A BSSRDF Model for Efficient Rendering of Fur with Global Illumination

Supplemental Materials

1 Micro-flakes approximation of scattered components

In this section, we elaborate on Section 4.4 of the main paper. We derive the parameters of a dipole model to approximate the scattered components of a hair/fur model based on the micro-flakes theory [Jakob et al. 2010]. Suppose $\bar{r}$ is the average radius of the hair/fur volume. We approximate each hair/fur fiber as a set of spherical micro-flakes with radius $\bar{r}$ compactly distributed along the hair/fur fiber as in Fig. 1. The phase function of each micro-flake is the normalized BCSDF of the hair/fur fiber that it approximates.

The micro-flakes theory estimates volume scattering parameters as follows:

$$\sigma_a = (1-\mu)A(\omega_i)\rho,\ \sigma_s = \mu A(\omega_i)\rho,\ g = \int_{S^2} p(\omega_i, \omega_r)(\omega_i \cdot \omega_r) d\omega_r \tag{1}$$

where μ is the albedo of each micro-flake, $A(\omega_i)$ is the projected area of each micro-flake along direction ω_i, the parenthesis is the dot product operator of ω_i and ω_r, ρ is the micro-flakes density and p is the phase function. ω_i and ω_r are the incoming and outgoing directions respectively.

Using this approximation, μ is the ratio of the energy scattered by the micro-flake to the total input energy, which can be estimated by a numerical integration of the BCSDF. Since the integral depends on the incoming longitudinal angle θ_i, we estimate the average of the integrals over all potential angles in the range $[0, \pi]$. Similarly, for g, we numerically estimate the integral and average over all potential longitudinal angles. The projected area $A(\omega_i)$ of each micro-flake is $\pi\bar{r}^2$, which is simply the projected area of the sphere. For the density ρ, we first estimate the number of micro flakes by $n = L/(2\bar{r})$, where L is the total length of all fibers. This is because each sphere takes up $2\bar{r}$ of the total fiber length and all spheres are compactly distributed. Then, we estimate the volume V that the fibers take up in a coarse voxel grid and $\rho = n/V$. At this point, we have all the components to obtain σ_a, σ_s and g for the dipole model.

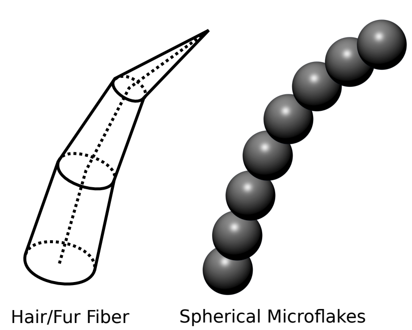

Figure 1: Approximating a hair fiber with spherical microflakes.

2 Importance sampling of dual scattering

Problem overview

Dual scattering is essentially adding 2 lobes to the original hair/fur BCSDF. To sample dual scattering, we are essentially sampling the overall BCSDF, including R, TT, TRT, TT^s and TRT^s lobes, together with the backward scattered lobe

$$S_b(\theta_i, \theta_r, \phi) = \frac{I_b(\phi)}{\pi \cos^2 \theta_i} \cdot d_b A_b \cdot G(\theta_r + \theta_i; \bar{\sigma_b}^2), \tag{2}$$

and the forward scattered lobe

$$S_f(\theta_i, \theta_r, \phi) = \frac{I_f(\phi)}{\cos^2 \theta_i} \cdot d_f A_f \cdot \sum_p \left(G(\theta_r + \theta_i; \bar{\beta}_p^2 + \bar{\sigma_f}^2) N_p^f \right). \tag{3}$$

Method overview

We follow the high-level importance sampling scheme in Yan et al. [2017]. We first select one of all these lobes based on the energy each lobe contains, then we sample the selected lobe. The selection probability is denoted as P, which is proportional to the energy each lobe carries.

When sampling the selected lobe, we separately sample it longitudinally and azimuthally with probability density functions (PDFs) p_M and p_N, respectively. Then, the PDF of sampling the entire sphere is simply converted as $p_M \cdot p_N / \cos(\theta_o)$. The sampling weight is the BCSDF value of the selected lobe at the sampled direction ω_o, divided by the probability P of selecting it.

Sampling S_b and S_f

First, we calculate the energy S_b and S_f carry, denoted as E_b and E_f. This is done simply by taking out the distribution related terms from Eqn. 2 and 3. We immediately have

$$E_b(\theta_i, \theta_r, \phi) = \frac{1}{\cos^2 \theta_i} \cdot d_b A_b, \tag{4}$$

and

$$E_f(\theta_i, \theta_r, \phi) = \frac{\pi}{\cos^2 \theta_i} \cdot d_f A_f \cdot \sum_p N_p^f. \tag{5}$$

Now, suppose we have already selected a lobe p. If $p \in R, TT, TRT, TT^s, TRT^s$, it is already handled by Yan et al. [2017] and we do not repeat the procedure here.

If the backward scattered lobe S_b is selected, we first importance sample it longitudinally according to the Gaussian $G(\theta_r + \theta_i; \bar{\sigma}_b^2)$, which is simply done by using the Box-Muller transform to sample a Gaussian centered at $-\theta_i$ with variance $\bar{\sigma}_b^2$. And the PDF p_M is the Gaussian value at the sampled position.

Then, we uniformly sample $\phi \in [-\pi/2, \pi/2]$ azimuthally with PDF $p_N = 1/\pi$, according to the backward hemisphere indicator I_b.

If the forward scattered lobe S_f is selected, since it consists of more sub-lobes (because the forward scattered lobe is essentially a modification of single hair/fur fibers' BCSDF with all lobes), we continue the lobe selection process within these sub-lobes to find one sub-lobe with probability Q (i.e. all probabilities of sampling the sub-lobes sum up to 1), then we're essentially sampling

$$\tilde{S}_f(\theta_i, \theta_r, \phi) = \frac{I_f(\phi)}{\cos^2 \theta_i} \cdot d_f A_f \cdot G(\theta_r + \theta_i; \bar{\beta}_p^2 + \bar{\sigma_f}^2) N_p^f / Q, \tag{6}$$

which is now similar to sampling the backward scattered lobe S_b.