Artificial Intelligence

CS 165A

Nov 10, 2020

Instructor: Prof. Yu-Xiang Wang

Today

→ Intro to RL

→ Markov Decision Processes

Announcement

- The TAs are still grading the midterm.

- We are hoping to release your midterm grades on Thursday.

- No discussion class this week.

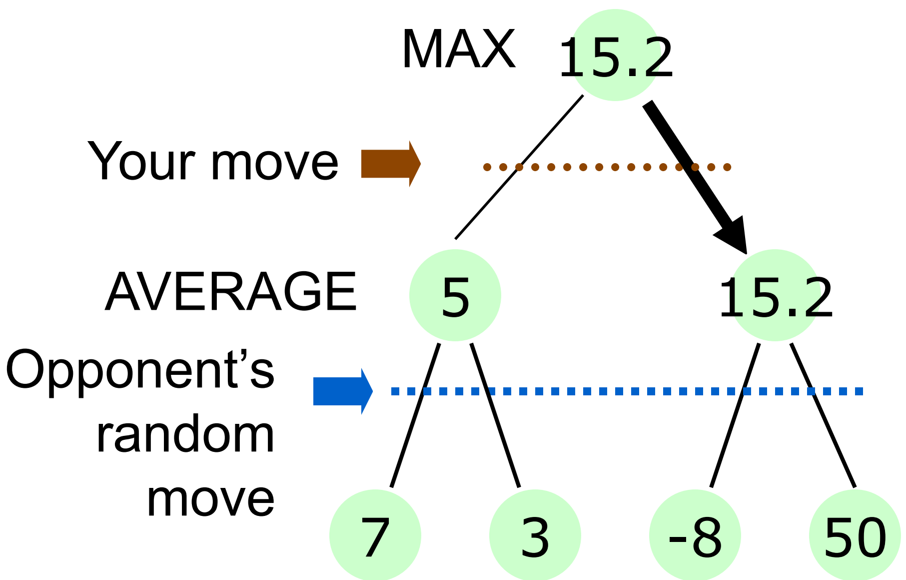
Announcement

- HW3 released last Thursday.

- Topics covered includes
  - Game playing
  - Markov Decision processes

- Programming question:
  - Solve PACMAN with ghosts moving around.

Recap: Expectimax



- Your opponent behave randomly with a given probability distribution,
- If you move left, your opponent will select actions with probability [0.5,0.5]
- If you move right, your opponent will select actions with [0.6,0.4]

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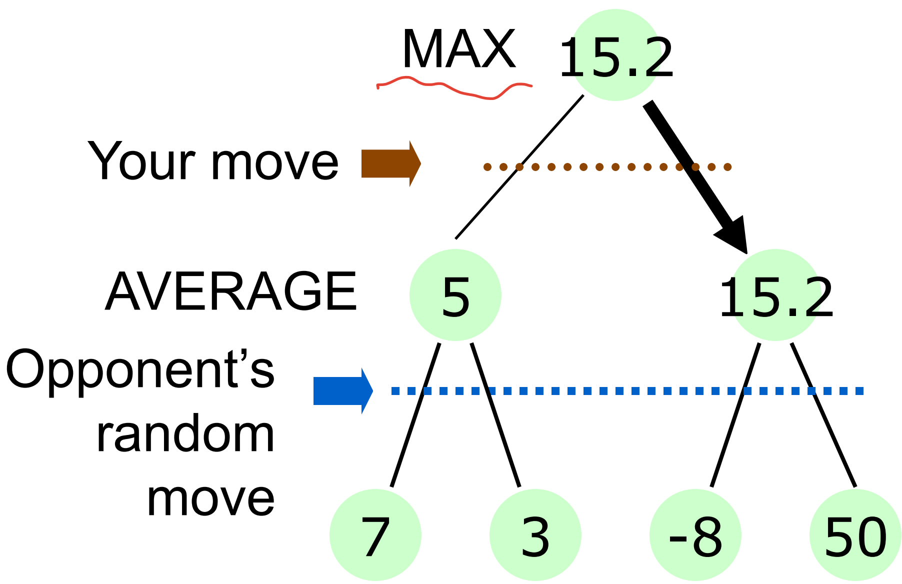

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From MAX point of view, she is playing against a stochastic environment.

Games: Modelling, Inference, Learning

- Modelling:
  - Formulating games as a search problem
  - Modeling your opponent
- Inference:
  - How to search for a strategy
  - Minimax, Expectimax (and Expectiminimax)
  - Pruning
  - Heuristic function and cut-off search
- Learning:
  - Learning heurtistic functions
  - Modeling your opponent from data

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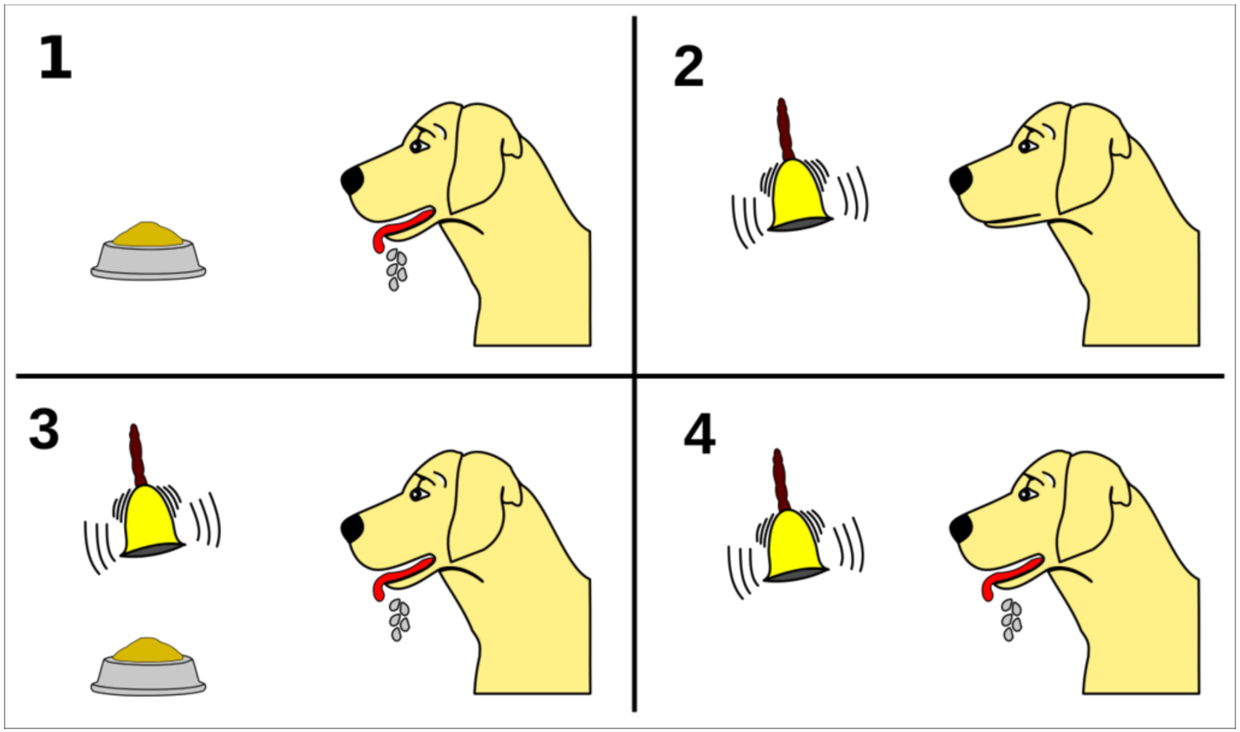
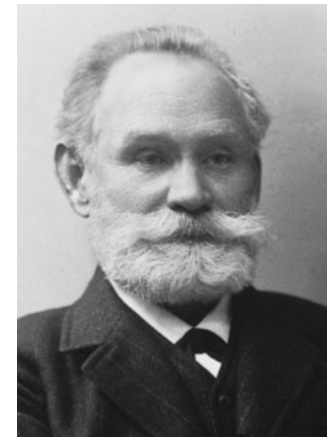
(Where are the data coming from?)

Reinforcement Learning Lecture Series

- Overview (Today)
- Markov Decision Processes (Today)
- Bandits problems and exploration
- Reinforcement Learning Algorithms

Reinforcement learning in the animal world





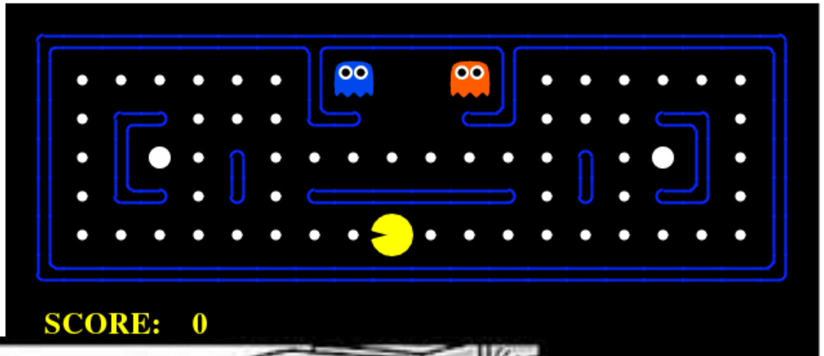
Ivan Pavlov

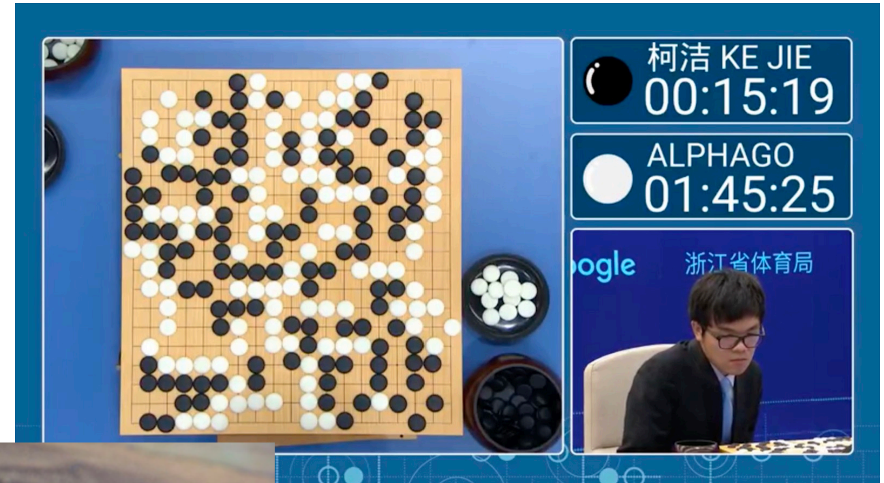
(1849 - 1936)

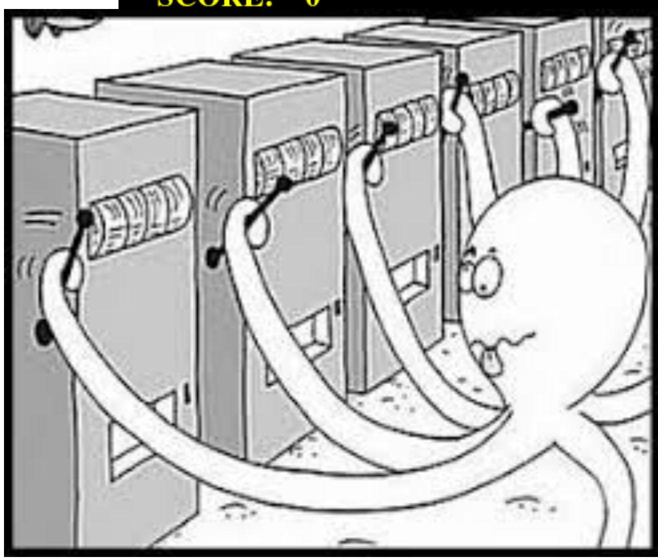
Nobel Laureate

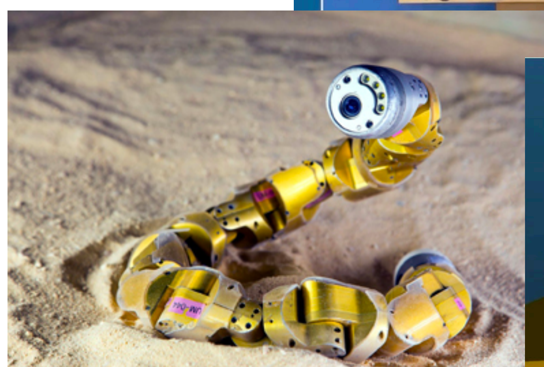
- Learn from rewards
- Reinforce on the states that yield positive rewards

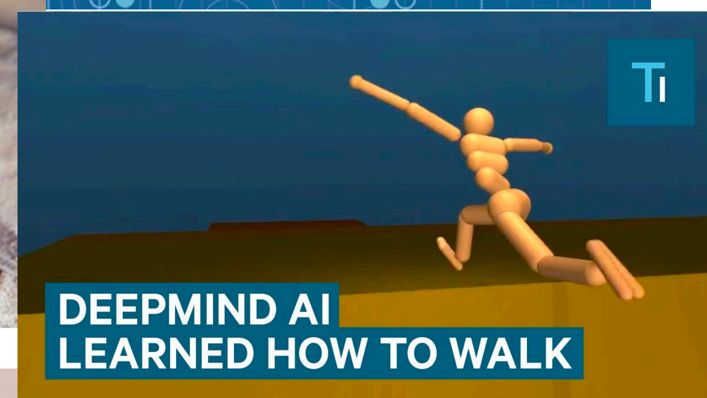
Reinforcement learning: Applications











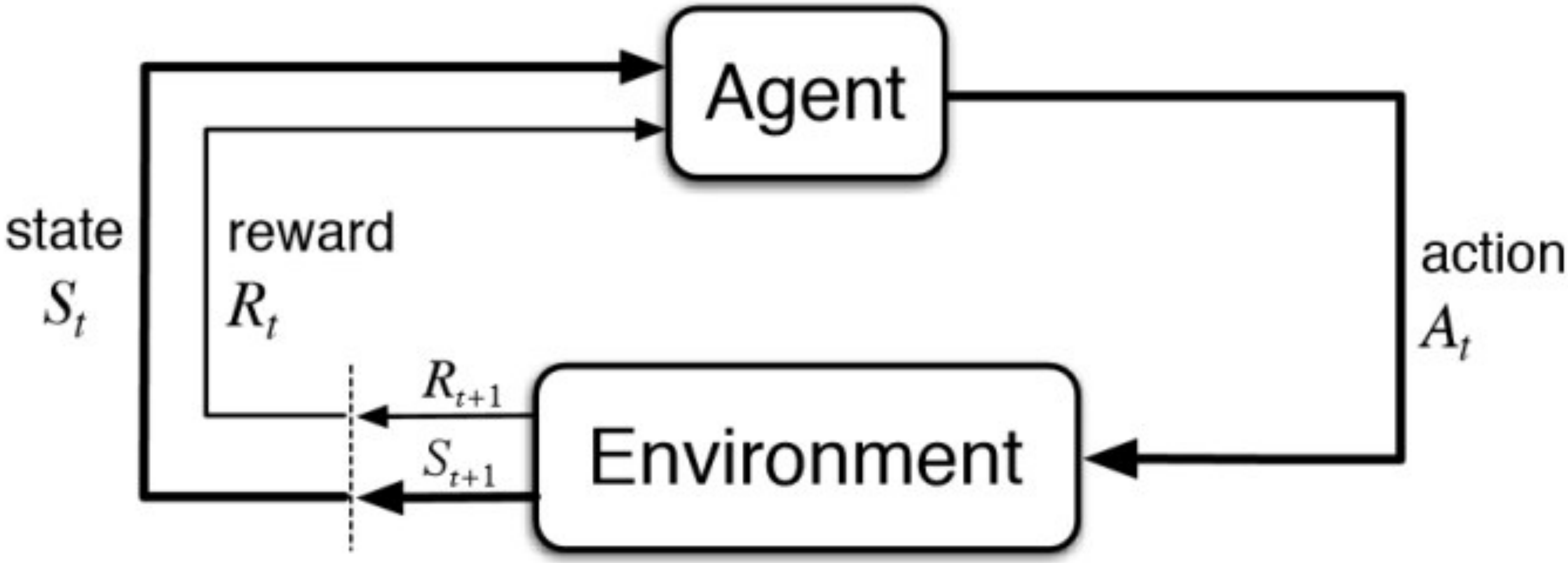
amazon

Recommendations

buy or not buy

Reinforcement learning problem setup

- State, Action, Reward
- Unknown reward function, unknown state-transitions.
- Agents might not even observe the state



Reinforcement learning problem setup

Reinforcement learning problem setup

- State, Action, Reward and Observation

$$S_t \in \mathcal{S} \quad A_t \in \mathcal{A} \quad R_t \in \mathbb{R} \quad O_t \in \mathcal{O}$$

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- Policy:
  - When the state is observable: $\pi : \mathcal{S} \to \mathcal{A}$
  - Or when the state is not observable

$$\pi_t : (\mathcal{O} \times \mathcal{A} \times \mathbb{R})^{t-1} \to \mathcal{A}$$

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- Learn the best policy that maximizes the expected reward
  - Finite horizon (episodic) RL: $\pi^* = \arg\max_{\pi \in \Pi} \mathbb{E}[\sum_{t=1}^{T} R_t]$
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T: horizon

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γ: discount factor

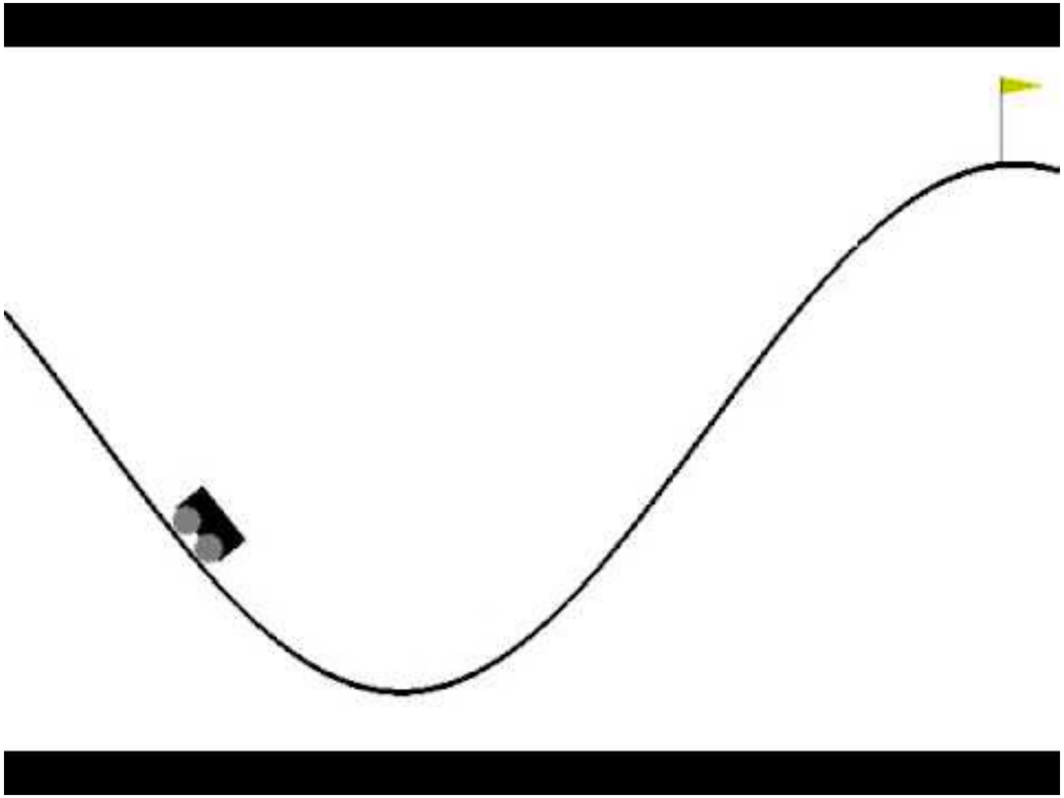
RL for robot control

- States: The physical world, e.g., location/speed/acceleration and so on.
- Observations: camera images, joint angles
- Actions: joint torques
- Rewards: stay balanced, navigate to target locations, serve and protect humans, etc.

RL for Inventory Management

- State: Inventory level, customer demand, competitor's inventory
- Observations: current inventory levels and sales history
- Actions: amount of each item to purchase
- Rewards: profit

Demonstrating the learning process



- Mountain car:

https://www.youtube.com/watch?v=U5w9PoKCOeM

Reading materials for RL

- Introduction:
  - Sutton and Barto: Chapter 1
- Markov Decision Processes
  - AIMA Section 17.1, Sutton and Barto: Ch 3
- Policy iterations / value iterations
  - AIMA Chapter 17.2-17.3, Sutton and Barto Ch 4.
- Bandits
  - Sutton and Barto Ch 2, AIMA Ch. 21.4 (AIMA Ch. 22.4) 4th Ed.
- RL Algorithms: Sutton and Barto Ch 4, Ch 5, Ch 6, Ch 13

- Next Tuesday:
  - Markov Decision Processes

Reinforcement learning is, arguably, the most general AI framework.

- RL: State, Action, Reward, Nothing is known.

- Simplified RL models:
  - iid state → Contextual bandits
  - No state, tabular action → Multi-arm bandits
  - iid state, no reward → Supervised Learning
  - Known dynamics / reward → Markov Decision Processes (Control/Cybernetics)
  - No reward / Unknown dynamics → System Identification

Reinforcement learning is very challenging

- The agent needs to:
  - Learn the state-transitions ----- How the world works
  - Learning the costs / rewards ----- Cost of actions
  - Learning how to search ----- Come up with a good strategy

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- All at the same time

Let us tackle different aspects of the RL problem one at a time

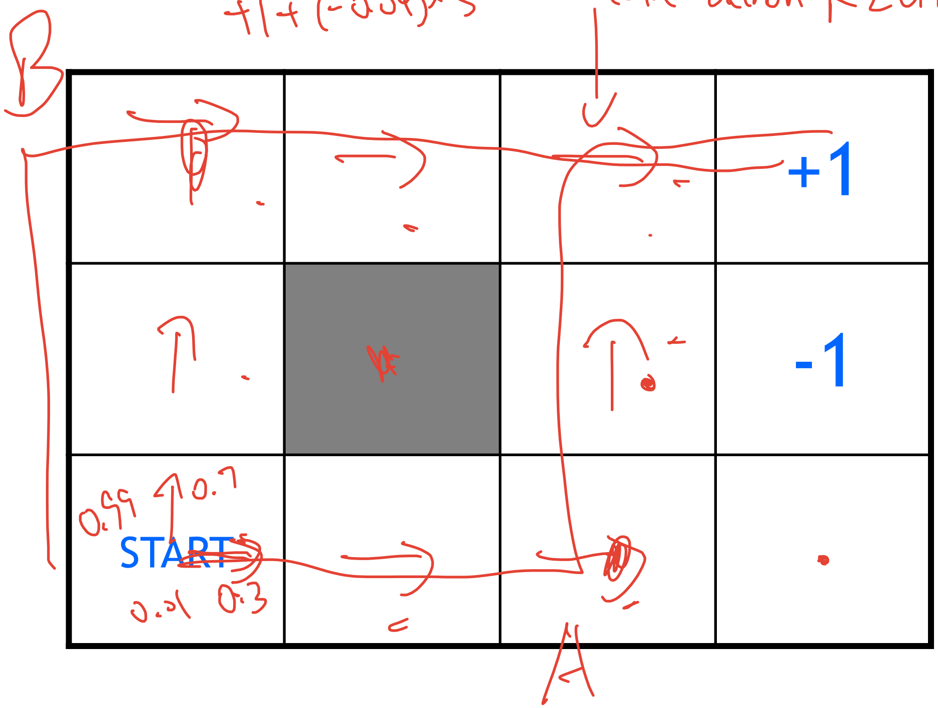
- **Markov Decision Processes:**
  - Dynamics are given no need to learn
- Bandits: Explore-Exploit in simple settings
  - RL without dynamics
- Full Reinforcement Learning
  - Learning MDPs

Robot in a room. (3 min discussion)

+1+(-0.04)×5 take action "RIGHT"

π: S -> A

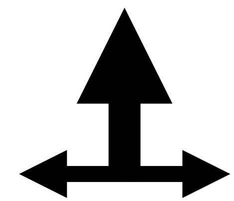
π: S -> P_A



actions: UP, DOWN, LEFT, RIGHT

e.g.,

UP



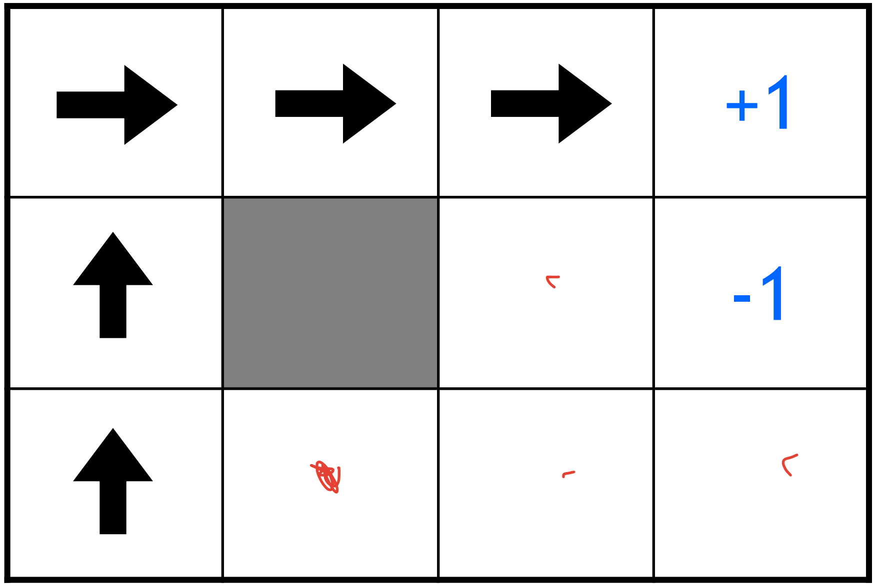
State-transitions with action **UP**:

80%	move up
10%	move left
10%	move right

*If you bump into a wall, you stay where you are.

- reward +1 at [4,3], -1 at [4,2]
- reward -0.04 for each step
- what's the strategy to achieve max reward?
- what if the transitions were deterministic?

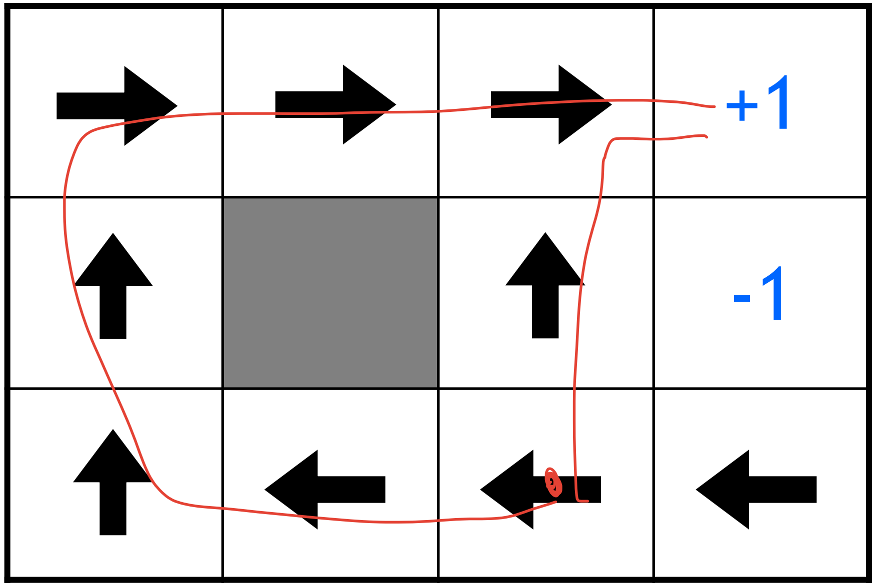
Is this a solution?



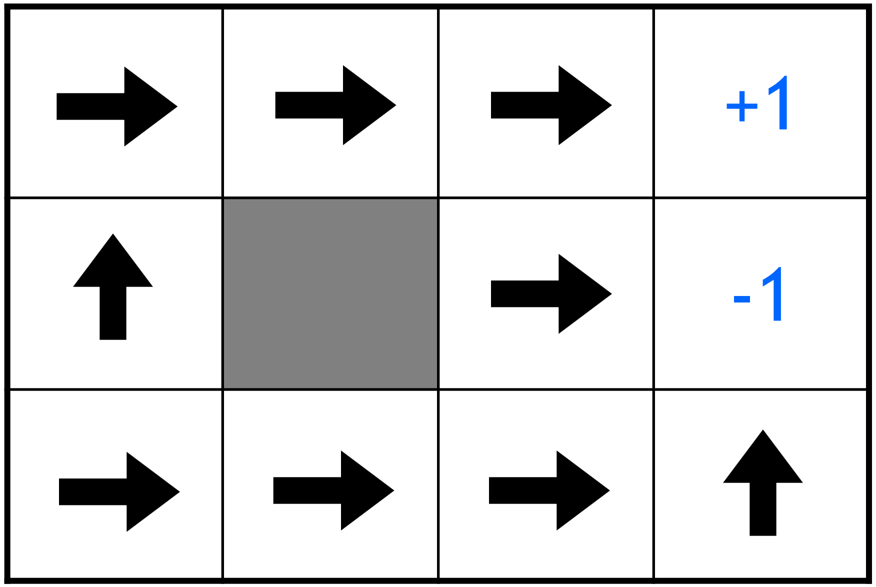
- only if transitions are deterministic
  - not in this case (transitions are stochastic)

- solution/policy
  - mapping from each state to an action

Optimal policy

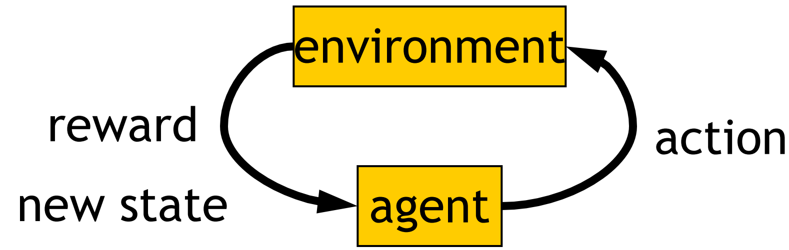


Reward for each step: -2



Markov Decision Process (MDP)

- set of states S, set of actions A, initial state S_0
- transition model P(s'| s,a)
  - P([1,2] | [1,1], up) = 0.8
- reward function r(s') r(s',a,s)
  - r([4,3]) = +1 (Sometimes also depend on s, a)
- goal: maximize cumulative reward in the long run



- policy: mapping from S to A
  - Overloading notation: π(s) outputs an actions (for deterministic policy), or a probability distribution of actions (for stochastic policy).
  - We also use π(a|s) as a short hand for $P_\pi(a|s)$ --- the conditional probability table under policy π

Tabular MDP

- Discrete State, Discrete Action, Reward and Observation

$$S_t \in \mathcal{S} \quad A_t \in \mathcal{A} \quad R_t \in \mathbb{R} \quad \sout{O_t \in \mathcal{O}}$$

- Policy:
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What is Markovian about MDPs?

- “Markov” generally means that given the present state, the future and the past are independent

- For Markov decision processes, “Markov” means action outcomes depend only on the current state

$$P(S_{t+1} = s' | S_t = s_t, A_t = a_t, S_{t-1} = s_{t-1}, A_{t-1}, \ldots S_0 = s_0)$$
$$=$$
$$P(S_{t+1} = s' | S_t = s_t, A_t = a_t)$$

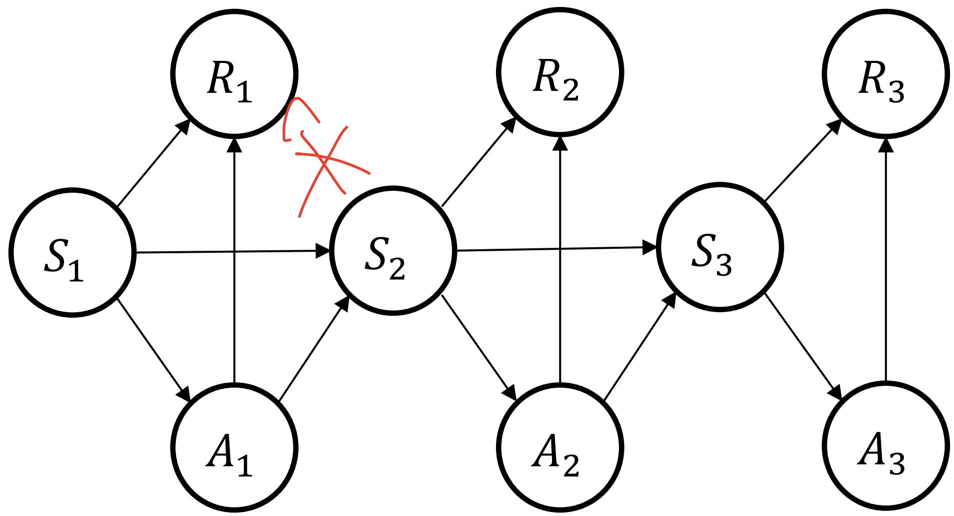
- This is just like search, where the future (available actions, states to transition to) could only depend on the current state (not the history)

Andrey Markov
(1856-1922)

(slide credit: Virtue and Rosenthal)

This is a **conditional independence** assumption!

- Example of a finite horizon MDP with H = 3, as a BayesNet



$P(A_t | S_t)$ ← policy π

$P(S_1)$ — d_0

$P(S_2 | S_1, A_1)$ — P_t, $t=1$

$E[R_1 | S_1, A_1]$ — $r(s_t, a_t)$

$\mathbb{E}[R_1 | S_1, A_1, S_2]$ — $r(s_t, a_t, s_{t+1})$

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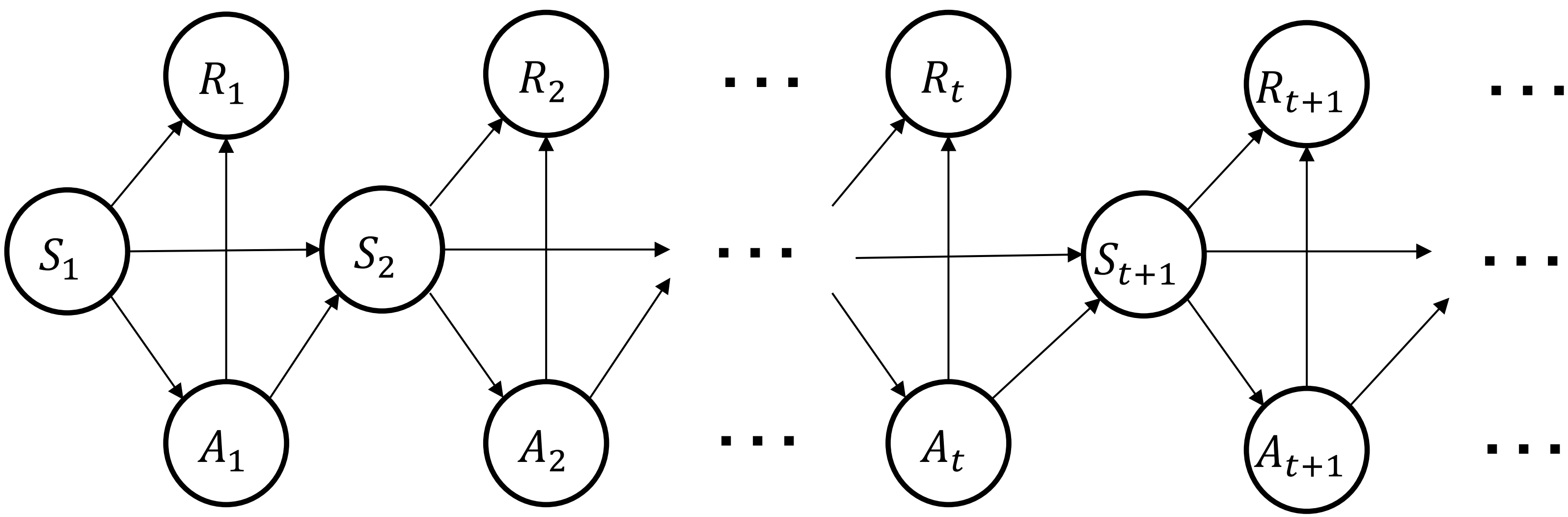
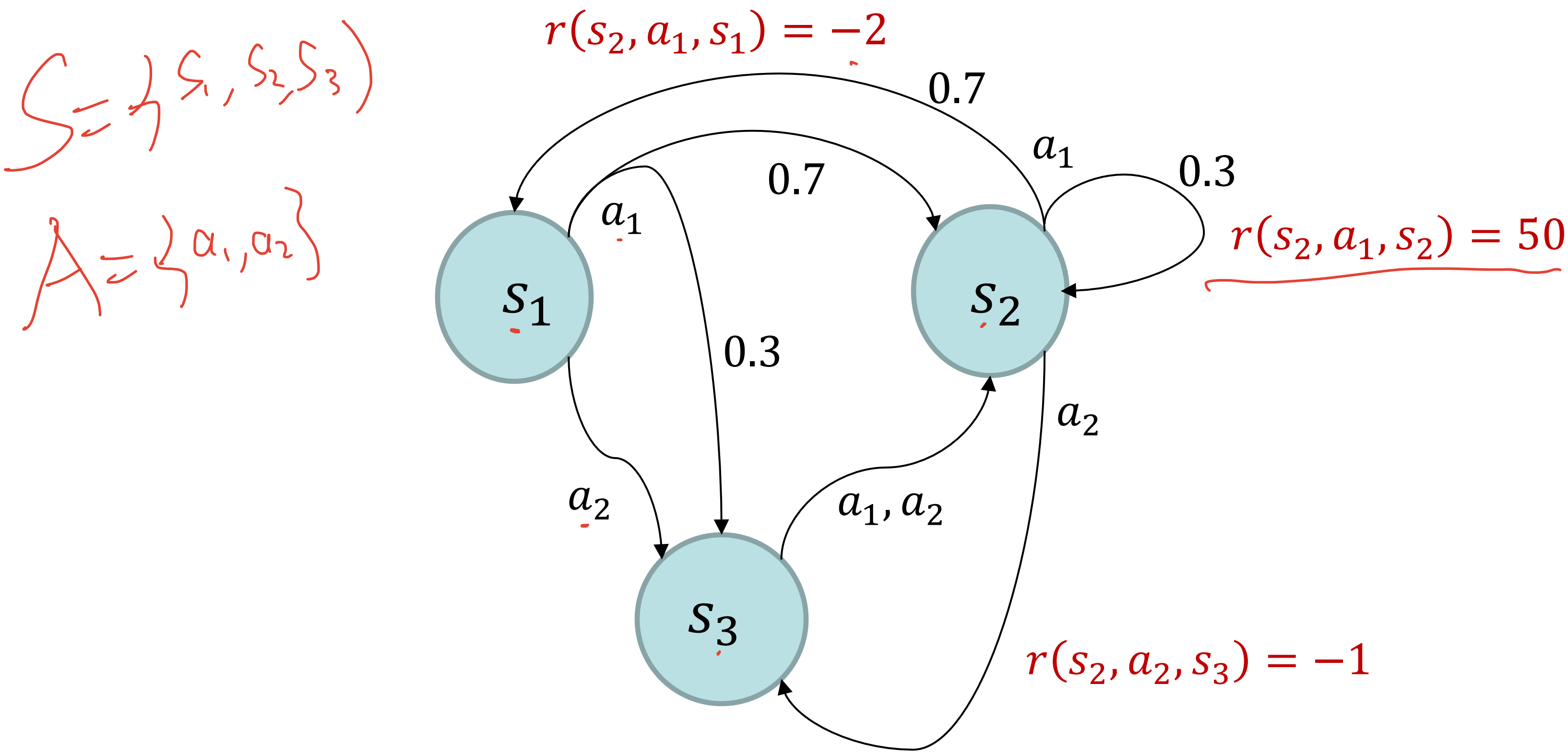
- Example of an infinite horizon MDP (as a BayesNet)



$P(s'|s,a)$ $E[R|S,A]$

State-space diagram representation of an MDP: An example with 3 states and 2 actions.

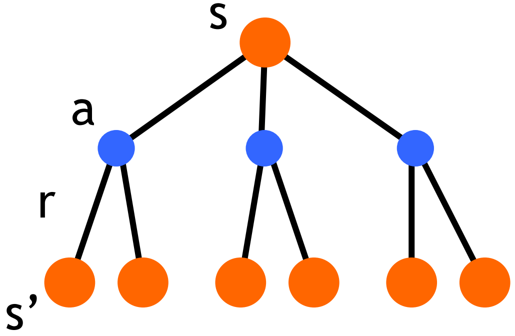


* The reward can be associated with only the state s' you transition into.

* Or the state that you transition from s and the action a you take.

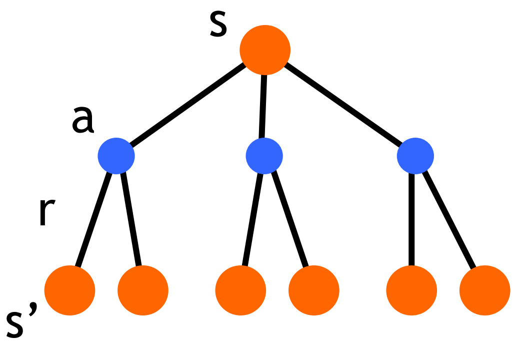
* Or all three at the same time.

Reward function and Value functions



Reward function and Value functions

- Immediate reward function r(s,a,s')
  - expected **immediate** reward



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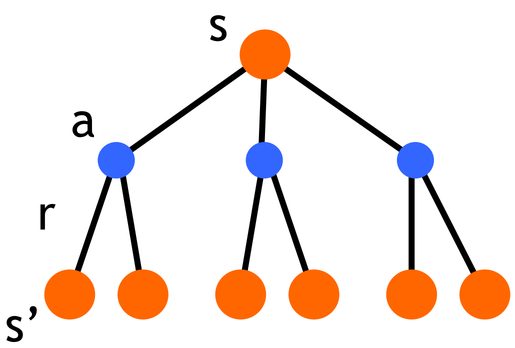
- state value function: $V^{\pi}(s)$
  - expected **long-term** return when starting in *s* and following π



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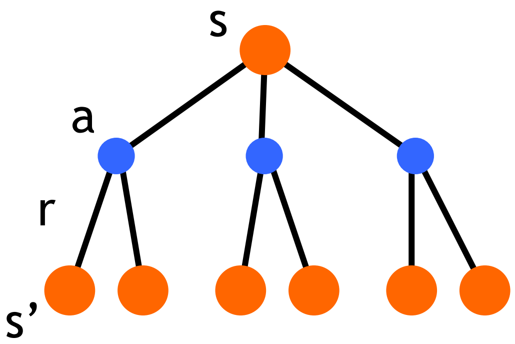

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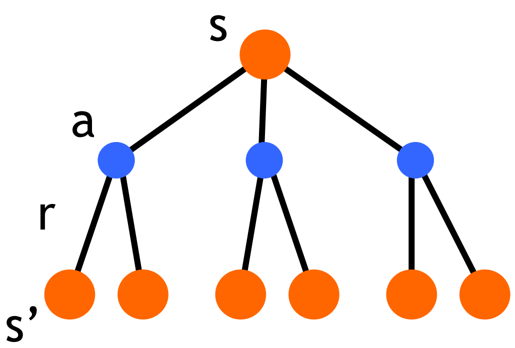
- useful for finding the optimal policy
  - can estimate from experience
  - pick the best action using $Q^{\pi}(s,a)$



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$$r(s, a, s') = \mathbb{E}[R_1 | S_1 = s, A_1 = a, S_2 = s']$$

$$r^\pi(s) = \mathbb{E}_{a\sim\pi(a|s)}[R_1 | S_1 = s]$$

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Bellman equations – the fundamental equations of MDP and RL

- An alternative, recursive and more useful way of defining the V-function and Q function

$V^\pi \in \mathbb{R}^{|S|}$

$$V^\pi(s) = \sum_a \pi(a|s) \sum_{s'} P(s'|s,a)[r(s,a,s') + \gamma V^\pi(s')] = \sum_a \pi(a|s) Q^\pi(s,a)$$

$E_{a \sim \pi(s)}$

$E_{\text{state transition}}$

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- Quiz:
  - Prove Bellman equation from the definition in the previous slide.
  - Write down the Bellman equation using Q function alone.

    $Q^\pi(s,a) = ?$

Bellman equations – the fundamental equations of MDP and RL

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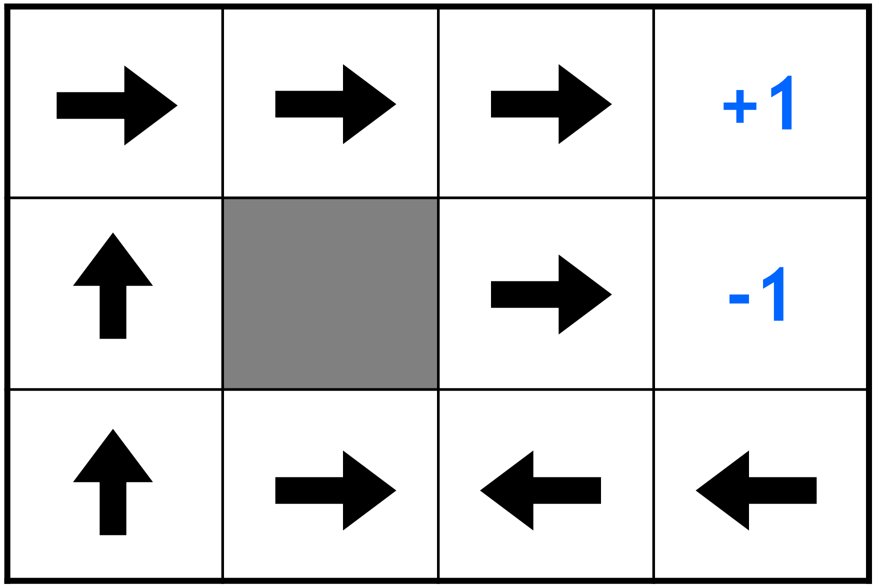
- More quiz:
  - On AIMA textbook, reward is only a function of the state your transition into (Think about we collect a reward when we transition into s'). What is the Bellman equation in this special case?
  - Sometimes, the reward is conditionally independent to s' given s, a. What is the Bellman equation in this special case?

Let's work out the Value function for a specific policy



actions: UP, DOWN, LEFT, RIGHT

e.g., **UP**

state-transitions with action **UP**:

80%	move UP
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*If you bump into a wall, you stay where you are.

- reward +1 at [4,3], -1 at [4,2]
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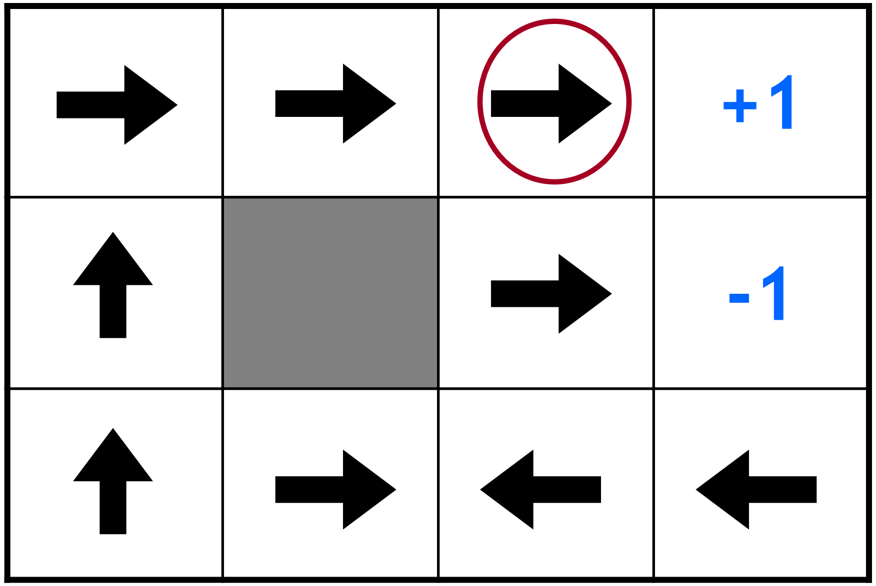

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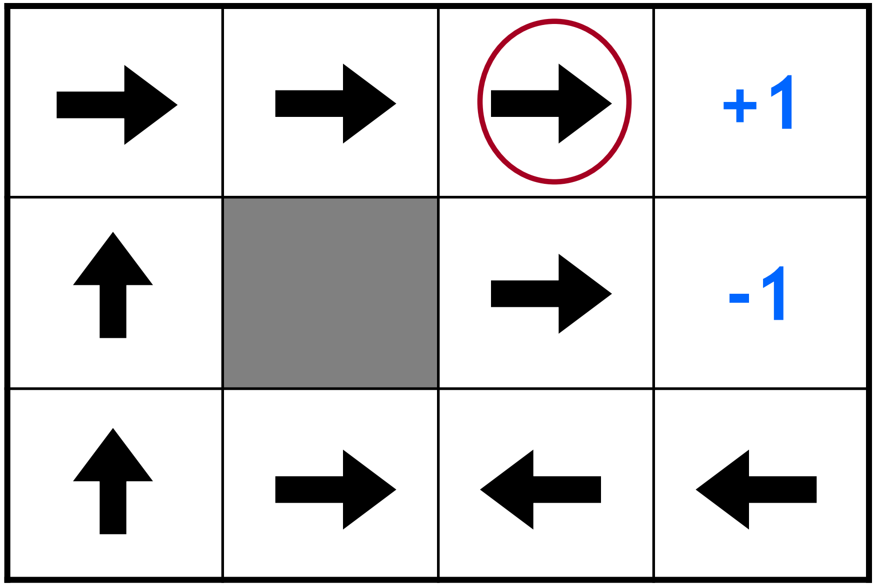

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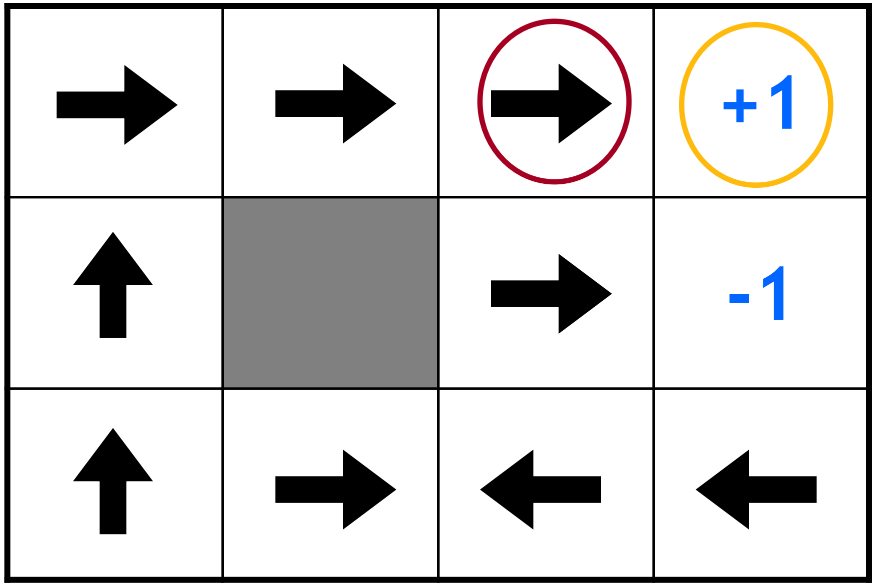
1.0 +

+

Let's work out the Value function for a specific policy



actions: UP, DOWN, LEFT, RIGHT

e.g., **UP**

state-transitions with action **UP**:

80%	move UP
10%	move LEFT
10%	move RIGHT

*If you bump into a wall, you stay where you are.

- reward +1 at [4,3], -1 at [4,2]
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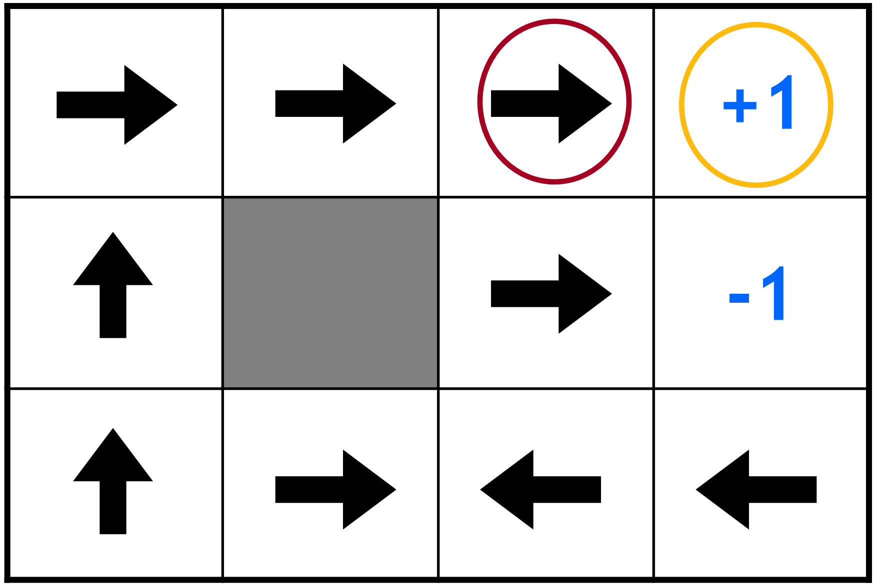

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1.0 + 0.8 * (+1-0.04

+

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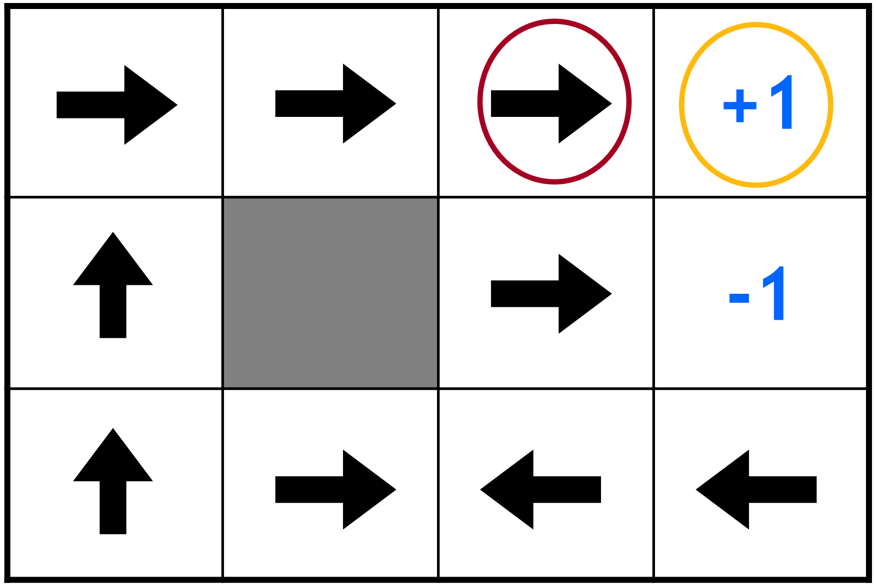

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1.0 0.8 * (+1-0.04 + 0)

+

+

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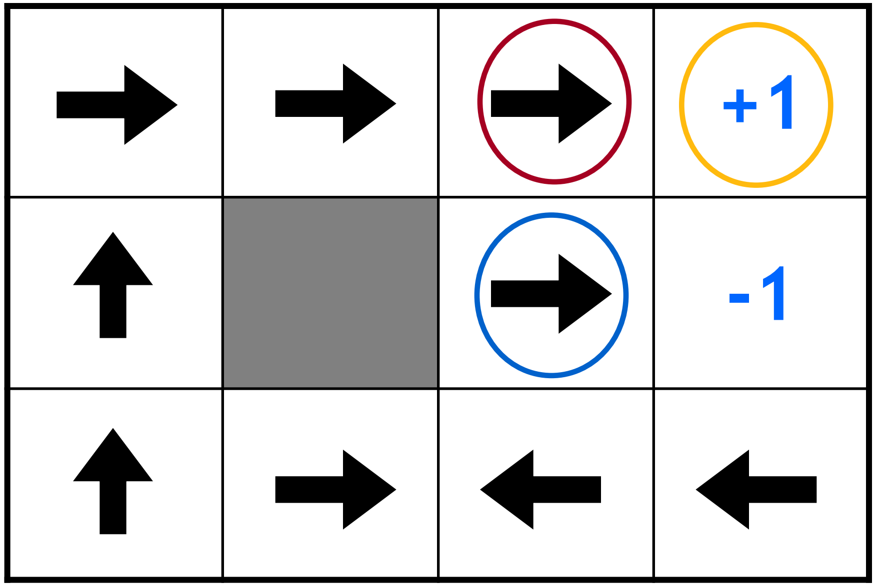

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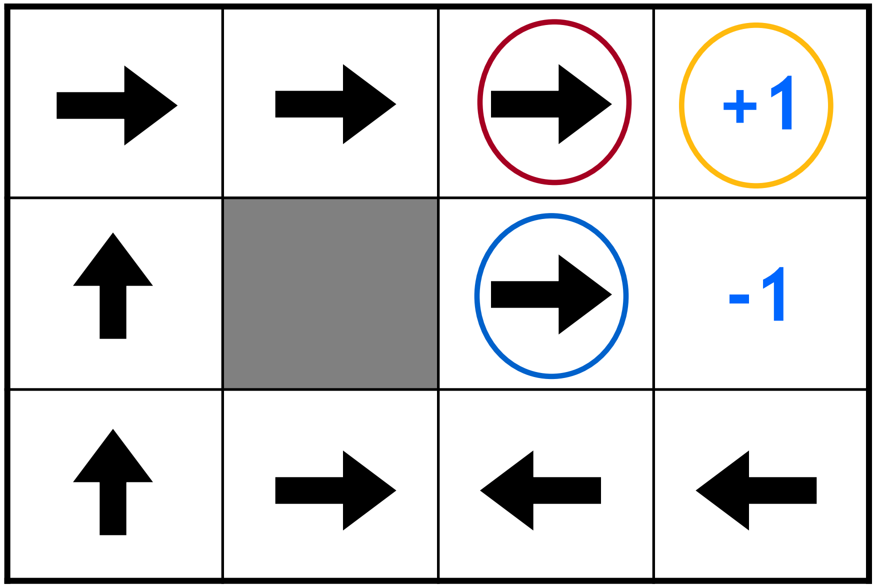

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1.0 + 0.8 * (+1-0.04 + 0)

+ 0.1 *

+

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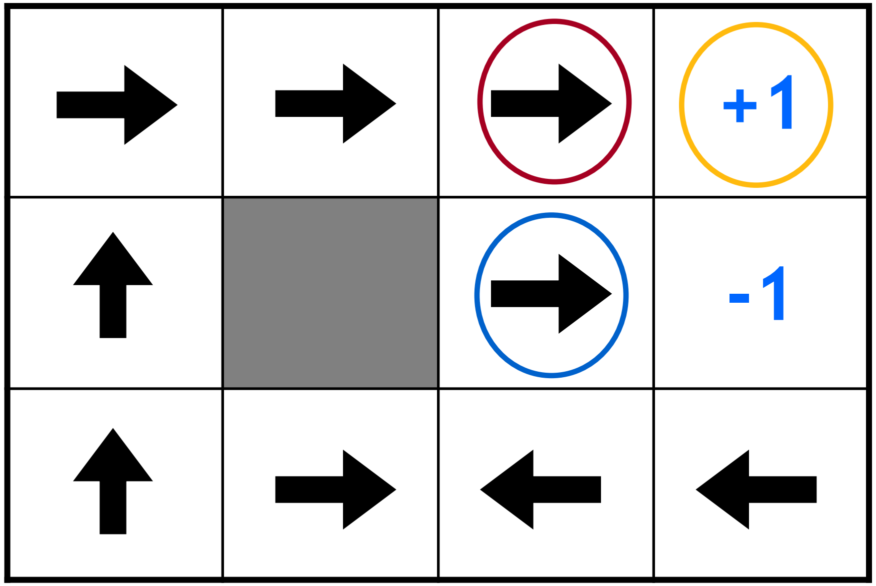

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1.0 0.8 * (+1-0.04 + 0)

\+ 0.1 * (-0.04 + V^{π}([3,2]))

\+

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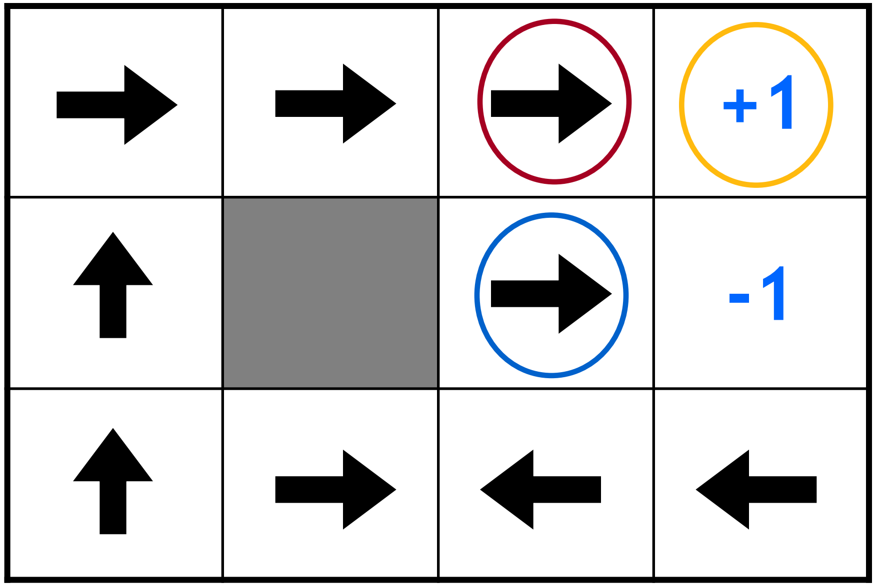

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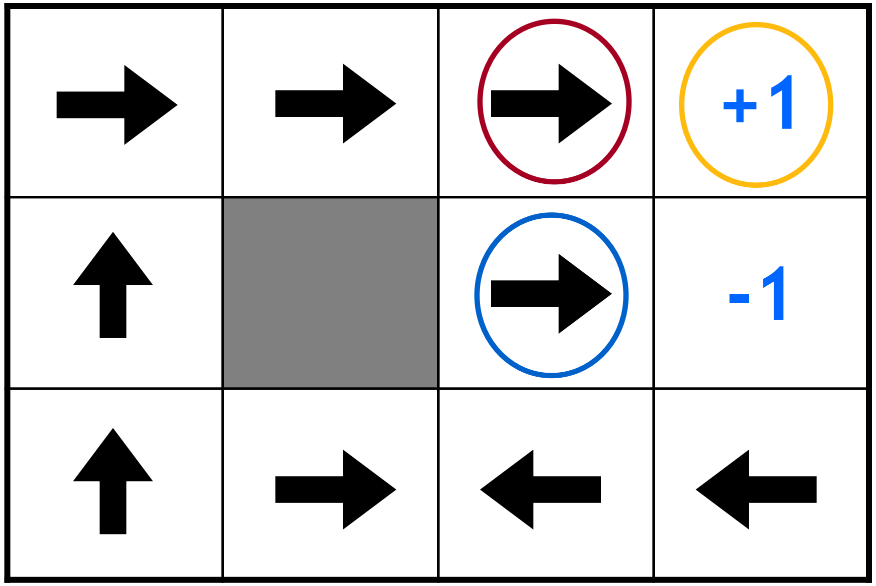
1.0   0.8 * (+1-0.04 + 0)

\+ 0.1 * (-0.04 + $V^{\pi}([3,2])$)

\+ 0.1 *

Let's work out the Value function for a specific policy



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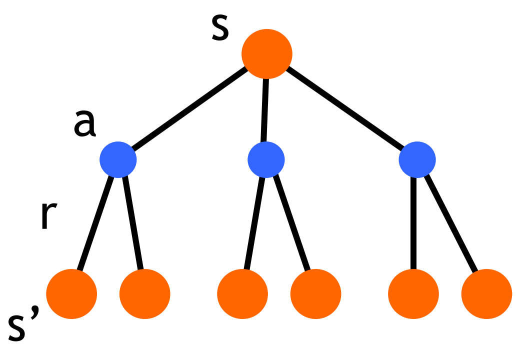
1.0 + 0.8 * (+1-0.04 + 0)

+ 0.1 * (-0.04 + V^π([3,2]))

+ 0.1 * (-0.04 + V^π([3,3]))

Optimal value functions



- there's a set of *optimal* policies
  - V^π defines partial ordering on policies
  - they share the same optimal value function

$$V^*(s) = \max_\pi V^\pi(s)$$

- Bellman optimality equation

$$V^*(s) = \max_a \sum_{s'} P(s'|s,a)[r(s,a,s') + \gamma V^*(s')]$$

  - system of n non-linear equations
  - solve for V*(s)
  - easy to extract the optimal policy

- having Q*(s,a) makes it even simpler

$$\pi^*(s) = \arg\max_a Q^*(s,a)$$

Inference problem: given an MDP, how to compute its optimal policy?

- It suffices to compute its Q* function, because:

$$\pi^*(s) = \arg\max_a Q^*(s,a)$$

- It suffices to compute its V* function, because:

$$Q^*(s,a) = \sum_{s'} P(s'|s,a)[r(s,a,s') + \gamma V^*(s')]$$

Algorithms for calculating the V* function

- Policy evaluation, policy-improvement

- Policy iterations

- Value iterations

Dynamic programming

- main idea
  - use value functions to structure the search for good policies
  - need a known model of the environment
- two main components
  - policy evaluation: compute V^{π} from π
  - policy improvement: improve π based on V^{π}

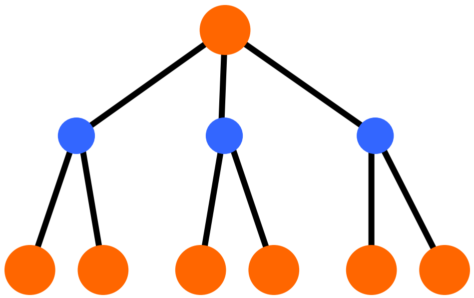
  - start with an arbitrary policy
  - repeat evaluation/improvement until convergence

Policy evaluation/improvement

- policy evaluation: π -> V^π
  - Bellman eqn's define a system of n eqn's
  - could solve, but will use iterative version

$$V^\pi_{k+1}(s) \leftarrow \sum_a \pi(a|s) \sum_{s'} P(s'|s,a)[r(s,a,s') + \gamma V^\pi_k(s')]$$

  - start with an arbitrary value function V_0, iterate until V_k converges



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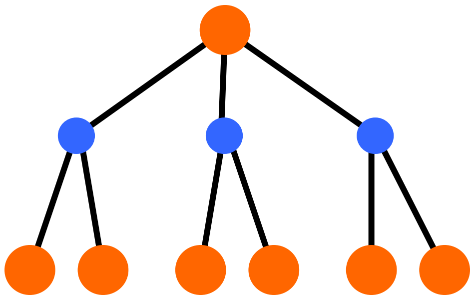
- policy improvement: V^{π} -> π'



Policy evaluation/improvement

- policy evaluation: π -> V^π
  - Bellman eqn's define a system of n eqn's
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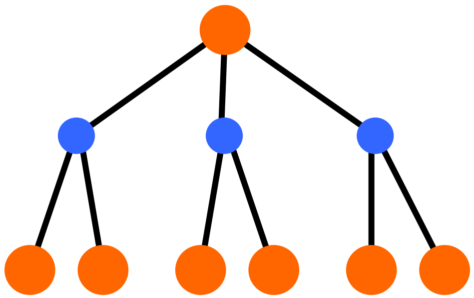
$$V_{k+1}^{\pi}(s) \leftarrow \sum_a \pi(a|s) \sum_{s'} P(s'|s,a)[r(s,a,s') + \gamma V_k^{\pi}(s')]$$

  - start with an arbitrary value function V_0, iterate until V_k converges



- policy improvement: V^π -> π'

$$\pi'(s) = \arg\max_a Q^{\pi}(s,a)$$
$$= \arg\max_a \sum_{s'} P(s'|s,a)[r(s,a,s') + \gamma V_k^{\pi}(s')]$$

Policy evaluation/improvement

- policy evaluation: π -> V^π
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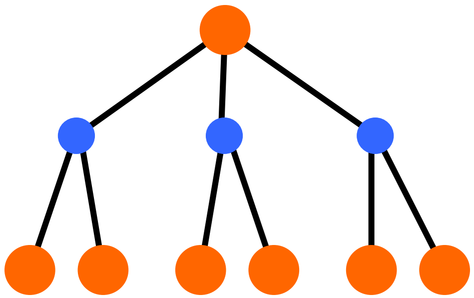


- policy improvement: V^π -> π'

$$\begin{aligned}\pi'(s) &= \arg\max_a Q^{\pi}(s,a) \\ &= \arg\max_a \sum_{s'} P(s'|s,a)[r(s,a,s') + \gamma V_k^{\pi}(s')]\end{aligned}$$

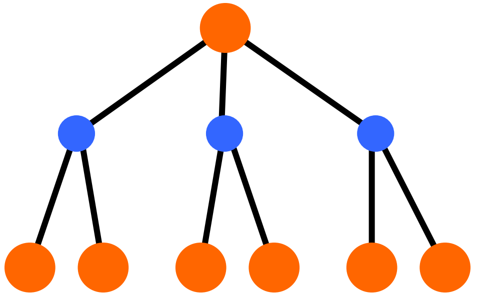
  - π' either strictly better than π, or π' is optimal (if $\pi = \pi$')

Policy/Value iteration

- Policy iteration

$$\pi_0 \rightarrow^E V^{\pi_0} \rightarrow^I \pi_1 \rightarrow^E V^{\pi_1} \rightarrow^I \ldots \rightarrow^I \pi^* \rightarrow^E V^*$$

  - two nested iterations; too slow
  - don't need to converge to V^{π_k}
    - just move towards it



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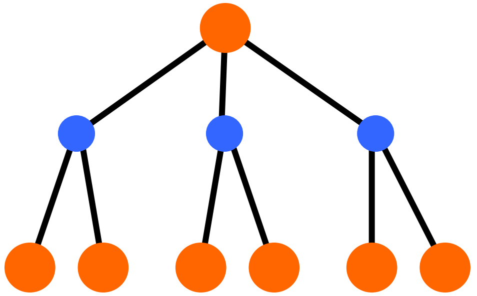

- Value iteration

$$V_{k+1}(s) \leftarrow \max_a \sum_{s'} P(s'|s,a)[r(s,a,s') + \gamma V_k(s')]$$

  – use Bellman optimality equation as an update
  – converges to V*

So far no learning at all. On Thursday:

- More on MDPs
- MDP inferences
- Start bandits and exploration