# Lecture 6 Advanced Composition (Part II), Gaussian mechanism

Yu-Xiang Wang

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# Recap: last lecture

- Private selection
  - Exponential mechanism
  - Report-Noisy-Max
- Application of Exponential mechanism
  - SmallDB algorithm
- Advanced Composition
  - Apply to linear query release
  - Privacy loss random variable

# Recap: Utility of Exponential Mechanism and small DB

- Utility of Exp-Mech

- Approximation error of a SmallDB

- Guarantee of SmallDB

# Recap: Advanced Composition

**Theorem:** The adaptive composition of k $(\varepsilon,\delta)$-DP mechanisms satisfies $(\tilde{\varepsilon}, \tilde{\delta}) - DP$ where

$$\tilde{\varepsilon} = \varepsilon\sqrt{2k\log(1/\delta')} + 2k\varepsilon^2, \qquad \tilde{\delta} = k\delta + \delta'$$

for any $\varepsilon, \delta \geq 0, \delta' > 0$

Slightly different, also not the tightest; but among the cleanest with a simple proof.

TLDR: For reasonable ranges of privacy parameter, total privacy loss scales as sqrt(k).

We will prove the version of this theorem for $\delta = 0$ today.

# Recap: Application of Advanced composition to linear query release.

- Advanced composition of Laplace mechanisms
  - For k times.

- Advanced composition of AboveThresh and Laplace Mechanism
  - For N times where N is the number of times to update the synthetic data

$$N = O\left(\frac{\log|\mathcal{X}|}{\alpha^2}\right), \quad \varepsilon_0 = \frac{\log 3k}{8}$$

$$\varepsilon_{total} = (\sqrt{N}\,\varepsilon_0)$$

# Summary of the problem of private query release

| | Laplace (release query) | Laplace (release data) | Private Multiplicative Weights *(Adaptive queries)* | SmallDB *(Fixed queries)* |
|---|---|---|---|---|
| Error under Pure-DP | $\frac{k\log(k/\beta)}{n\epsilon}$ | $\frac{\sqrt{\lvert\mathcal{X}\rvert}\log\frac{k}{\beta})}{n\epsilon}$ | $\left(\frac{\log\lvert\mathcal{X}\rvert\log(k/\beta)}{n\epsilon}\right)^{1/3}$ | $\left(\frac{\log\lvert\mathcal{X}\rvert\log\lvert\mathcal{Q}\rvert}{n\epsilon}\right)^{1/3} + \frac{\log\frac{1}{\beta}}{n\epsilon}$ |
| Error under approx-DP | $\frac{\sqrt{k\log\frac{1}{\delta}}\log\frac{k}{\beta}}{n\epsilon}$ | Same as above | $\left(\frac{\log\frac{k}{\beta}\sqrt{\log\lvert\mathcal{X}\rvert\log\frac{1}{\delta}}}{n\epsilon}\right)^{1/2}$ | Same as above |
| Computational complexity (per query) | $O(n)$ | $O(\lvert\mathcal{X}\rvert)$ | $O(\max\{\lvert\mathcal{X}\rvert, n\})$ | $O(\lvert\mathcal{X}\rvert)$ |

# Recap: Privacy loss random variable

- PLRV is the log probability ratio as a random variable

$p, p'$ density of $M(x), M(x')$

$$\boldsymbol{\varepsilon}_{\mathcal{M}}^{x,x'} = \log\left(\frac{p(\mathbf{y})}{p'(\mathbf{y})}\right) \text{ where random variable } \mathbf{y} \sim \mathcal{M}(x).$$

- Tail bound of privacy loss r.v. implies DP

**Lemma 1** (Tail bound to $(\epsilon, \delta)$-DP conversion). *Let* $\boldsymbol{\varepsilon}_{\mathcal{M}}^{x,x'}$ *be the privacy loss RV defined above. If*

$$\mathbb{P}(\boldsymbol{\varepsilon}_{\mathcal{M}}^{x,x'} > \epsilon) \leq \delta$$

*for* all *pair of neighboring* $x, x'$ *then* $\mathcal{M}$ *satisfies* $(\epsilon, \delta)$*-DP.*

(You are to prove this in HW1.)

# This lecture

- Advanced composition (Part II)
  - Proof of advanced composition for pure DP mechanisms
- Gaussian mechanism
  - PLRV of the Gaussian mechanism
  - Composition of Gaussian mechanisms via Adv. composition
- Renyi Differential Privacy
  - Deriving RDP from PLRVs
  - Improved composition of Gaussian mechanism

# Readings:

- Advanced Composition for pure-DP
  - Lecture notes
- Gaussian mechanism
  - Balle and W., 2018
- Probability inequalities and subgaussian tail bounds
  - Larry's notes: https://www.stat.cmu.edu/~larry/=stat705/Lecture2.pdf
- Renyi Differential Privacy
  - Bun and Steinke, 2017
  - Mironov, 2017

# Adaptive Composition = Sum of privacy loss random variables

- Fix two neighboring datasets, consider a sequence of adaptively chosen pure-DP mechanisms

$M_1, M_2, \cdots, M_k$, $\quad y_1 = M_1(x) \cdots y_k \sim M_k(x)$

$M_i(x, y_1, \cdots, y_{i-1}) \quad \forall\, y_1 \cdots y_{i-1} \in \mathcal{Y}^{i-1}$

$M_i(x, y_1 \cdots y_{i-1})$ satisfy $\varepsilon$-DP?

$$PLRV_{(M_1,\cdots,M_k)}^{x,x'} = \log \frac{P_x(y_1 \sim y_m)}{P_{x'}(y_1 \sim y_m)} = \log \frac{P_x(y_1) \cdot P_x(y_2|y_1) \cdot P_x(y_3|y_1,y_2) \cdots P_x(y_k|y_{1:k-1})}{P_{x'}(y_1) \cdots}$$

$$= \sum_{i=1}^{k} \varepsilon_{M_i(x, y_1 \cdots y_{i-1})}^{x,x'} \le k\varepsilon$$

# Proof Idea of Advanced Composition

- Observation 1: sometimes PLRV is positive, other times negative. They cancel with each other.

- Observation 2: as k gets larger, the sum of PLRV concentrates around its mean.
  - Calculate their mean
  - Bound the deviation from the mean

- Observation 3: the adaptivity means that the PLRV will depend on the past

# Martingale

- We say that a sequence of r.v. $X_1,\ldots,X_n,\ldots$ is a Martingale if for any n

$$\mathbf{E}(|X_n|) < \infty$$

$$\mathbf{E}(X_{n+1} \mid X_1, \ldots, X_n) = X_n.$$

- Example:
  - Random-walk: Total number of heads minus tails in n coin tosses

$X_n$ = # of heads − # of tails after n tosses

# Azuma-Hoeffding's inequality

- **Azuma-Hoeffding's inequality**: Assume $X_1, \ldots, X_n$ are Martingale differences

$$S_n = X_1 + \ldots + X_n$$

$$\mathbb{P}\left[S_n \geq \epsilon\right] \leq e^{-\frac{2\epsilon^2}{\sum_{i=1}^{n}(b_i - a_i)^2}}$$

- Apply Azuma-Hoeffding's inequality to our problem
  - What are these martingale differences?
  - What are these bounds?

# Proof for the advanced composition for pure DP mechanisms

- Fix x, x', apply Azuma-Hoeffding's inequality

$$P\left(\sum \varepsilon_i - E\left[\sum \varepsilon_i\right] > t\right) \le e^{\frac{-2t^2}{4k\varepsilon^2}} = \delta$$

$(M_1 \cdots M_k)$ satisfy $(\varepsilon, \delta)$-DP with

$$\varepsilon = E\left[\sum_i \varepsilon_i\right] + \varepsilon\sqrt{2k \log\frac{1}{\delta}}$$

$$2k\varepsilon^2 + \varepsilon\sqrt{2k \log\frac{1}{\delta}}$$

$$\frac{t^2}{2k\varepsilon^2} = \log\frac{1}{\delta}$$

$$t = \sqrt{2k\varepsilon^2 \log\frac{1}{\delta}}$$

# Bounding the KL-divergence

$$\mathbb{E}_{x\sim P}\left[\log\frac{P(x)}{Q(x)}\right] = \int P(x)\log\frac{P(x)}{Q(x)}\,dx = D_{KL}(P\|Q)$$

- **Lemma** (Pinsker's inequality)

$$\int |P(x)-Q(x)|\,dx = \|P-Q\|_1 \le \sqrt{2D_{KL}(P\|Q)}$$

- **Corollary**: KL-divergence is nonnegative.
- Now's let's bound the expected value of PLRV:

Assumption $P, Q$ are $M(x), M(x')$, $M$ is $\varepsilon$-DP.

$$\begin{aligned} D_{KL}(P\|Q) &= \int P(x)\log\frac{P(x)}{Q(x)}\,dx + D_{KL}(Q\|P) - D_{KL}(Q\|P) \\ &\le \int P(x)\log\frac{P(x)}{Q(x)}\,dx + \int Q(x)\log\frac{Q(x)}{P(x)}\,dx \quad (D_{KL}(Q\|P)\ge 0) \\ &= \int (P(x)-Q(x))\log\frac{P(x)}{Q(x)}\,dx \\ &\le \int |P(x)-Q(x)|\,dx \cdot \operatorname*{ess\,sup}_x \left|\log\frac{P(x)}{Q(x)}\right| \le \varepsilon\cdot\sqrt{2D_{KL}(P\|Q)} \end{aligned}$$

($\int|P(x)-Q(x)|\,dx = \|P-Q\|_1$; $\operatorname*{ess\,sup}_x \le \varepsilon$; Pinsker)

$$D_{KL}(P\|Q) \le 2\varepsilon^2$$

# Improved bounds of the KL-divergence and tighter version of Advanced composition.

Condition: P,Q satisfies that the log-odds ratio ≤ $\varepsilon$.

1. Bound from Dwork and Roth book

$$D_{KL}(P\|Q) \leq \epsilon(e^{\epsilon}-1)$$

2. Bound from Bun and Steinke

$$D_{KL}(P\|Q) \leq \frac{\epsilon^2}{2}$$

← Use this for theory / get asymptotic rate

3. Tight bound from Adam Smith (also in the proof of Bun and Steinke):

$$D_{KL}(P\|Q) \leq \epsilon \cdot \frac{e^{\epsilon}-1}{e^{\epsilon}+1} = \epsilon \cdot \tanh(\epsilon/2)$$

Implement this one (already in autodp)

Proofs are left as an exercise (maybe one question in HW2).

# Checkpoint

- A simple proof of advanced composition for pure-DP mechanisms via PLRV

**Theorem:** The adaptive composition of k $(\varepsilon,\delta)$-DP mechanisms satisfies $(\tilde{\varepsilon},\tilde{\delta}) - DP$ where

$$\tilde{\varepsilon} = \varepsilon\sqrt{2k\log(1/\delta')} + 2k\varepsilon^2, \qquad \tilde{\delta} = k\delta + \delta'$$

for any $\varepsilon, \delta \geq 0, \delta' > 0$

Can be improved to 0.5.

- Proof of the approx DP mechanism is similar but requires providing a PLRV that works for all approx. DP mechanisms.
- We will (hopefully) cover that in the next lecture as a natural by product.

# Gaussian mechanism

- Releasing low-sensitivity query f

$f: \mathbb{N}^{|\mathcal{X}|} \to \mathbb{R}^d$, $GM(G)$: outputs $f(x) + N(0, G^2 I_d)$

- L2-Sensitivity of f

$\Delta_2^f := \max_{x, x' \text{ neighbors}} \|f(x) - f(x')\|_2$

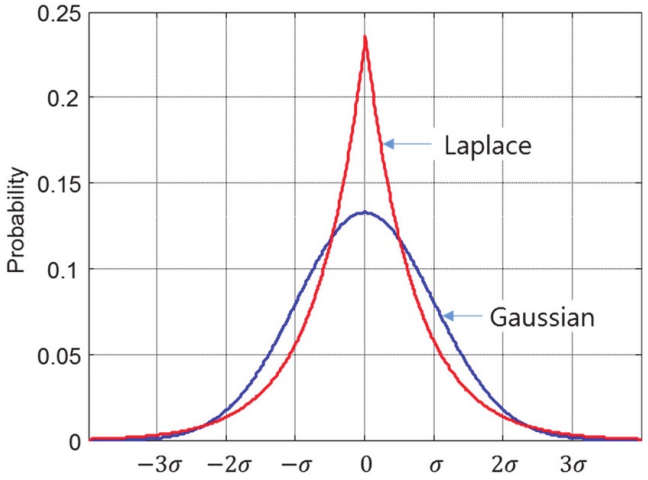
$\|x\|_2 = \sqrt{\sum x_i^2}$

$\|x\|_1 = \sum |x_i|$

$\|x\|_2 \leq \|x\|_1$

# Gaussian noise is more concentrated than Laplace noise

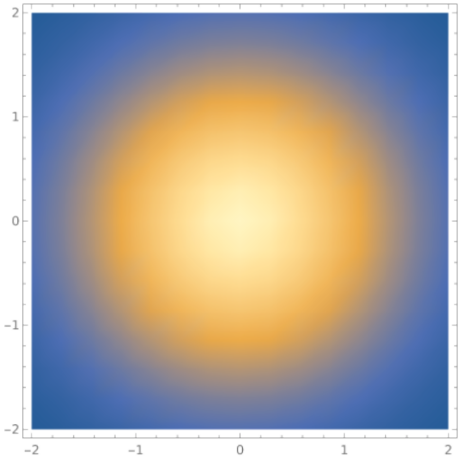
- N(0, $\sigma$) vs Lap(0,b)

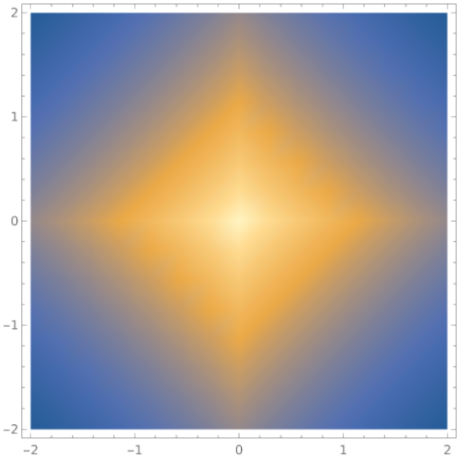



99.7%

- In multiple dimensions

isotropic







Credit: Damien Desfontaines https://desfontain.es/privacy/gaussian-noise.html

# Examples of queries and their sensitivities

1. Histogram under "add/remove"
2. Histogram under "replace"
3. Voting when each individual has k-ballots
4. Uncentered sample covariance ("Gram") matrix

# Privacy Loss Random Variable of the Gaussian mechanism is Gaussian

- Recall: $\varepsilon_{\mathcal{M}}^{x,x'} = \log(\frac{p(\mathbf{y})}{p'(\mathbf{y})})$ where random variable $\mathbf{y} \sim \mathcal{M}(x)$.

- The privacy loss RV of a Gaussian mechanism is

$$\mathcal{N}(\eta, 2\eta) \text{ with } \eta = D^2/2\sigma^2, \text{ where } D = \|f(x) - f(x')\|.$$

$$\log \frac{(\sqrt{2\pi}\sigma)^d e^{-\frac{\|f(x)-y\|^2}{2\sigma^2}}}{(\sqrt{2\pi}\sigma)^d e^{-\frac{\|f(x')-y\|^2}{2\sigma^2}}} = \frac{-\|f(x)-y\|^2 + \|f(x')-y\|^2}{2\sigma^2} = \frac{1}{2\sigma^2}\left(-\|f(x)-y\|^2 + \|f(x')-f(x)+f(x)-y\|^2\right)$$

$$= \frac{1}{2\sigma^2}\left(\underbrace{\|f(x')-f(x)\|^2}_{D^2} + \underbrace{2(f(x')-f(x))^T(f(x)-y)}_{N(0,\,\sigma^2 \cdot 4\|f(x')-f(x)\|^2)}\right) \quad \because y = f(x)+z,\ z \sim N(0,\sigma^2)$$

$$\sim \frac{D^2}{2\sigma^2} + N\left(0, \frac{4D^2\sigma^2}{4\sigma^4}\right) \sim N(\eta, 2\eta)$$

$4D^2\sigma^2$, $2\cdot\frac{D^2}{2\sigma^2}$

# The privacy analysis of Gaussian mechanism

**Lemma 1** (Tail bound to $(\epsilon, \delta)$-DP conversion). *Let* $\boldsymbol{\varepsilon}_{\mathcal{M}}^{x,x'}$ *be the privacy loss RV defined above. If*

$$\mathbb{P}(\boldsymbol{\varepsilon}_{\mathcal{M}}^{x,x'} > \epsilon) \leq \delta$$

*for* all *pair of neighboring* $x, x'$ *then* $\mathcal{M}$ *satisfies* $(\epsilon, \delta)$*-DP.*

- Useful lemma: Gaussian tail bound

Let $X \sim \mathcal{N}(\mu, \sigma^2)$, we have:

$$\mathbb{P}(X - \mu \geq u) \leq \exp(-u^2/(2\sigma^2)).$$


$\mathcal{N}(\eta, 2\eta)$ *with* $\eta = D^2/2\sigma^2$, *where* $D = \|f(x) - f(x')\|$.

# The privacy analysis of Gaussian mechanism

- The Gaussian mechanism with variance $\sigma^2$ for a query with L2-sensitivity $\Delta$ satisfies $(\varepsilon,\delta)$-DP with
  - $\varepsilon = \frac{\Delta_2^2}{2\sigma^2} + \frac{\Delta_2}{\sigma}\sqrt{2\log\frac{1}{\delta}}$
  - $\delta = \delta > 0$

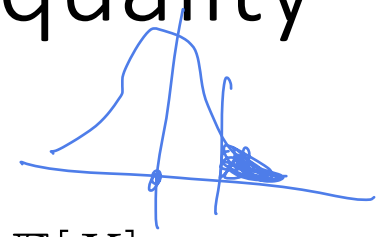
**Classical Gaussian mechanism:** For all $0 < \varepsilon, \delta \leq 1$, The mechanism obeys $(\varepsilon,\delta)$-DP if we choose

$$\sigma = \frac{\Delta}{\epsilon}\sqrt{2\log(1.25/\delta)}$$

# Remainder of the lecture

- Detour on centration inequalities
- Concentrated Differential Privacy
- The composition of Gaussian Mechanism

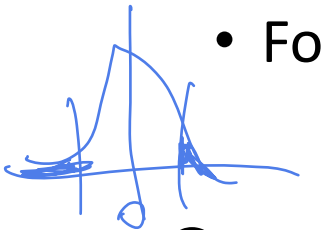
# Detour: Concentration inequality



- Markov's inequality
  - For any non-negative r.v. X: $\mathbb{P}(X \geq t) \leq \dfrac{\mathbb{E}[X]}{t}$

$$P(X \geq t) = \int_t^\infty p(x)\,dx = \frac{1}{t}\int_t^\infty t\,p(x) \leq \frac{1}{t}\int_t^\infty x\,p(x)\,dx \leq \frac{1}{t}\int_0^\infty x\,p(x)\,dx = \frac{\mathbb{E}[X]}{t} \quad \square$$

- Chebychev's inequality
  - For any r.v. X with variance $\sigma^2$ : $\mathbb{P}\Big(|X - \mathbb{E}[X]| \geq t\sigma\Big) \leq \dfrac{1}{t^2}$



$$P(|X-\mathbb{E}[X]| > t\sigma) = P(|X-\mathbb{E}[X]|^2 \geq t^2\sigma^2) \leq \frac{\mathbb{E}[(X-\mathbb{E}[X])^2]}{t^2\sigma^2} = \frac{1}{t^2}$$

(Markov)

- Generalizing Chebychev:

$$P(|X-\mathbb{E}[X]|^k > (t\sigma)^k) \leq \frac{\mathbb{E}[|X-\mathbb{E}[X]|^k]}{t^k}$$

$k$-th Central Moment of R.V. $X$

# Detour: Chernoff's method

Define, $\mu = \mathbb{E}[X]$. For any $t > 0$, we have that,

$$\mathbb{P}((X - \mu) \geq u) = \mathbb{P}(\exp(t(X - \mu)) \geq \exp(tu)) \leq \frac{\mathbb{E}[\exp(t(X - \mu))]}{\exp(tu)}$$

- Chernoff's bound:

$$\mathbb{P}((X - \mu) \geq u) \leq \inf_{0 \leq t \leq b} \exp(-t(u + \mu))\mathbb{E}[\exp(tX)].$$

# Subgaussian random variables

- We say a r.v. X with mean μ is $\sigma$-subgaussian if

$$\mathbb{E}[\exp(t(X-\mu))] \leq \exp(\sigma^2 t^2/2), \text{ for all } t \in \mathbb{R}.$$

  - We say that X is subgaussian if there exists constant $\sigma$.

- Example 1: N(μ, $\sigma^2$) is subgaussian.

- Example 2: Bounded random variables are subgaussian.
  - Exercise: what is $\sigma$ parameter if the range is [a,b]?

# Tail bound of subgaussian random variables

- By the Chernoff's bound we get that

$$\mathbb{P}(X - \mu \geq u) \leq \exp(-u^2/(2\sigma^2)).$$

- Proof: By the Chernoff's method

$$\mathbb{P}(X-\mu \geq u) = \mathbb{P}(e^{t(X-\mu)} \geq e^{tu}) \leq e^{-tu}\mathbb{E}[e^{t(X-\mu)}]$$
$$\leq e^{-tu} e^{\frac{\sigma^2 t^2}{2}} = e^{\frac{\sigma^2 t^2}{2} - tu}$$
$$t = \frac{u}{\sigma^2} \quad = e^{\frac{\sigma^2 u^2}{2\sigma^4} - \frac{u^2}{\sigma^2}} = e^{-\frac{u^2}{2\sigma^2}}$$

$$\min_t \frac{\sigma^2 t^2}{2} - tu$$
$$\sigma^2 t - u = 0 \Rightarrow t = \frac{u}{\sigma^2}$$

# Average of n independent $\sigma$-subgaussian RVs is $\frac{\sigma}{\sqrt{n}}$-subgaussian.

- Why?

$$\mathbb{E}[\exp(t(\widehat{\mu}-\mu))] = \mathbb{E}[\exp(t/n\sum_{i=1}^{n}(X_i-\mu)]$$
$$= \prod_{i=1}^{n}\mathbb{E}[\exp(t(X_i-\mu)/n)]$$
$$\leq \exp(t^2\sigma^2/(2n)).$$

- which implies

$$\mathbb{P}(|\widehat{\mu}-\mu| \geq k\sigma/\sqrt{n}) \leq 2\exp(-k^2/2).$$

# Idea: let's handle mechanisms that are Gaussian-mechanism-like, in a sense that

1. For any neighboring datasets, the PLRV is $\sigma$-subgaussian

2. Then composition is straightforward
   - The sum of $k$ PLRVs is $\sigma\sqrt{k}$ - subgaussian

# From Moment Generating Functions to Renyi divergence

- Moment Generating function of PLRV

$$\mathbb{E}_{x\sim p}\left[e^{t\log\frac{p(x)}{q(x)}}\right] = \mathbb{E}_{x\sim p}\left[\left(\frac{p(x)}{q(x)}\right)^t\right] = \int p(x)\left(\frac{p(x)}{q(x)}\right)^t dx = \int q(x)\frac{(p(x))^{t+1}}{(q(x))^{t+1}}dx = \mathbb{E}_{x\sim q}\left[\left(\frac{p(x)}{q(x)}\right)^{t+1}\right]$$

MGF.

- Renyi Divergence $\alpha \in (0,1)\cup(1,\infty)$

$$D_\alpha(P\|Q) = \frac{1}{\alpha-1}\ln\int p^\alpha q^{1-\alpha}\,\mathrm{d}\mu.$$

$\alpha = t+1$

$\alpha = 2 \rightarrow$ $\chi^2$-Divergence

$\alpha = \frac{1}{2} \rightarrow$ Hellinger's Distance

$$D_1(P\|Q) = D(P\|Q) = \text{Kullback-Leibler divergence}$$

$$D_\infty(P\|Q) = \ln\left(\operatorname*{ess\,sup}_P \frac{p}{q}\right)$$

Log-odd Ref.

Van Erven, T., & Harremos, P. (2014). Rényi divergence and Kullback-Leibler divergence. *IEEE Transactions on Information Theory*, *60*(7), 3797-3820.

# Example of Renyi Divergence

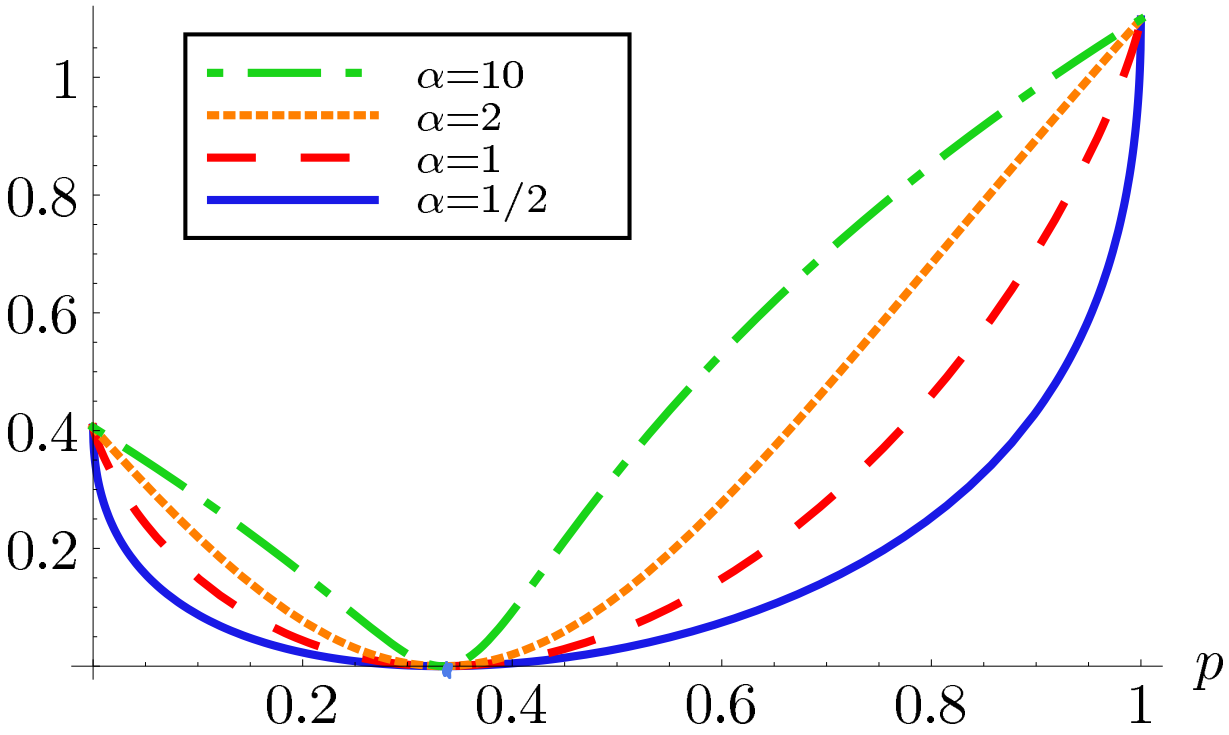



Fig. 2. Rényi divergence as a function of $P = (p, 1-p)$ for $Q = (1/3, 2/3)$

# Renyi Differential Privacy and Concentrated Differential Privacy

- (Mironov, 2017) We say that a mechanism satisfies $(\alpha, \epsilon)$-Renyi DP, if

$$\max_{x,x'} D_\alpha(\mathcal{M}(x)\|\mathcal{M}(x')) \leq \epsilon$$

- (Dwork and Rothblum / Bun and Steinke, 2016) We say a mechanism satisfies $\rho$-zCDP, if

$$\sup_{x,x'} D_\alpha(\mathcal{M}(x)\|\mathcal{M}(x')) \leq \rho\alpha, \forall\alpha > 1$$

- Connection to PLRV:

$$\mathbb{E}\left[e^{(\alpha-1)\varepsilon_M^{x,x'}}\right] \leq e^{(\alpha-1)\rho\alpha} \quad \text{for all } \alpha \geq 1$$

$\varepsilon_M^{x,x'}$ for $\mathcal{M}$ $\quad$ $O(\rho)$-subgaussian

# Examples of Renyi-DP/ CDP mechanisms

| Mechanism | Differential Privacy | Rényi Differential Privacy for $\alpha$ |
|---|---|---|
| Randomized Response | $\left\|\log \frac{p}{1-p}\right\|$ | $\alpha > 1$: $\frac{1}{\alpha-1}\log\left(p^\alpha(1-p)^{1-\alpha} + (1-p)^\alpha p^{1-\alpha}\right)$<br>$\alpha = 1$: $(2p-1)\log\frac{p}{1-p}$ |
| Laplace Mechanism | $1/\lambda$ | $\alpha > 1$: $\frac{1}{\alpha-1}\log\left\{\frac{\alpha}{2\alpha-1}\exp(\frac{\alpha-1}{\lambda}) + \frac{\alpha-1}{2\alpha-1}\exp(-\frac{\alpha}{\lambda})\right\}$<br>$\alpha = 1$: $1/\lambda + \exp(-1/\lambda) - 1 = .5/\lambda^2 + O(1/\lambda^3)$ |
| Gaussian Mechanism | $\infty$ | $\alpha/(2\sigma^2)$ |

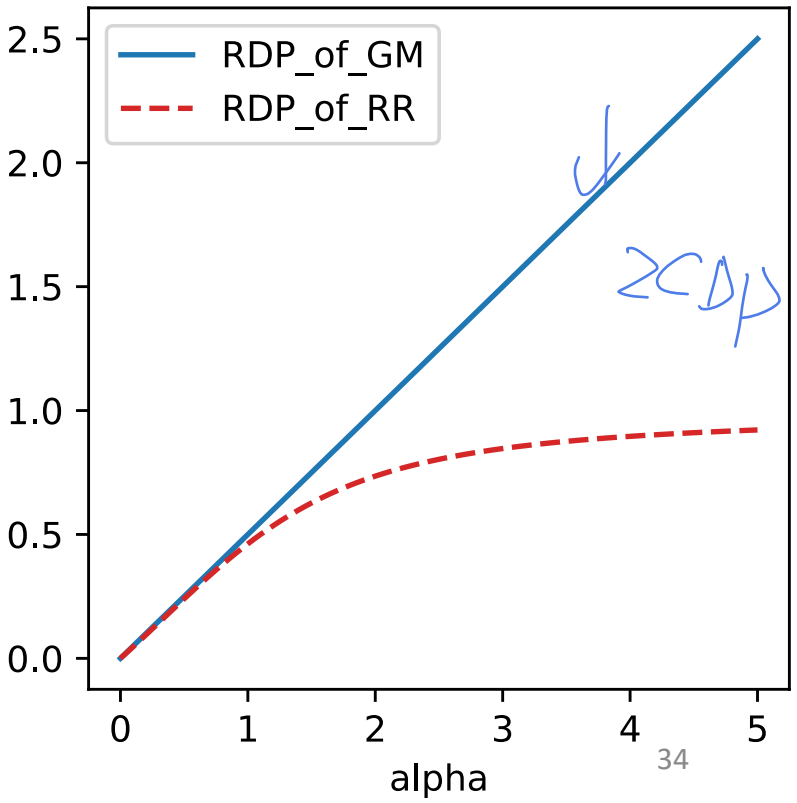
- Gaussian mechanism

$\frac{\Delta^2}{2\sigma^2}$ -zCDP

- Pure-DP mechanism

$\varepsilon$-

$\rho = \frac{1}{2}\varepsilon^2$ -zCDP



# Properties of Renyi DP / CDP

- Adaptive Composition
- Conversion to approximate DP

$$(\epsilon, \alpha)\text{-RDP implies } (\epsilon(\alpha) + \frac{\log(1/\delta)}{\alpha - 1}, \delta)\text{-DP}$$

$$\textit{If } M \textit{ provides } \rho\textit{-zCDP, then } M \textit{ is } (\rho + 2\sqrt{\rho \log(1/\delta)}, \delta)\textit{-DP.}$$

- Other properties: Postprocessing, risk multiplier, group privacy (see Mironov, 2017)

# Composition, revisited

- Composition of pure-DP mechanism via zCDP
- Composition of Gaussian mechanism via zCDP
- Baseline: Composition of Gaussian mechanism via Advanced Composition

# Next Lecture

- More on Renyi Differential Privacy
- Alternative characterization of DP
  - Privacy-profiles
  - Tradeoff functions
- Modern tool:  autodp