithm with a 32-round Feistel structure and unspecified S-boxes; see Charney et al. [241].

RC2 is a block cipher proprietary to RSA Data Security Inc. (as is the stream cipher RC4). WAKE is a block cipher due to Wheeler [1237] employing a key-dependent table, intended for fast encryption of bulk data on processors with 32-bit words. TEA (Tiny Encryption Algorithm) is a block cipher proposed by Wheeler and Needham [1238].