# Artificial Intelligence

CS 165A

May 19, 2022

Instructor: Prof. Yu-Xiang Wang

Today

→ Reinforcement Learning

# Recap: Tabular MDP

- Discrete State, Discrete Action, Reward and Observation

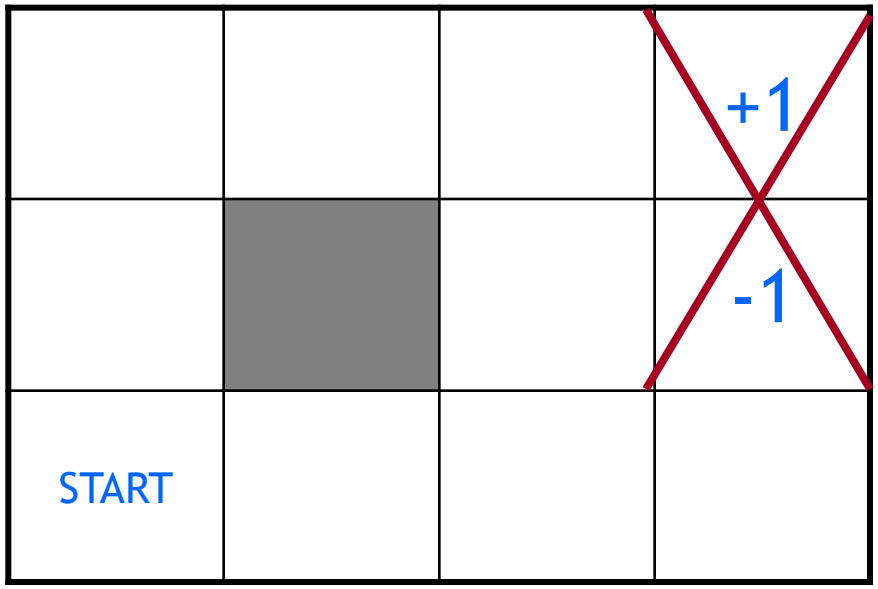
$$S_t \in \mathcal{S} \quad A_t \in \mathcal{A} \quad R_t \in \mathbb{R} \quad \sout{O_t \in \mathcal{O}}$$

- Policy:
  - When the state is observable: $\pi : \mathcal{S} \to \mathcal{A}$
  - ~~Or when the state is not observable~~

$$\sout{\pi_t : (\mathcal{O} \times \mathcal{A} \times \mathbb{R})^{t-1} \to \mathcal{A}}$$

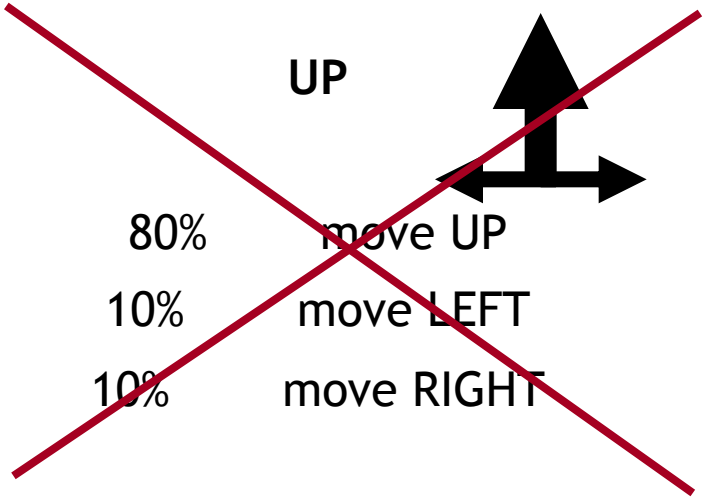
- Learn the best policy that maximizes the expected reward
  - Finite horizon (episodic) RL: $\pi^* = \arg\max_{\pi \in \Pi} \mathbb{E}[\sum_{t=1}^{T} R_t]$ T: horizon
  - Infinite horizon RL: $\pi^* = \arg\max_{\pi \in \Pi} \mathbb{E}[\sum_{t=1}^{\infty} \gamma^{t-1} R_t]$ γ: discount factor

# Recap: Example: Frozen Lake




**Action 1, Action 2, Action 3, Action 4**

actions: ~~UP, DOWN, LEFT, RIGHT~~




- ~~reward +1 at [4,3], -1 at [4,2]~~
- ~~reward -0.04 for each step~~
- what's the strategy to achieve max reward?

# Recap: Plug in the estimated MDP parameters and then do offline planning.

$$V_{k+1}^{\pi}(s) \leftarrow \sum_{a} \pi(a|s) \sum_{s'} \hat{P}(s'|s,a)[\hat{r}(s,a,s') + \gamma V_k^{\pi}(s')]$$

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* Note the **"hat"**. Usually it indicates empirical estimates.

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* These iterations will produce $\hat{V}^*$ and $\hat{Q}^*$ functions, and then $\hat{\pi}^*$

$\hat{\pi}^*(s) \leftarrow \arg\max_a \hat{Q}^*(s,a)$

# Recap: Three ideas for solving RL

- Idea 1: Model-based approach
  - Estimated the CPTs of MDP by their empirical frequency
  - Plug-in the estimate to Bellman equations for VI / PI

- Idea 2: Model-free approach: Directly estimate V function and Q function
  - Monte Carlo: Run many episodes of the MDP
  - First-visit MC: first time you visit State s, keep all subsequent rewards, then average over many such episodes

- Idea 3: Better model-free approach: Combining MC with VI / PI directly.
  - Temporal difference (TD) learning

# This lecture

- Monte Carlo Methods
- Temporal Difference Learning
- Features and linear function approximation
- Policy Gradient method

# Idea 2: **Model-free** Reinforcement Learning

- Do we need the model? Can we learn the Q function directly?
  - **How many free parameters are there to represent the Q-function?**
  - **Ans: SA << O(S^2A)**

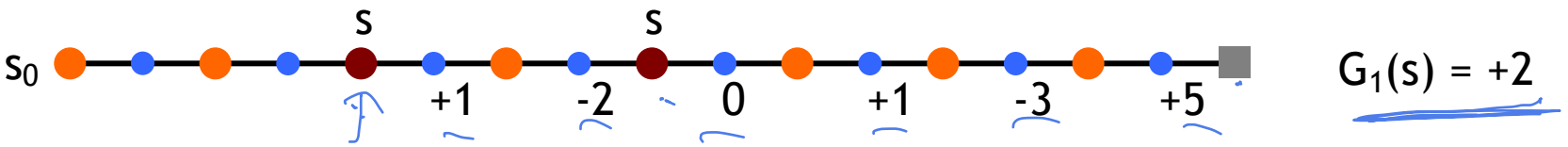
- Recall: Policy iterations

$$\pi_0 \rightarrow^E V^{\pi_0} \rightarrow^I \pi_1 \rightarrow^E V^{\pi_1} \rightarrow^I \ldots \rightarrow^I \pi^* \rightarrow^E V^*$$

  - Maybe we can do policy evaluation without estimating the model?

# Monte Carlo Policy Evaluation (Prediction)

- want to estimate $V^{\pi}(s)$
  - = expected return starting from s and following π
  - – estimate as average of observed returns in state s
- We can execute the policy $\pi$
- first-visit MC
  - – average returns following the first visit to state s



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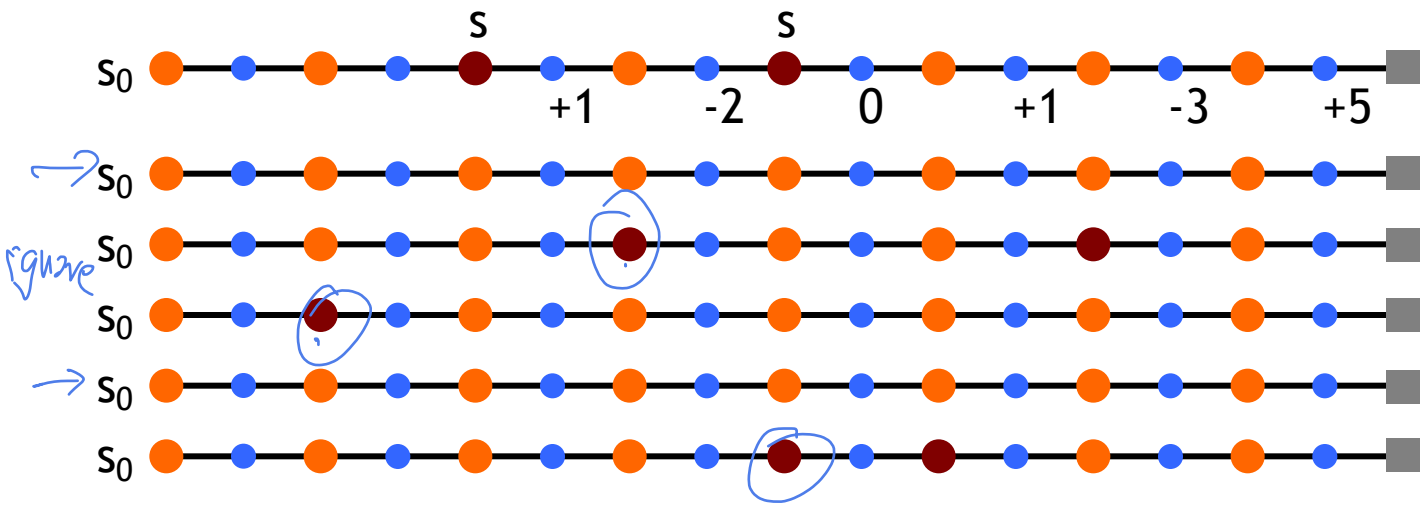
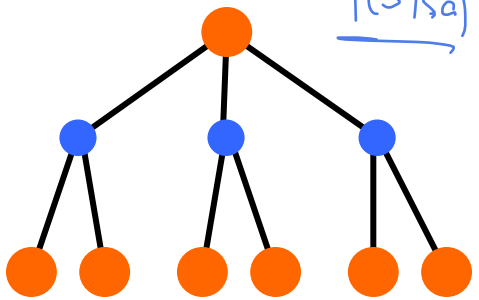


# Monte Carlo Policy Optimization (Control)

- $V^\pi$ not enough for policy improvement
  - need exact model of environment



- estimate $Q^\pi(s,a)$

$$\pi'(s) = \arg\max_a Q^\pi(s,a)$$

- MC control

$$\pi_0 \rightarrow^E Q^{\pi_0} \rightarrow^I \pi_1 \rightarrow^E Q^{\pi_1} \rightarrow^I \ldots \rightarrow^I \pi^* \rightarrow^E Q^*$$

  - update after each episode

- Two problems
  - greedy policy won't explore all actions
  - Requires many independent episodes for the estimated value function to be accurate.

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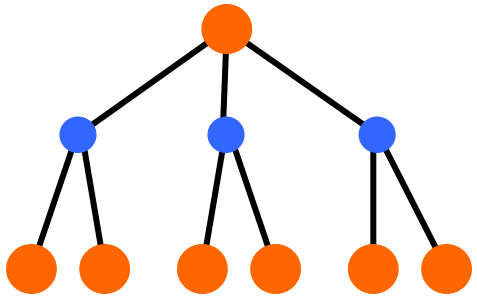

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# Improved Monte-Carlo Q-function estimate using Bellman equations

- Recall:

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MC estimate

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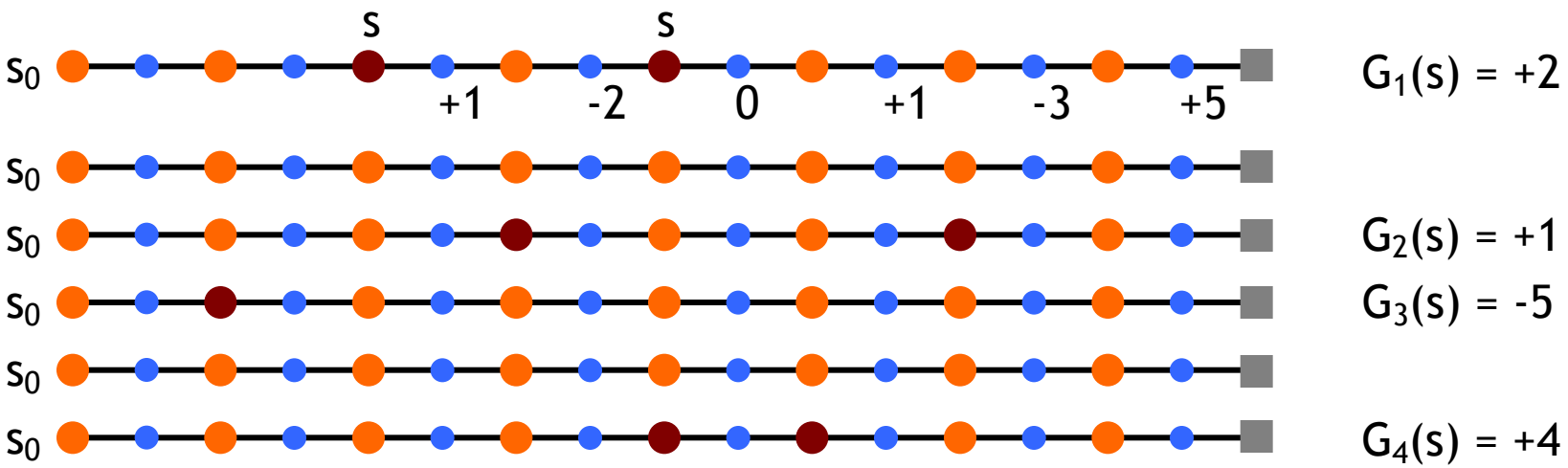
*No need to estimate P(s' | s,a) or r(s,a,s') as intermediate steps.

*Require only O(SA) space, rather than O(S^2A)

# Online averaging representation of MC




$V^{\pi}(s) \approx (2 + 1 - 5 + 4)/4 = 0.5$

$\bar{X}_t = \frac{1}{t}\sum_{i=1}^{t} X_i$ receive $X_{t+1}$

$\bar{X}_{t+1} = \frac{1}{t+1}\sum_{i=1}^{t+1} X_i = \frac{1}{t+1}\left(t\bar{X}_t + X_{t+1}\right) = \bar{X}_t + \frac{1}{t+1}X_{t+1} - \frac{1}{t+1}\bar{X}_t$

$\frac{1}{t+1}(X_{t+1} - \bar{X}_t)$

- Alternative, *online averaging* update

$$V(S_t) \leftarrow V(S_t) + \alpha\left[G_t - V(S_t)\right], \quad \text{where } \alpha = 1/N_{S_t}$$

# DP + MC = Temporal Difference Learning

- Monte Carlo

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$$\mathbb{E}_\pi[G_t] = \mathbb{E}_\pi[R_t|S_t] + \mathbb{E}_\pi[\gamma V^\pi(S_{t+1})]$$

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- TD-Policy evaluation

$(S_t, \pi(S_t), S_{t+1}, R_t)$ ← Data point

$$V(S_t) \leftarrow V(S_t) + \alpha\Big[R_{t+1} + \gamma V(S_{t+1}) - V(S_t)\Big]$$

Realized at time $t+1$ | your estimate of at time $t$

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**Bootstrapping!**

$$V(S_t) \leftarrow V(S_t) + \alpha\Big[R_{t+1} + \gamma V(S_{t+1}) - V(S_t)\Big]$$

# Bootstrap's origin

- "The Surprising Adventures of Baron Münchausen"
  - Rudolf Erich Raspe, 1785


**PULL YOURSELF UP BY THE BOOT STRAPS!!!**

- In statistics: Brad Efron's resampling methods
- In computing: Booting…
- In RL: It simply means TD learning

# TD policy optimization (TD-control )

- SARSA (On-Policy TD-control)
  - Update the Q function by bootstrapping Bellman Equation

$$Q(S,A) \leftarrow Q(S,A) + \alpha\left[R + \gamma Q(S',A') - Q(S,A)\right]$$

  - Choose the next A' using Q, e.g., eps-greedy.

- Q-Learning (Off-policy TD-control)
  - Update the Q function by bootstrapping Bellman Optimality Eq.

$$Q(S,A) \leftarrow Q(S,A) + \alpha\left[R + \gamma \max_a Q(S',a) - Q(S,A)\right]$$

  - Choose the next A' using Q, e.g., eps-greedy, or any other policy.

Remarks:

- These are **proven to converge** asymptotically.
- Much more data-efficient in practice, than MC.
- Regret analysis is still active area of research.

# Advantage of TD over Monte Carlo

- Given a trajectory, a roll-out, of T steps.
  - MC updates the Q function only once
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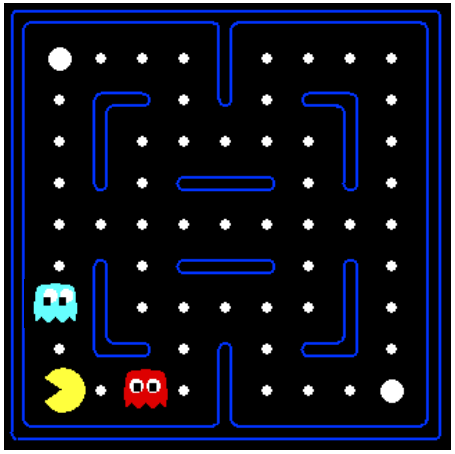
(Optional reading: Sutton and Barton 9.3 Semi-gradient methods)

# The problem of large state-space is still there

- We need to represent and learn SA parameters in Q-learning and SARSA.
- S is often large
  - 9-puzzle, Tic-Tac-Toe: 9! = 362,800, S^2 = 1.3*10^11
  - PACMAN with 20 by 20 grid. S = O(2^400), S^2 = O(2^800)
- O(S) is not acceptable in some cases.
- Need to think of ways to "generalize"/share information across states.

# Example: Pacman

Let's say we discover through experience that this state is bad:



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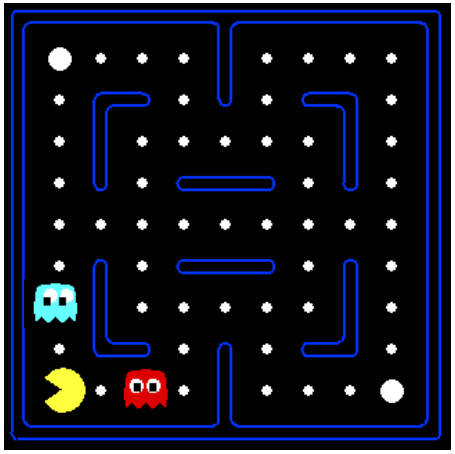
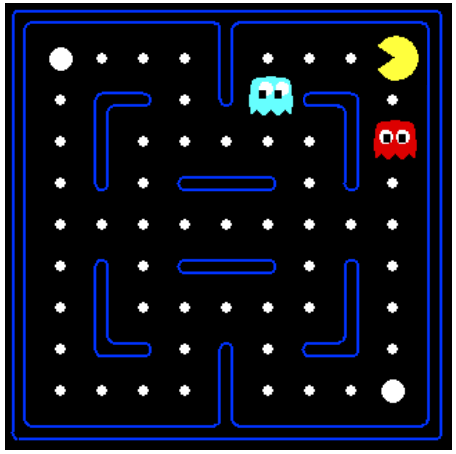


In naïve q-learning, we know nothing about this state:



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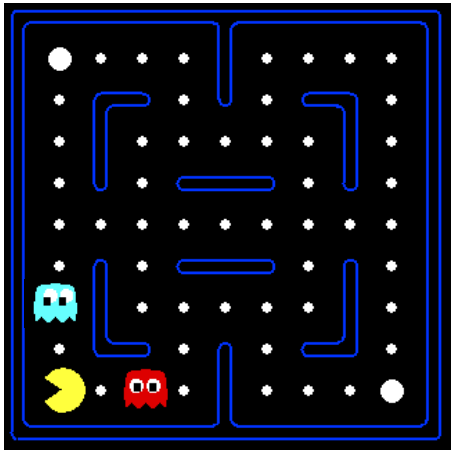

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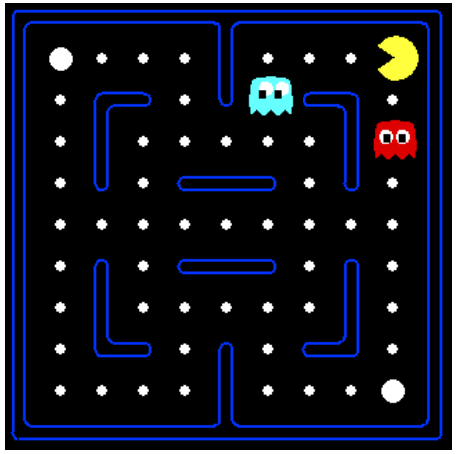
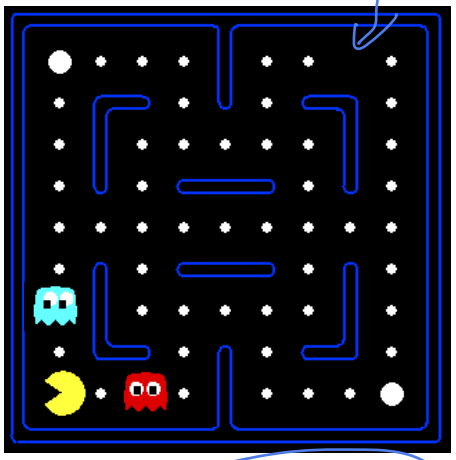


Or even this one!





(From Dan Klein and Pieter Abbeel)

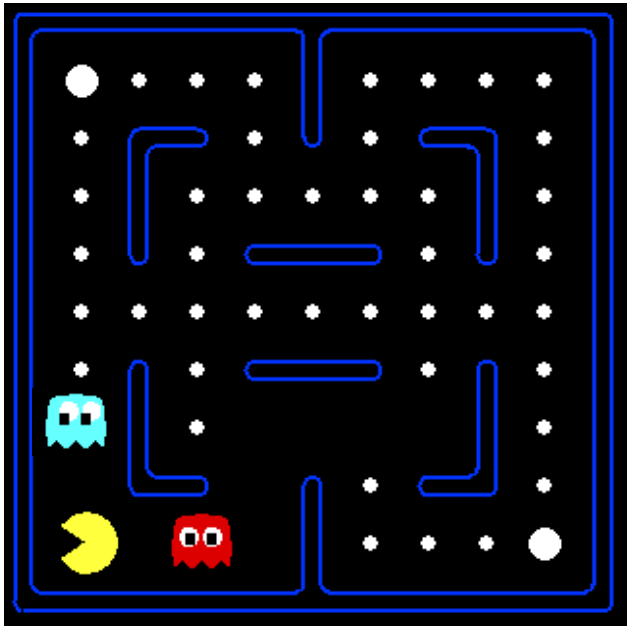
# Video of Demo Q-Learning Pacman – Tiny – Watch All

# Video of Demo Q-Learning Pacman – Tiny – Silent Train

# Video of Demo Q-Learning Pacman – Tricky – Watch All

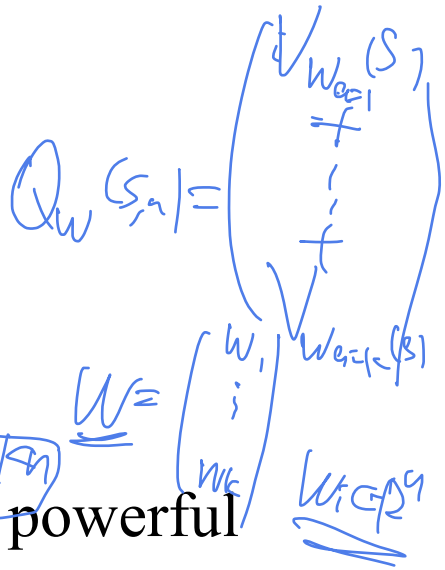
# Why not use an evaluation function? A Feature-Based Representations

- Solution: describe a state using a vector of features (properties)
  - Features are functions from states to real numbers (often 0/1) that capture important properties of the state
  - Example features:
    - Distance to closest ghost
    - Distance to closest dot
    - Number of ghosts
    - 1 / (dist to dot)$^2$
    - Is Pacman in a tunnel? (0/1)
    - …… etc.
    - Is it the exact state on this slide?
  - Can also describe a q-state (s, a) with features (e.g. action moves closer to food)



# Linear Value Functions

- Using a feature representation, we can write a q function (or value function) for any state using a few weights:
  - $V_{\mathbf{w}}(s) = w_1 f_1(s) + w_2 f_2(s) + \ldots + w_n f_n(s)$
  - $Q_{\mathbf{w}}(s,a) = w_1 f_1(s,a) + w_2 f_2(s,a) + \ldots + w_n f_n(s,a)$
- Advantage: our experience is summed up in a few powerful numbers
- Disadvantage: states may share features but actually be very different in value!



# Updating a linear value function

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- Original Q learning rule tries to reduce prediction error at s, a:

$$Q(s,a) \leftarrow Q(s,a) + \alpha \cdot [R(s,a,s') + \gamma \max_{a'} Q(s',a') - Q(s,a)]$$

# Updating a linear value function

$$\langle a, b \rangle = a^T b = \sum_{i=1}^{n} a_i b_i$$

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$$\langle w, f(s,a) \rangle \qquad \langle w, f(s,a) \rangle$$

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$$w_i \leftarrow w_i + \alpha \cdot [R(s,a,s') + \gamma \max_{a'} Q(s',a') - Q(s,a)] \partial Q_{\mathbf{w}}(s,a)/\partial w_i$$
$$= w_i + \alpha \cdot [R(s,a,s') + \gamma \max_{a'} Q(s',a') - Q(s,a)] f_i(s,a)$$

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- Qualitative justification:
  - Pleasant surprise: increase weights on positive features, decrease on negative ones
  - Unpleasant surprise: decrease weights on positive features, increase on negative ones

# Q-Learning with function approximation

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Exact Q's

# Q-Learning with function approximation

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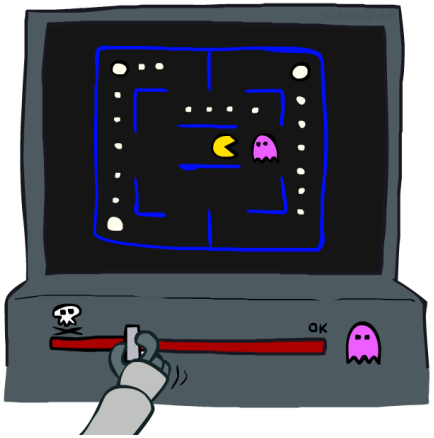

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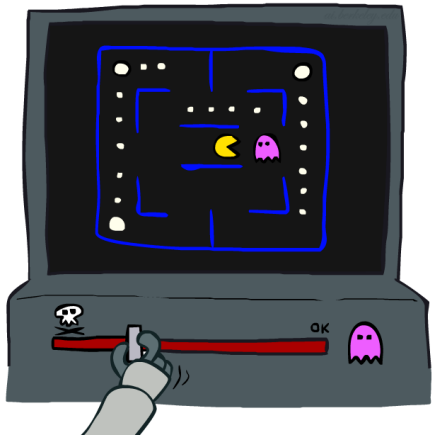

- Intuitive interpretation:
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- Formal justification: online least squares (Read the textbook!)

# PACMAN Q-Learning (Linear function approx.)

# So far, in RL algorithms

- Model-based approaches
  - Estimate the MDP parameters.
  - Then use policy-iterations, value iterations.
- Monte Carlo methods:
  - estimating the rewards by empirical averages
- Temporal Difference methods:
  - Combine Monte Carlo methods with Dynamic Programming
- Linear function approximation in Q-learning
  - Similar to SGD
  - Learning heuristic function

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*Question: What is the policy class $\Pi$ of interest in these methods?

# Policy gradient

- Let's not worry about states, dynamics, Q function.
  - We might not even observe the true state.
  - Let's specify a class of parametrized policy and hope to compare to the best within this class

- Objective function to maximize: $J(\boldsymbol{\theta}) \doteq v_{\pi_{\boldsymbol{\theta}}}(s_0)$,

*Note how this theorem is non-trivial… The first two terms depends on $\pi$, but we did not take the gradient w.r.t. them.

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***Show that this is an unbiased estimator of the gradient.**

# The REINFORCE algorithm (Williams, 1987)

**REINFORCE, A Monte-Carlo Policy-Gradient Method (episodic)**

Input: a differentiable policy parameterization $\pi(a|s,\boldsymbol{\theta})$
Initialize policy parameter $\boldsymbol{\theta} \in \mathbb{R}^{d'}$
Repeat forever:
  Generate an episode $S_0, A_0, R_1, \ldots, S_{T-1}, A_{T-1}, R_T$, following $\pi(\cdot|\cdot,\boldsymbol{\theta})$
  For each step of the episode $t = 0, \ldots, T-1$:
    $G \leftarrow$ return from step $t$
    $\boldsymbol{\theta} \leftarrow \boldsymbol{\theta} + \alpha\gamma^t G \nabla_{\boldsymbol{\theta}} \ln \pi(A_t|S_t,\boldsymbol{\theta})$

- From Sutton and Barto Ch .13.
- Note the $\gamma^t$ term. This is for the discounted (episodic) case
- Updating the parameter T times for each episode!
- Easy to implement easy to understand from SGD theory.

# Elements of State-of-the-Art Reinforcement Learning

- Use a deep neural network to parameterize Q-function
- Use a deep neural network to parameterize the policy \pi
- Run  a combination of Q-learning and Policy Gradient.
  - Actor-Critics, A3C, etc…
- Heuristic-based exploration:  curiosity, reward shaping, etc..
- Experience replay to generate more data from existing data.
- Multi-agent RL: modeling your opponents

# Example of State-of-the-Art RL for Hide-n-Seek

Lowe, Wu et al. (2017)
https://proceedings.neurips.cc/paper/2017/hash/68a9750337a418a86fe06c1991a1d64c-Abstract.html

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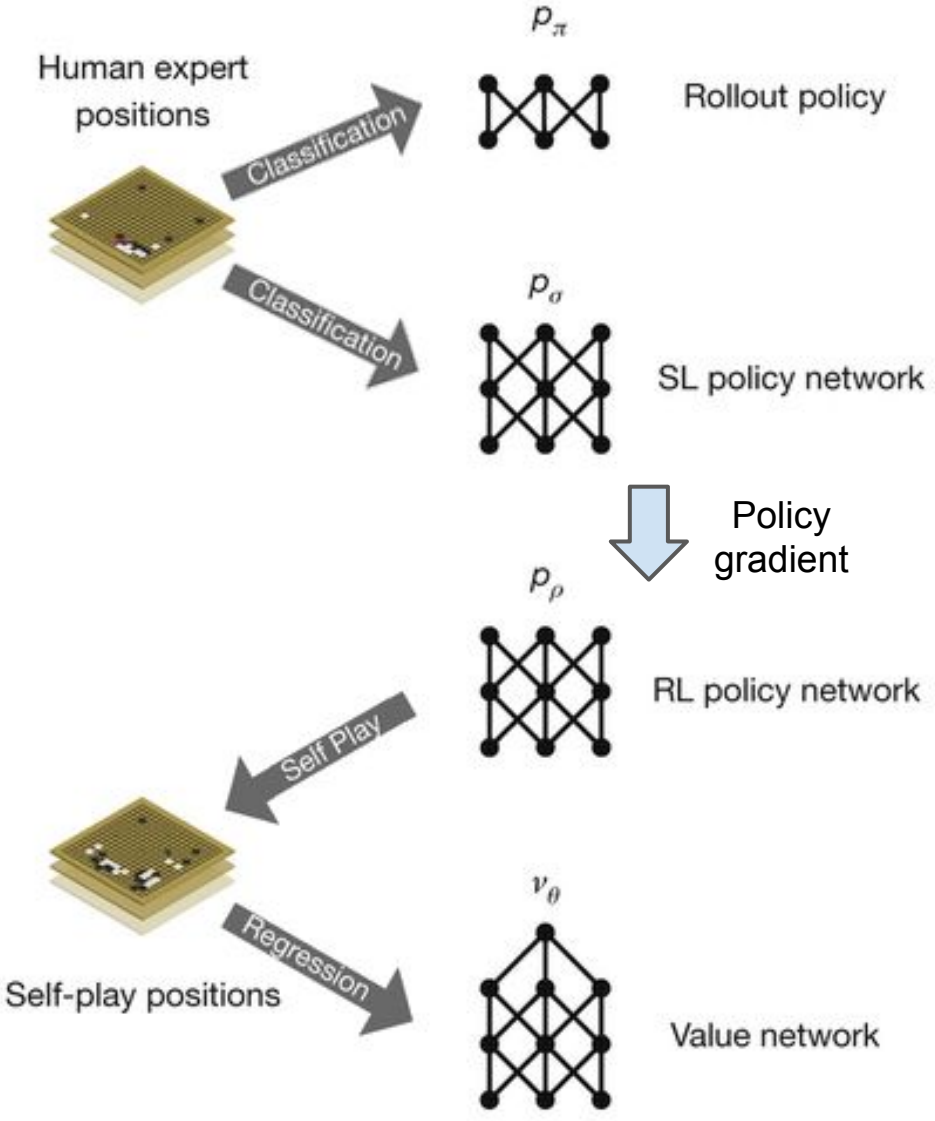
# Alpha-Go and Alpha-Zero

- Parameterize the policy networks with CNN
- Supervised learning initialization
- RL using Policy gradient
- Fit Value Network (This is a heuristic function!)
- Monte-Carlo Tree Search

https://www.youtube.com/watch?v=4D5yGiYe8p4

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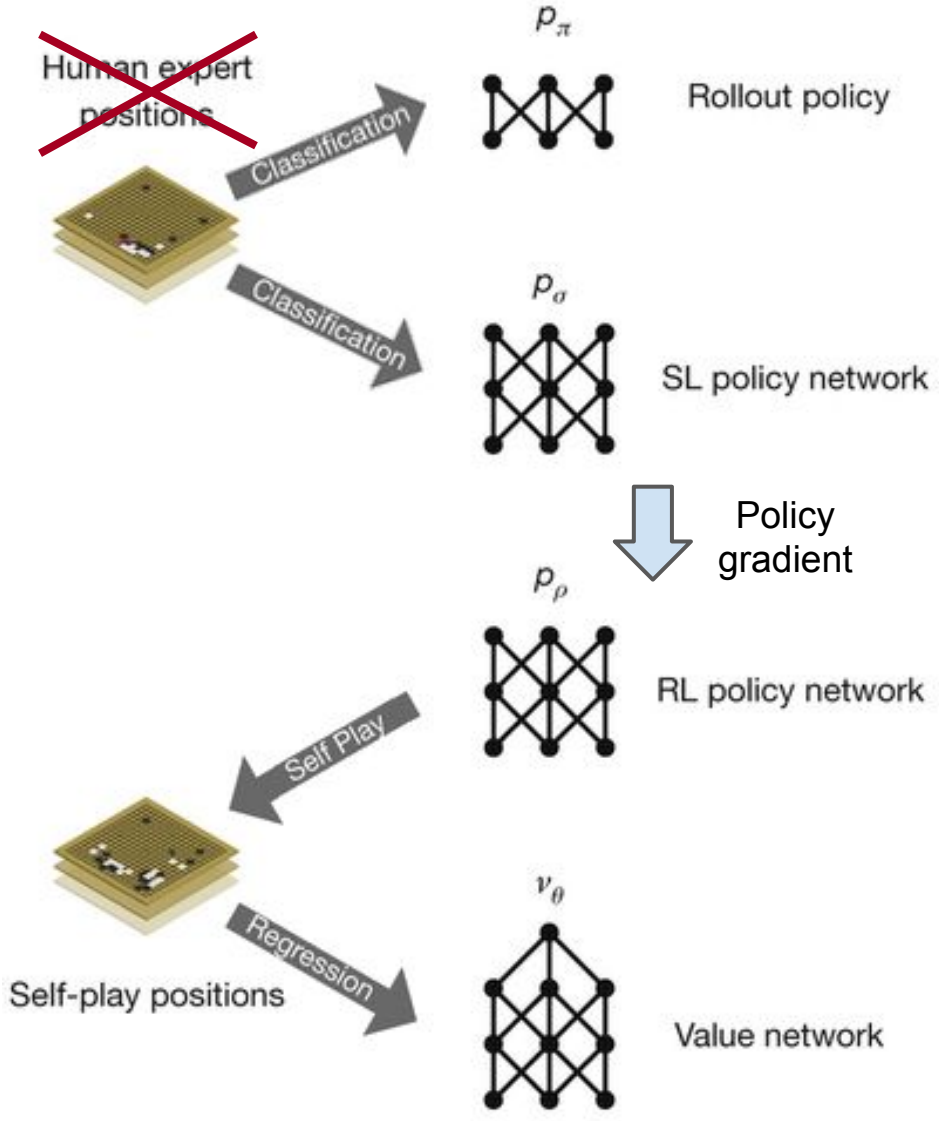


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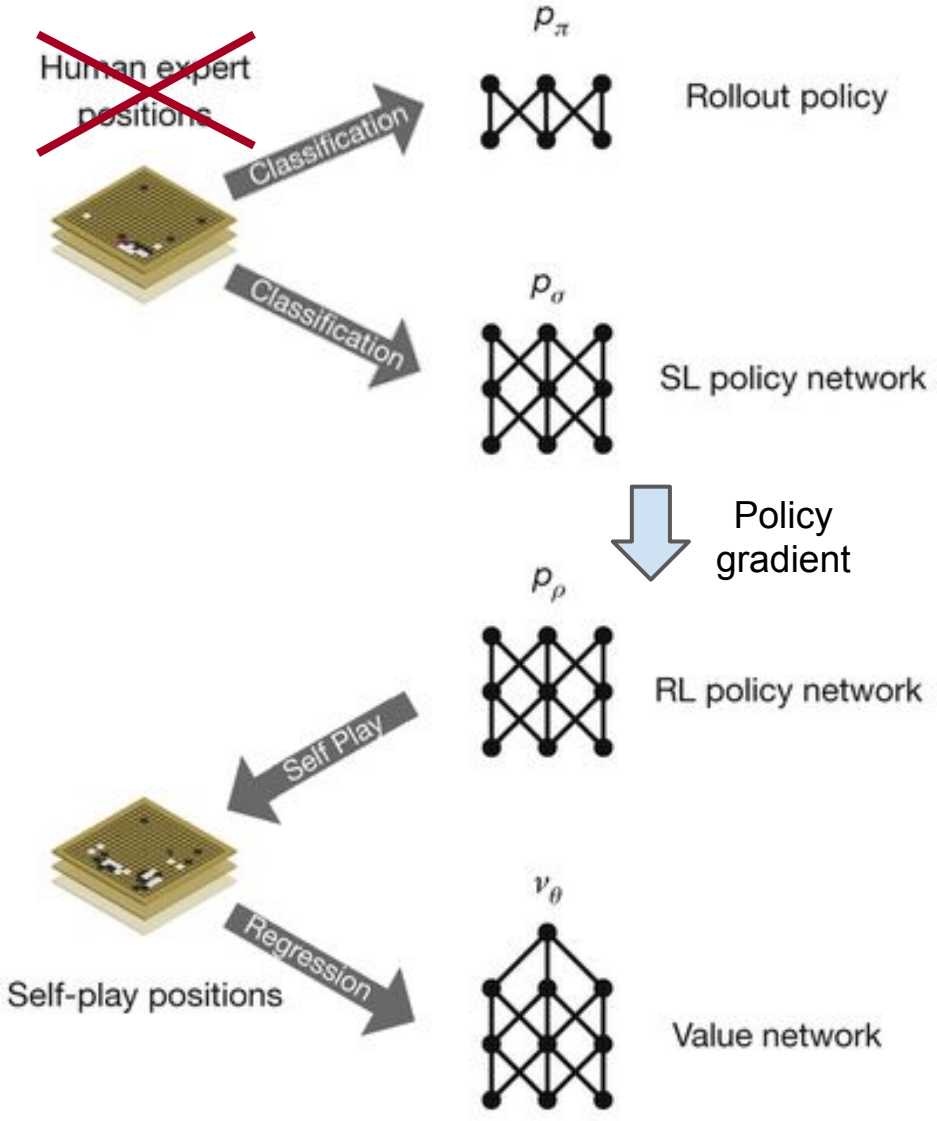


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D. Silver, et al. "Mastering the game of go without human knowledge." *Nature* 550.7676 (2017): 354-359.

# Summary of RL algorithms

- Model-based:
  - Policy iteration / Value iteration
  - Need to estimate the dynamics (MDP parameters)

- Model-free: (no need to "explicitly" estimate dynamics)
  - TD learning: SARSA, Q-learning
  - Function approximation (Share information across states)

- Absolutely model-free (do not even need an MDP model)
  - Policy gradient

# Next lecture

- Start Logical Agent
- Logical inference for propositional logic
- What you should do:
  - Read Chapter 7 of AIMA textbook.
  - Start working on Project 3 if you haven't yet.