Artificial Intelligence

CS 165A

May 23, 2022

Instructor: Prof. Yu-Xiang Wang

Today

→ Exploration (Part 2)

→ Reinforcement Learning

Notes

- Instructor OH is on 3pm Tuesday from this point onwards!
- Project 3 (Learning PACMAN) on the course website!
  - The MDP part (Q1,Q2,Q3) is already ready for you to complete
  - The RL algorithm part (Q6 onwards this week's lectures / next Tuesday)
- Grading for leaderboard / Report for Project 1 almost done
- Grades for Project 2 published on Gradescope
  - You may still submit for partial credits
  - Challenge the autograder, request TA for a manual evaluation if you believe something is wrong with the autograder.

Recap: Multi-arm bandits: Problem setup

- No state. k-actions $a \in \mathcal{A} = \{1, 2, ..., k\}$
- You decide which arm to pull in every iteration

$$A_1, A_2, ..., A_T$$

- You collect a cumulative payoff of $\sum_{t=1}^{T} R_t$
- The goal of the agent is to maximize the expected payoff.
  - For future payoffs?
  - For the expected cumulative payoff?

Recap: How do we measure the performance of an **online learning agent**?

- The notion of "Regret":
  - I wish I have done things differently.
  - Comparing to the best actions in the hindsight, how much worse did I do.

- For MAB, the regret is defined as follow

$$T \max_{a \in [k]} \mathbb{E}[R_t|a] - \sum_{t=1}^{T} \mathbb{E}_{a \sim \pi} \left[\mathbb{E}[R_t|a] \right]$$

Recap: MAB Algorithms

- Idea: Plug-in estimate of the reward value
- Greedy: Regret = O(T)
- Explore-first: Regret = O($T^{2/3}$)
- More algorithms today

Recap: Hoeffding's inequality and how quickly empirical averages converge to the mean.

- Statistics is about using **samples** from a distribution to infer the properties of the distribution itself (**population**)
  - $X_1, X_2, X_3, \ldots, X_n \sim P$

- **(Simplified version of the) Hoeffding's Inequality**:
  - with high probability,

$$\left| \frac{1}{n} \sum_{i=1}^{n} X_i - \mathbb{E}[X_1] \right| = O(1/\sqrt{n})$$

*If you try each arm multiple times, then you have a good idea what the expected reward is.

This lecture

- More algorithms for exploration under multi-armed bandits
- Contextual bandits
- Reinforcement Learning algorithmss

Recap: How does Explore First work?

- First try out all arms each for N/k times.

- Then commit to one of the arm

- Key message: You *per-step* regret is proportional to how well you can estimate the value of that arm.

ε-Greedy strategy: one way to balance exploration and exploitation

- You choose with probability 1- ε

$$A_t \doteq \arg\max_a Q_t(a),$$

- With probability ε, choose an action uniformly at random!
  - Including the argmax.

- Carefully choose ε parameter.

Let's analyze ε-Greedy!

- By Hoeffding's inequality, at every t
  - w.h.p. each arm is chosen at least 0.5 ε t / k times.
  - w.h.p.,

$$Q_t(a) \approx q_*(a) \pm C\sqrt{k/\epsilon t}$$

- Regret bound is

$$\epsilon T + \sum_{t=1}^{T} C\sqrt{\frac{k}{\epsilon t}}$$

Convergent and divergent series

$$1 + 1/2^2 + 1/3^2 + ... + 1/T^2 = ?$$

$$1 + 1/2 + 1/3 + ... + 1/T = ?$$

$$1 + \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{3}} + ... + \frac{1}{\sqrt{T}} = ?$$

On the side, check out Riemann's Zeta Function…

Let's analyze ε-Greedy!

- Regret bound

$$\epsilon T \quad + \sum_{t=1}^{T} C\sqrt{\frac{k}{\epsilon t}} \leq \epsilon T + O\left(\sqrt{\frac{Tk}{\epsilon}}\right)$$

- Choose the optimal ε to minimize the bound?

- Work it out yourself that you get T^{2/3} just like in Explore-First.

Upper Confidence Bound algorithm (UCB)

- At time t, choose the action

$$A_t \leftarrow \underset{a}{\operatorname{argmax}} \left[Q_t(a) + c\sqrt{\frac{\log(1+t)}{N_t(a)}} \right]$$

- Idea: Be optimistic
  - Choose an option that maximizes the upper confidence bound.

$$\mathbb{E}[\text{Regret}] = O(\sqrt{Tk})$$

- The proof is out of the scope of this course. For those who are interested, please look up. It's not difficult.

Example: Two-Armed Bandits

- k=2. Expected reward $r(a = 1) = 0.8,\ r(a = 2) = 0.5$

- Question: what is the Q-function for this problem?
  - If episodical with horizon T= 1.
  - If discounted with $0 \leq \gamma < 1$

- What is the optimal policy?

Example: UCB on a Two Armed Bandit

- A run of UCB algorithm with c = 2.
  1. Initialize $\mathrm{Q}_1(a) = \infty, \forall\, a \in \{1,2\}$. UCB: $\bar{Q}_1(a) = \infty$.
  2. Pick action $\mathrm{A}_1 = 1$. (break ties arbitrarily), receive a reward of 0.
     a. Update $N_2(a) \leftarrow N_1(a) + 1(\mathrm{A}_1 = \mathrm{a})$
     b. Update $\mathrm{Q}_2(1) = 0/1 = 0,\ Q_2(2) = \infty$
     c. Update $\bar{Q}_2(1) \leftarrow Q_2(1) + 2\sqrt{\frac{\log 2}{1}} \approx 0 + 1.67, \bar{Q}_2(2) = \infty$
  3. Pick action $a = 2$ (because of what?), receive a reward of 1.
     a. Update $N_3(2) \leftarrow N_2(2) + 1, \mathrm{Q}_3(2) \leftarrow 1$
     b. Update $\bar{Q}_3(2) \leftarrow Q_3(2) + 2\sqrt{\frac{\log 2}{1}} \approx 1 + 1.67$
     c. Keep $N_3(1), Q_3(1), \bar{Q}_3(1)$ unchanged.
  4. Which action to pick at next?
  5. …

A 10-armed bandits benchmark

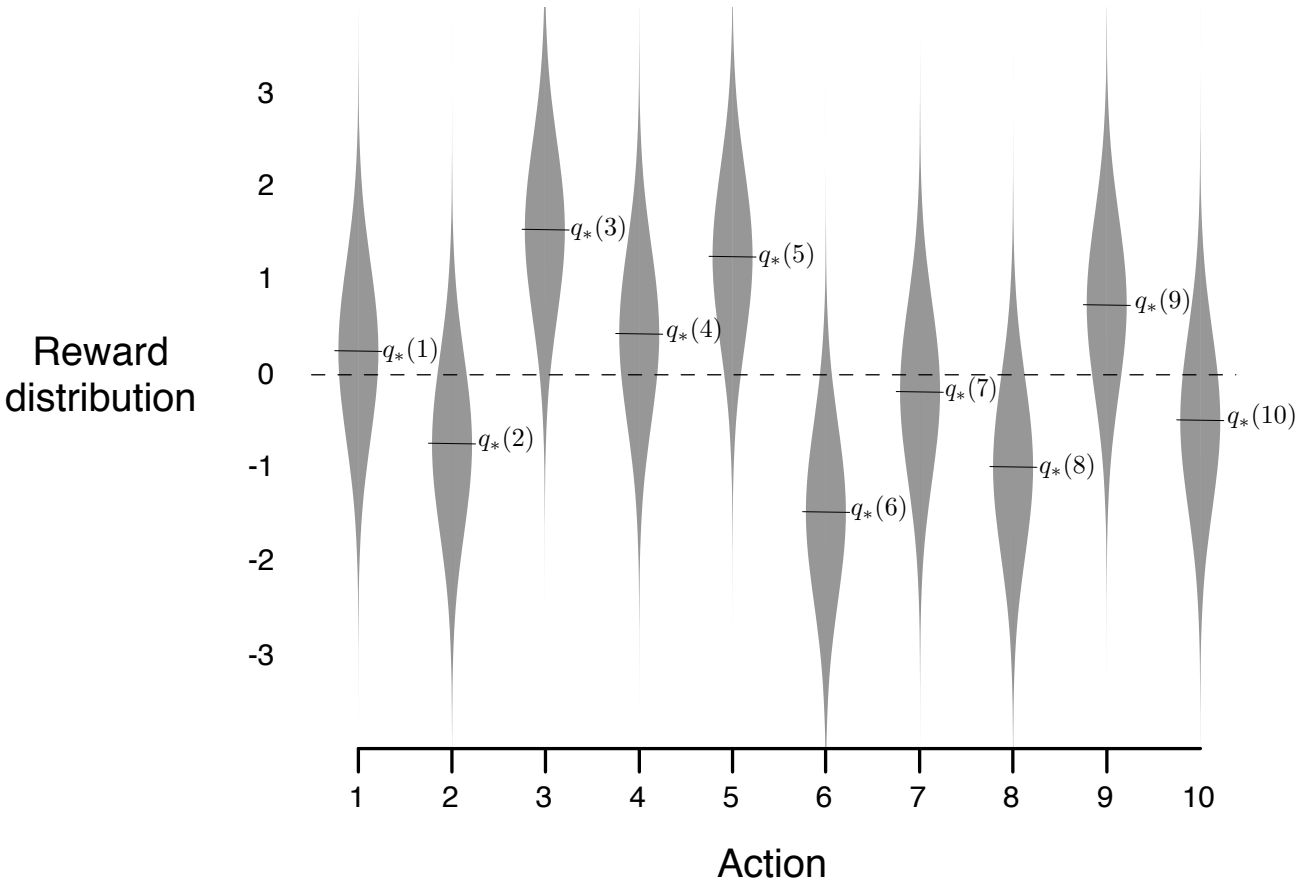
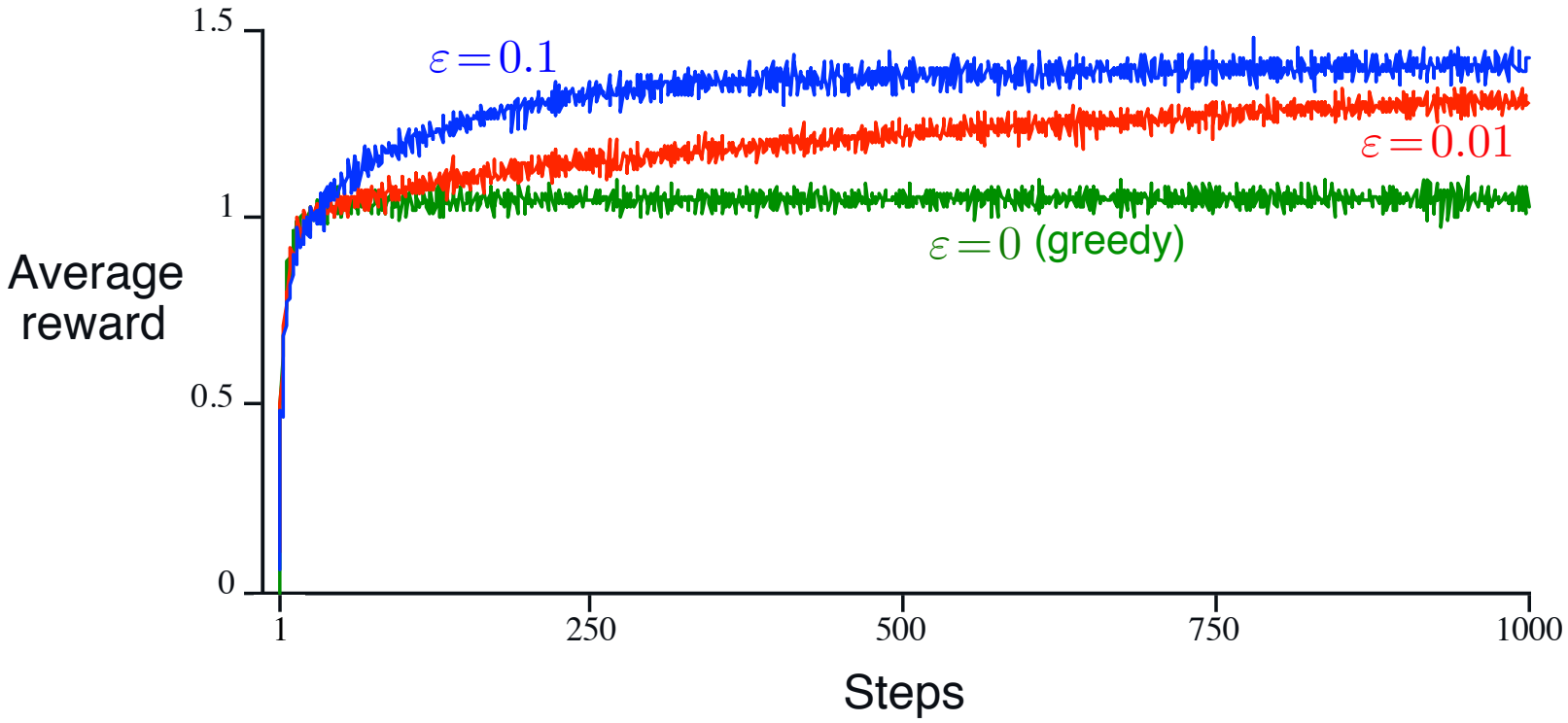
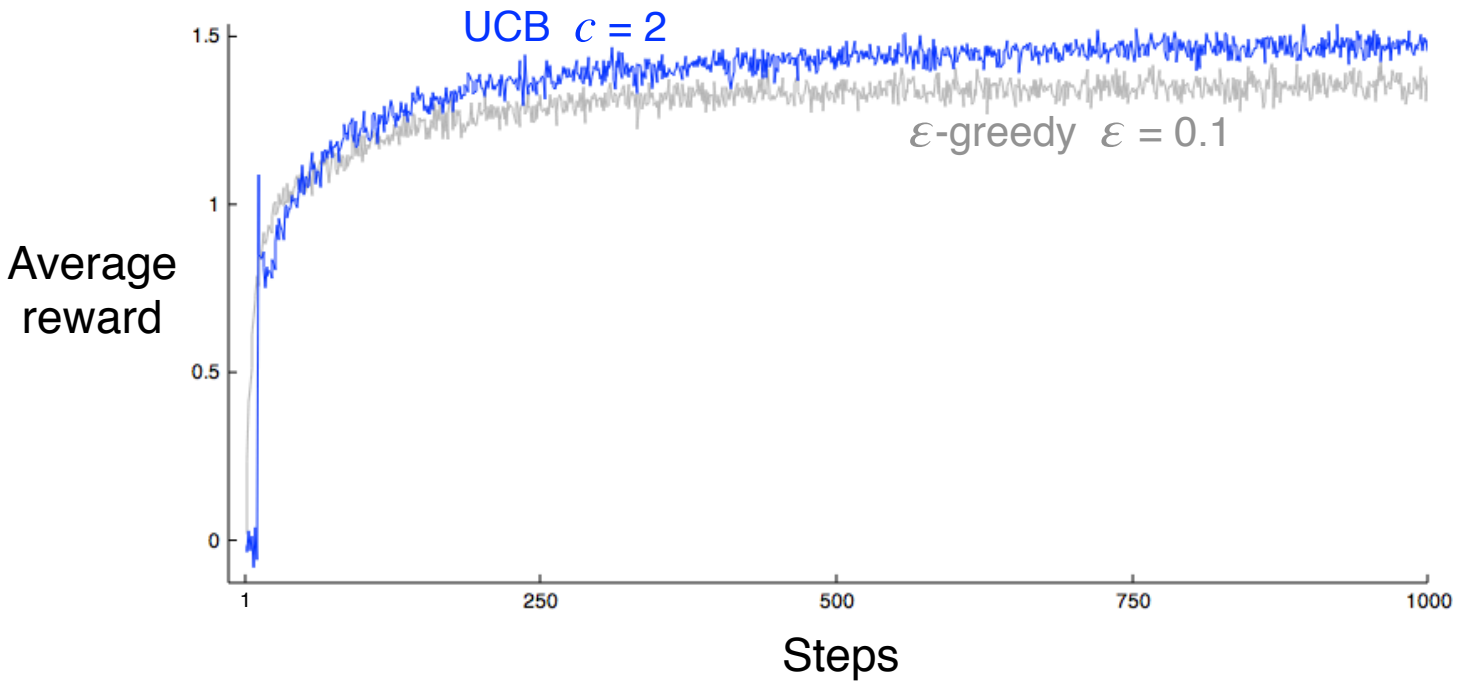


Figure 2.1: An example bandit problem from the 10-armed testbed. The true value $q_*(a)$ of each of the ten actions was selected according to a normal distribution with mean zero and unit variance, and then the actual rewards were selected according to a mean $q_*(a)$ unit variance normal distribution, as suggested by these gray distributions.

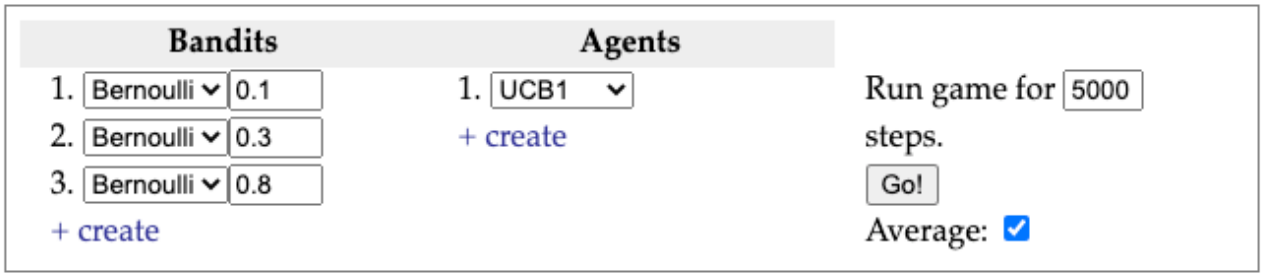
Comparing the different algorithms

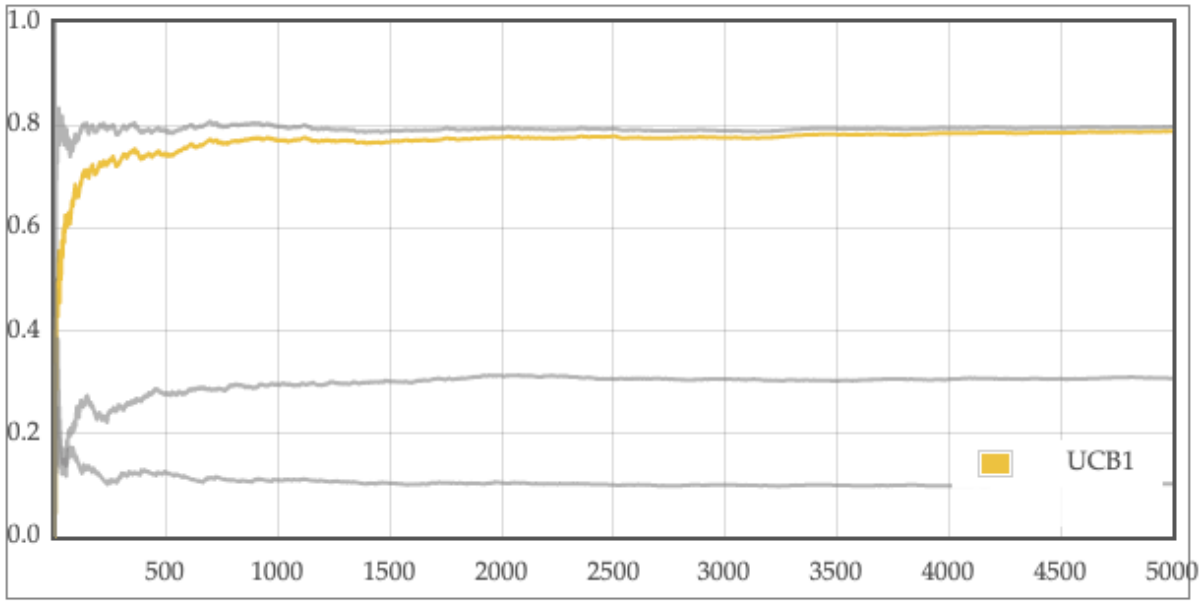


UCB vs. ε-Greedy



Live Demo of an Agent Solving Multi-Armed Bandits





Credit Mark Reid: http://mark.reid.name/code/bandits/

Variants of Bandits problems

- Online Learning from Expert Advice
  - Adversarial chooses the outcome
  - You observe outcome of other arms as well
  - Compare against the best arm in the hindsight

Remark: In all these problems, there are algorithms with provably low-regret.

- Adversarial k-Armed Bandits
  - Same as above. But you observe only your arm.

- Nonstationary Bandits
  - Stochastic but the reward distribution changes over time.
  - Compare against the best arm for each time.

- Contextual bandits: you have a state in each time point.

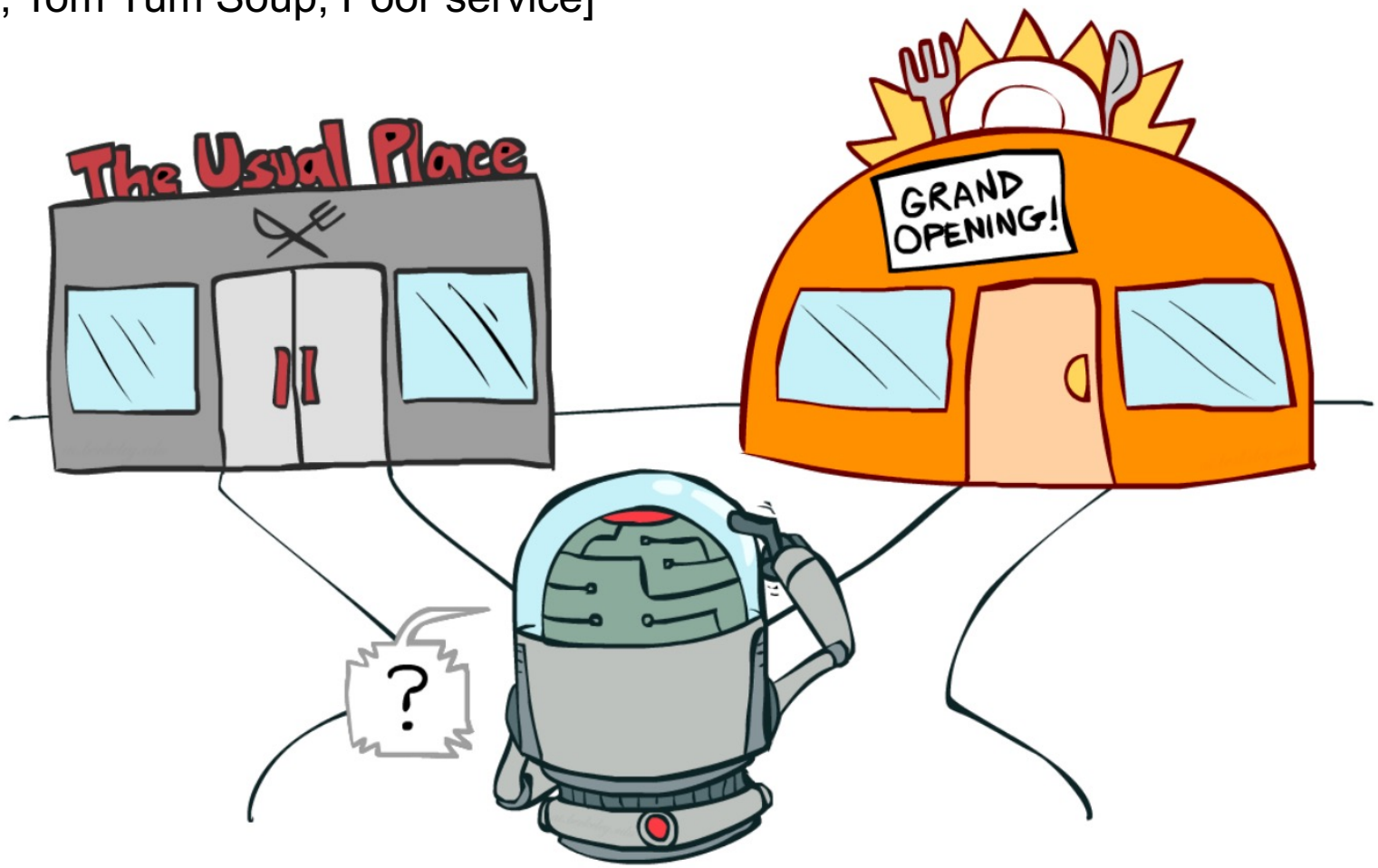
Do I have to try, if I have features?

Features:

[Noodles, Tom Yum Soup, Poor service]

Features:

[Burger, Fries, Onion Ring, Fried Chicken]



(Illustration from Dan Klein and Pieter Abbeel's course in UC Berkeley)

We know how to use with features, don't we?

- Classifier agent
  - Take features of a restaurant as input
  - Output a prediction of "will I like the food?"
- Train with supervised learning
  - Using the my previous visits to the restaurants
  - Using Yelp reviews

Why can't we just use that?

How to explore?

Contextual Bandits: Problem Setup

- For each round t = 1, 2, 3, …, T:
  - A context $x_t \sim$ unknown distribution i.i.d.
  - Agent picks an action $a_t = 1,2,3,..,K$
  - Reward $r_t \sim D(. \mid x_t, a_t)$

- Agent's goals:
  - Learn the best policy out of many policies Π
  - Minimize the cumulative regret

A finite family of policies

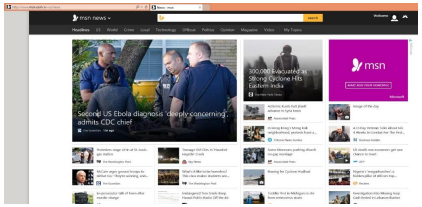
$$T \cdot \max_{\pi \in \Pi} \mathbb{E}_{\pi}[r_t(x_t, a_t)] - \mathbb{E}_{\text{Agent's policy}} \left[\sum_{t=1}^{T} r_t(x_t, a_t) \right]$$

Reward from the best policy

Reward collected by the Agent

Applications of Contextual Bandits

Personalized news?

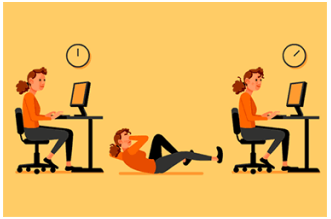


Repeatedly:

1. Observe features of user+articles
2. Choose a news article.
3. Observe click-or-not

Goal: Maximize fraction of clicks

Health advice?



Repeatedly:

1. Observe features of user+advice
2. Choose an advice.
3. Observe steps walked

Goal: Healthy behaviors (e.g. step count)

amazon

Recommendations

buy or not buy

Exploration vs. Exploitation in Contextual Bandits.

- Challenging because:
  - Infinite state space, never see the same context again.
  - Exponentially large policy space
- Ideas:
  - ExploreFirst, ε-Greedy $O(T^{2/3})$
  - UCB? But how do we construct Confidence Interval for an exponentially large set of policies?
- Optimal regret:

$$O(\sqrt{KT \log |\Pi|})$$

Remainder of the lecture today

- Reinforcement learning for MDPs
  - Model-based vs model-free algorithms
  - Online policy iterations
  - Temporal difference learning

- Readings:
  - AIMA Ch. 21.1-21.3 (Ch 22.1- 22.3 in 4th Edition)
  - Sutton and Barto: Ch 4-6
  - Maybe: Sutton and Barto: Ch 6, Ch 13

Let us tackle different aspects of the RL problem one at a time

- Markov Decision Processes:
  - Dynamics are given no need to learn
- Bandits: Explore-Exploit in simple settings
  - RL without dynamics
- **Full Reinforcement Learning**
  - Learning MDPs

Recap: Tabular MDP

- Discrete State, Discrete Action, Reward and Observation

$$S_t \in \mathcal{S} \quad A_t \in \mathcal{A} \quad R_t \in \mathbb{R} \quad \sout{O_t \in \mathcal{O}}$$

- Policy:
  - When the state is observable: $\pi : \mathcal{S} \rightarrow \mathcal{A}$
  - ~~Or when the state is not observable~~

$$\sout{\pi_t : (\mathcal{O} \times \mathcal{A} \times \mathbb{R})^{t-1} \rightarrow \mathcal{A}}$$

- Learn the best policy that maximizes the expected reward
  - Finite horizon (episodic) RL: $\pi^* = \arg\max_{\pi \in \Pi} \mathbb{E}[\sum_{t=1}^{T} R_t]$ T: horizon
  - Infinite horizon RL: $\pi^* = \arg\max_{\pi \in \Pi} \mathbb{E}[\sum_{t=1}^{\infty} \gamma^{t-1} R_t]$ γ: discount factor

Recap: Policy Iterations and Value Iterations

- What are these algorithms for?
  - Algorithms of computing the V* and Q* functions from MDP parameters

- Policy Iterations

$$\pi_0 \rightarrow^E V^{\pi_0} \rightarrow^I \pi_1 \rightarrow^E V^{\pi_1} \rightarrow^I \ldots \rightarrow^I \pi^* \rightarrow^E V^*$$

- Value iterations

$$V_{k+1}(s) \leftarrow \max_a \sum_{s'} P(s'|s,a)[r(s,a,s') + \gamma V_k(s')]$$

- How do we make sense of them?
  - Recursively applying the Bellman equations until convergence.

*These methods are called “Dynamic Programming” approaches in Chap 4 of Sutton and Barto.

Revisit the dynamic programming approach

- Policy Evaluation

$$V_{k+1}^{\pi}(s) \leftarrow \sum_{a} \pi(a|s) \sum_{s'} P(s'|s,a)[r(s,a,s') + \gamma V_k^{\pi}(s')]$$

- Policy improvement

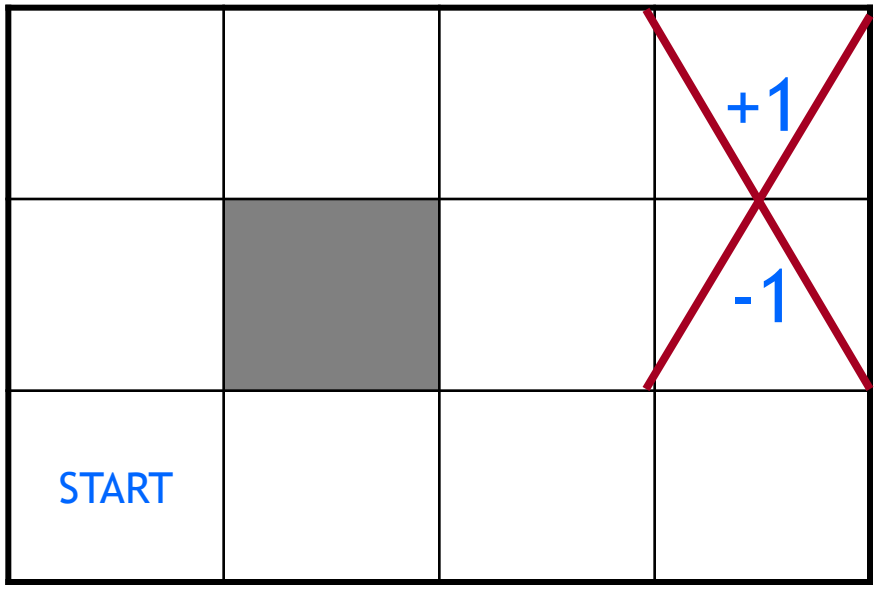
$$\pi'(s) = \arg\max_{a} Q^{\pi}(s,a)$$
$$= \arg\max_{a} \sum_{s'} P(s'|s,a)[r(s,a,s') + \gamma V_k^{\pi}(s')]$$

- Value iterations

$$V_{k+1}(s) \leftarrow \max_{a} \sum_{s'} P(s'|s,a)[r(s,a,s') + \gamma V_k(s')]$$

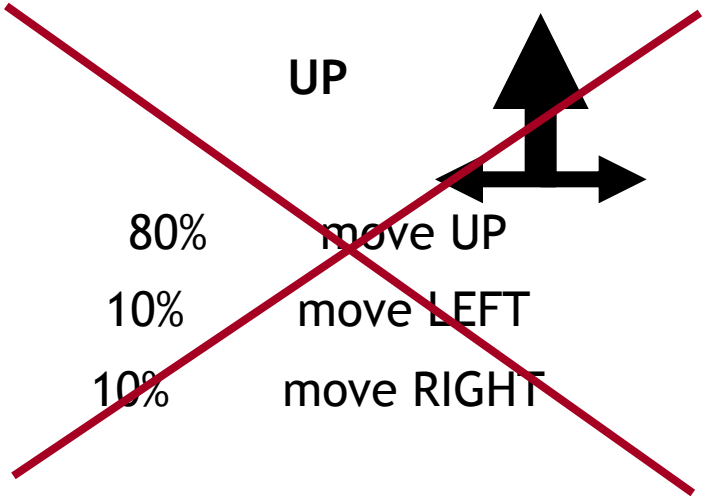
***We do not have the MDP parameters in RL!**

Example: Frozen Lake



Action 1, Action 2, Action 3, Action 4

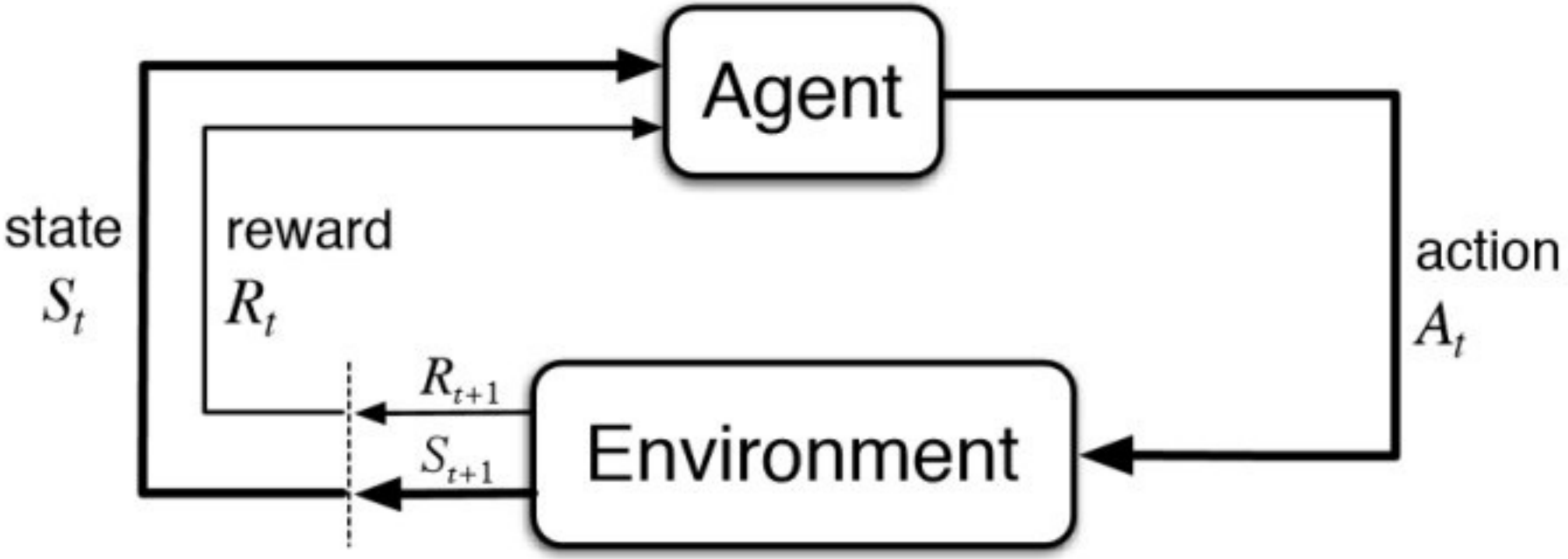
actions: ~~UP, DOWN, LEFT, RIGHT~~



- ~~reward +1 at [4,3], -1 at [4,2]~~
- ~~reward -0.04 for each step~~
- what's the strategy to achieve max reward?

Instead, reinforcement learning agents have "online" access to an environment

- State, Action, Reward
- Unknown reward function, unknown state-transitions.
- Agents can "act" and "experiment", rather than only doing offline planning.



Idea 1: **Model-based** Reinforcement Learning

- Model-based idea
  - Let's approximate the model based on experiences
  - Then solve for the values as if the learned model were correct

- Step 1: Get data by running the agent to explore
  - Many data points of the form:
    $\{(s_1, a_1, s_2, r_1), \ldots, (s_N, a_N, s_{N+1}, r_N)\}$

- Step 2: Estimate the model parameters
  - $\hat{P}(s'|s, a)$ --- again this is a CPT we need to observe the transition many times for each s, a
  - $\hat{r}(s', a, s)$ --- this is an estimate of the empirical rewards.

Then we can plug in these estimates and then use dynamic programming for policy evaluation / improvements.

$$V_{k+1}^{\pi}(s) \leftarrow \sum_{a} \pi(a|s) \sum_{s'} \hat{P}(s'|s,a)[\hat{r}(s,a,s') + \gamma V_k^{\pi}(s')]$$

$$\pi' \leftarrow \arg\max_{a} \sum_{s'} \hat{P}(s'|s,a)[\hat{r}(s,a,s') + \gamma V_k^{\pi}(s')]$$

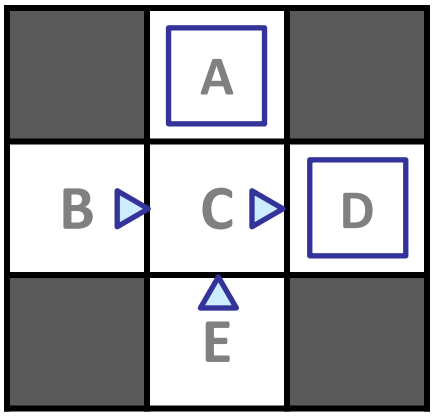
$$V_{k+1}(s) \leftarrow \max_{a} \sum_{s'} \hat{P}(s'|s,a)[\hat{r}(s,a,s') + \gamma V_k(s')]$$

* Note the **"hat".** Usually it indicates empirical estimates.

* These iterations will produce $\hat{V}^*$ and $\hat{Q}^*$ functions, and then $\hat{\pi}^*$

Example: Model-Based RL (2 min exercise)

Input Policy π



Assume: $\gamma = 1$

Observed Episodes (Training)

Episode 1

B, east, C, -1

C, east, D, -1

D, exit, x, +10

Episode 2

B, east, C, -1

C, east, D, -1

D, exit, x, +10

Episode 3

E, north, C, -1

C, east, D, -1

D, exit, x, +10

Episode 4

E, north, C, -1

C, east, A, -1

A, exit, x, -10

Learned Model

$\hat{P}(s'|s,a)$

T(B, east, C) =
T(C, east, D) =
T(C, east, A) =
…

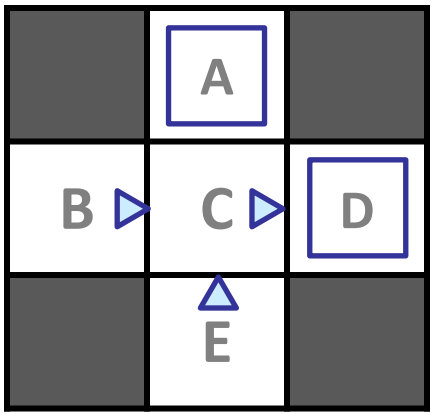
$\hat{r}(s,a,s')$

R(B, east, C) =
R(C, east, D) =
R(D, exit, x) =
…

Slide credits: Pat and Stephanie

Example: Model-Based RL (2 min exercise)

Input Policy π



Assume: $\gamma = 1$

Observed Episodes (Training)

Episode 1

B, east, C, -1
C, east, D, -1
D, exit, x, +10

Episode 2

B, east, C, -1
C, east, D, -1
D, exit, x, +10

Episode 3

E, north, C, -1
C, east, D, -1
D, exit, x, +10

Episode 4

E, north, C, -1
C, east, A, -1
A, exit, x, -10

Learned Model

$\hat{P}(s'|s,a)$

T(B, east, C) = 1.00
T(C, east, D) = 0.75
T(C, east, A) = 0.25
…

$\hat{r}(s,a,s')$

R(B, east, C) = -1
R(C, east, D) = -1
R(D, exit, x) = +10
…

Slide credits: Pat and Stephanie

This is simply the "Exploration-First" strategy! But there are complications.

- In bandits problems
  - Uniformly sample the actions for N rounds.
  - Guarantees that each choice is explored O(N/k) times.

- For MDPs
  - Often we need to take a carefully chosen sequence of actions to reach a state
  - The chance of randomly running into a state can be **exponentially small.**
  - **Question: What is an example of this?**

*Need to somehow update the "exploration policy" on the fly!

More caveats

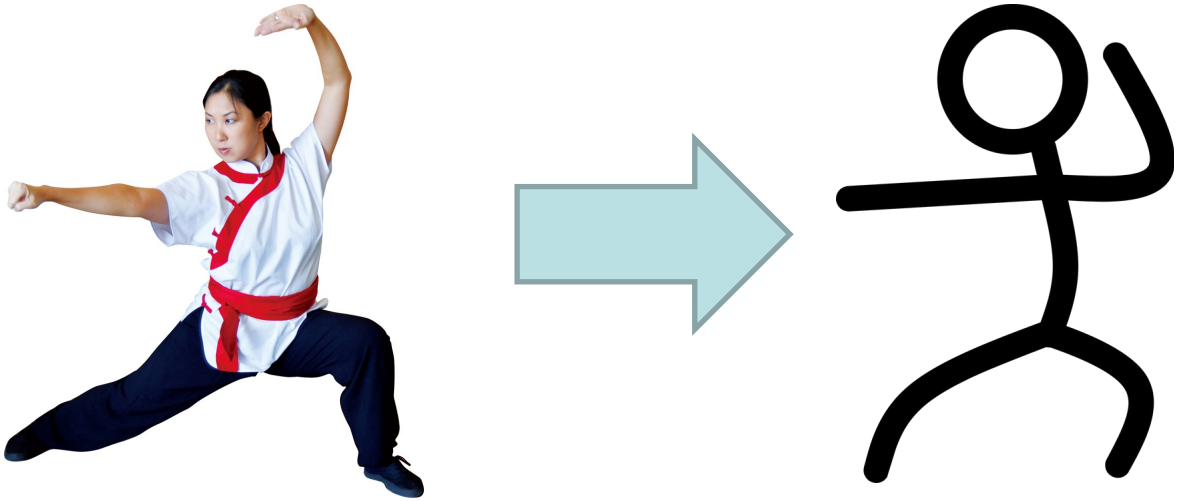
- The fitted model is just an approximation of the environment.
- How does the error in the fitted MDP translate into the error in the estimated value functions V* and Q*?
- How does the error in the estimated Q* function affect the suboptimality of the policy that maximizes $\hat{Q}^*$?
- Answered by "Simulation Lemma" (Kearns and Singh, 2002)
  - Resurgence of research on this more recently: Yin and W. (2020), Yin, Bai and W. (2020)

Even more caveats

- How many free parameters are there to represent an MDP?
  - Ans: O(S^2A)

- S is often large
  - 9-puzzle, Tic-Tac-Toe: 9! = 362,800, S^2 = 1.3*10^11
  - PACMAN with 20 by 20 grid. S = O(2^400), S^2 = O(2^800)

- In practice, we often have to use an approximate model.



Idea 2: **Model-free** Reinforcement Learning

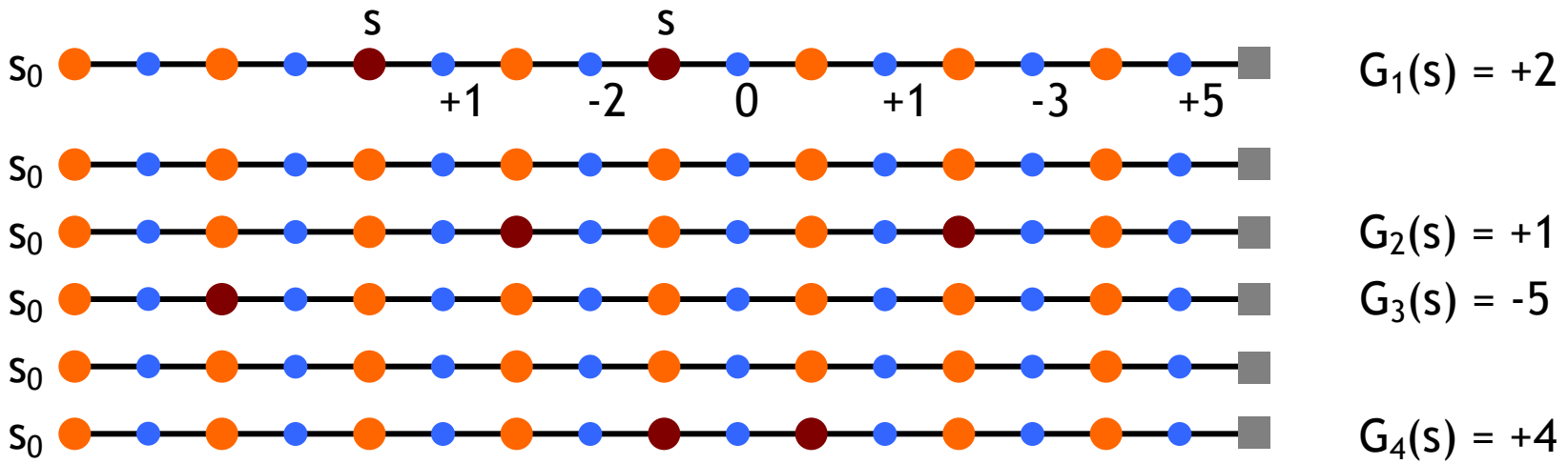
- Do we need the model? Can we learn the Q function directly?
  - **How many free parameters are there to represent the Q-function?**
  - **Ans: SA << O(S^2A)**
- Recall: Policy iterations

$$\pi_0 \rightarrow^E V^{\pi_0} \rightarrow^I \pi_1 \rightarrow^E V^{\pi_1} \rightarrow^I \ldots \rightarrow^I \pi^* \rightarrow^E V^*$$

  - Maybe we can do policy evaluation without estimating the model?

Monte Carlo Policy Evaluation (Prediction)

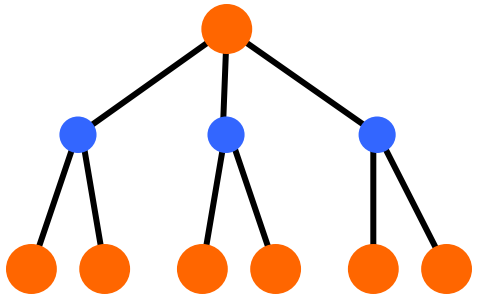
- want to estimate $V^{\pi}(s)$
  = expected return starting from s and following π
  – estimate as average of observed returns in state s
- We can execute the policy π
- first-visit MC
  – average returns following the first visit to state s



$V^{\pi}(s) \approx (2 + 1 - 5 + 4)/4 = 0.5$

Monte Carlo Policy Optimization (Control)

- V^π not enough for policy improvement
  - need exact model of environment



- estimate $Q^\pi(s,a)$

$$\pi'(s) = \arg\max_a Q^\pi(s,a)$$

- MC control

$$\pi_0 \rightarrow^E Q^{\pi_0} \rightarrow^I \pi_1 \rightarrow^E Q^{\pi_1} \rightarrow^I \ldots \rightarrow^I \pi^* \rightarrow^E Q^*$$

  - update after each episode

- Two problems
  - greedy policy won't explore all actions eps-greedy!
  - Requires many independent episodes for the estimated value function to be accurate.

Improved Monte-Carlo Q-function estimate using Bellman equations

- Recall:

$$Q^{\pi}(s,a) = \sum_{s'} P(s'|s,a)[r(s,a,s') + \gamma \sum_{a'} \pi(a'|s')Q^{\pi}(s',a')]$$

$$Q^{\pi}(s,a) = r^{\pi}(s,a) + \gamma \mathbb{E}_{s' \sim P(s'|s,a)}[V^{\pi}(s')]$$

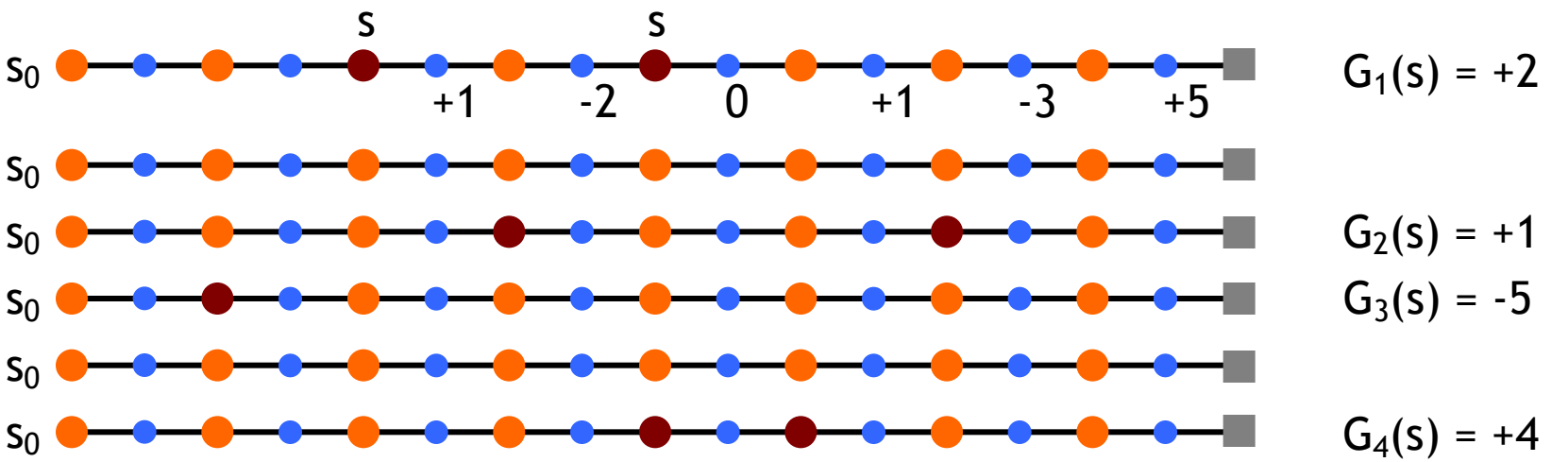
- We can use the empirical (Monte Carlo) estimate.

$$\widehat{Q^{\pi}}(s,a) = \widehat{r^{\pi}}(s,a) + \gamma \widehat{\mathbb{E}}_{s' \sim P(s'|s,a)}[\widehat{V}^{\pi}(s')]$$

*No need to estimate P(s' | s,a) or r(s,a,s') as intermediate steps.

*Require only O(SA) space, rather than O(S^2A)

Online averaging representation of MC



$V^{\pi}(s) \approx (2 + 1 - 5 + 4)/4 = 0.5$

- Alternative, *online averaging* update

$$V(S_t) \leftarrow V(S_t) + \alpha\Big[G_t - V(S_t)\Big], \quad \text{where } \alpha = 1/N_{S_t}$$

So far, in RL algorithms

- Model-based approaches
  - Estimate the MDP parameters.
  - Then use policy-iterations, value iterations.

- Monte Carlo methods:
  - estimating the rewards by empirical averages

Next lecture

- Wrap up RL lectures
  - Temporal difference methods and boostrapping
  - Linear function approximation and Q-learning