Lecture 9 Differentially Private Machine Learning

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COMPUTER SCIENCE
UC SANTA BARBARA
Computing. ReInvented.

Logistic notes

- Submit your HW1 and Project proposal if you haven't yet

- HW2 is on the course website
  - Except a code template for Q4 which I am still finalizing.

  - You should start working on the theoretical parts

Recap: Last lecture

- Composition of mechanism-specific representations
  - RDP accountant
  - Fourier accountant
- Unified treatment via a dominating privacy loss random variable
  - And its characteristic function
- autodp: How you would represent DP mechanism and compute privacy loss

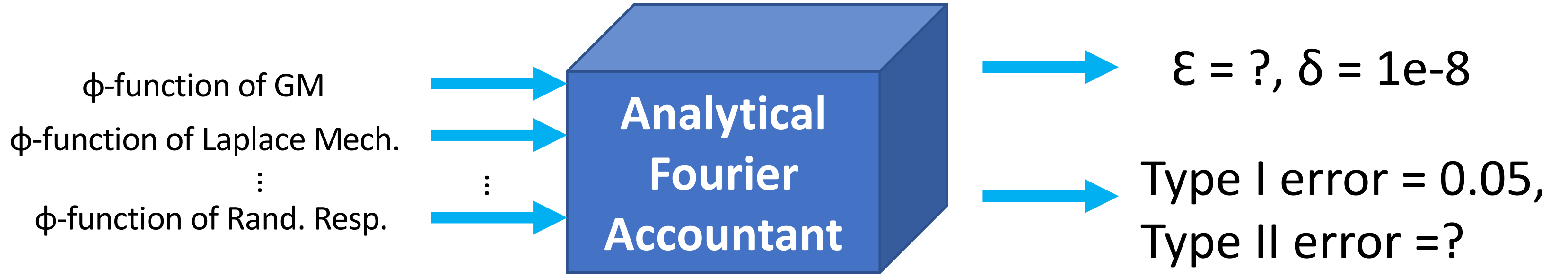
Recap: Mechanism specific analysis and privacy accounting

	Functional view	Pros	Cons
Renyi DP [Mironov, 2017]	$D_\alpha(P\|Q) \leq \epsilon(\alpha), \forall\alpha \geq 1$	Natural composition	lossy conversion to (ϵ, δ)-DP.
Privacy profile [Balle and Wang, 2018]	$\mathbb{E}_q[(\frac{p}{q} - e^\epsilon)_+] \leq \delta(e^\epsilon), \forall\epsilon \geq 0$	Interpretable.	messy composition.
f-DP[Dong et al., 2021]	Trade-off function f	Interpretable, CLT	messy composition.
PLD [Sommer et al., 2019, Koskela et al., 2020]	Probability density of $\log(p/q)$	Natural composition via FFT	Limited applicability.

Table 1: Modern functional views of DP guarantees and their pros and cons.

- Renyi DP is qualitatively different from approximate DP. Composition is quite natural with RDP.
- The composition of privacy-profiles and tradeoff functions are equivalent and somewhat messy.
  - The key to get it to work is to find a dominating pair
  - Using ϕ-function representation, we get the natural composition of RDP, and the tightness of privacy-profile / tradeoff functions.

Recap: Analytical Fourier accountant



- Composition: simply add up the log of phi functions
- Conversion to approx. DP via Levy's formula
- Conversion to tradeoff function via duality.

Zhu, Dong and W. (2020) https://arxiv.org/abs/2106.08567

Recap: autodp: automating differential privacy computation (for both laypersons and experts)

- Users describe their randomized algorithm to autodp

- autodp focuses on computing privacy losses

Open source project:
https://github.com/yuxiangw/autodp

pip install autodp

This lecture

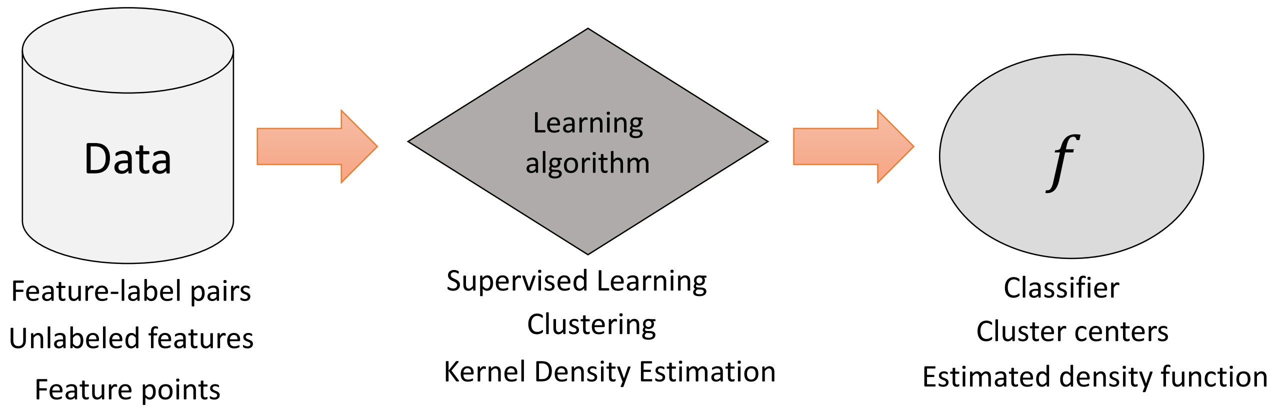
- Introduction to Differentially Private Machine Learning
  - Problem setup and notations
  - Examples
  - A learning theoretic study of the problem

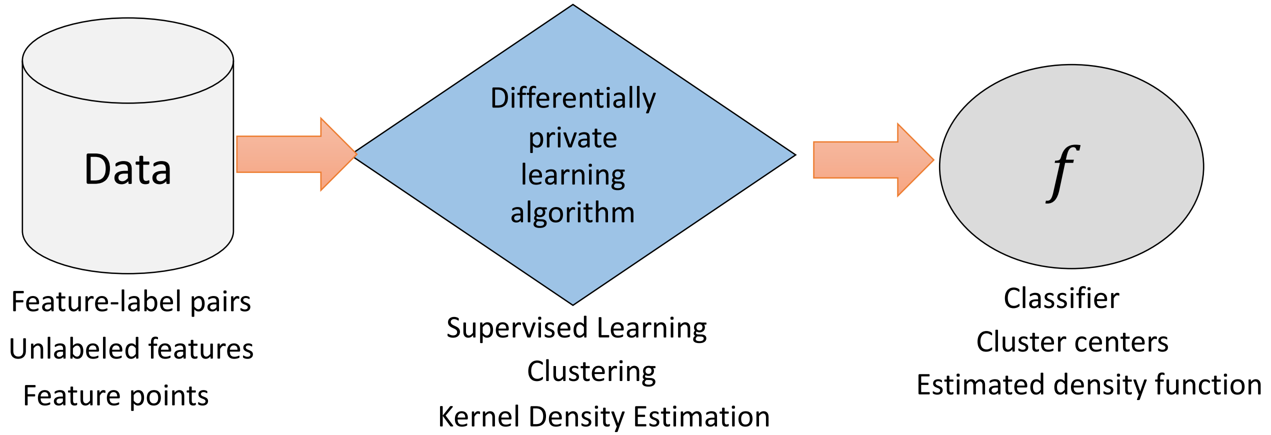
- Posterior sampling mechanism
  - When the loss functions are bounded
  - A new analysis of the when they are not

(Optional) reading materials

- "Learning with differential privacy": https://arxiv.org/pdf/1502.06309.pdf
- "Privacy-for-free: Posterior Sampling and Stochastic Gradient Monte Carlo" https://arxiv.org/abs/1502.07645
- "Differential privacy without sensitivity" , Minami et al. http://papers.neurips.cc/paper/6050-differential-privacy-without-sensitivity.pdf
- Other (not covered in this lecture)
  - "What can we learn privately?" https://arxiv.org/abs/0803.0924

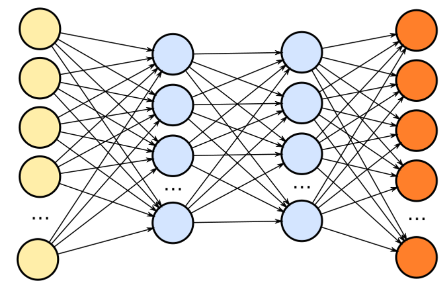
Differentially Private Machine Learning



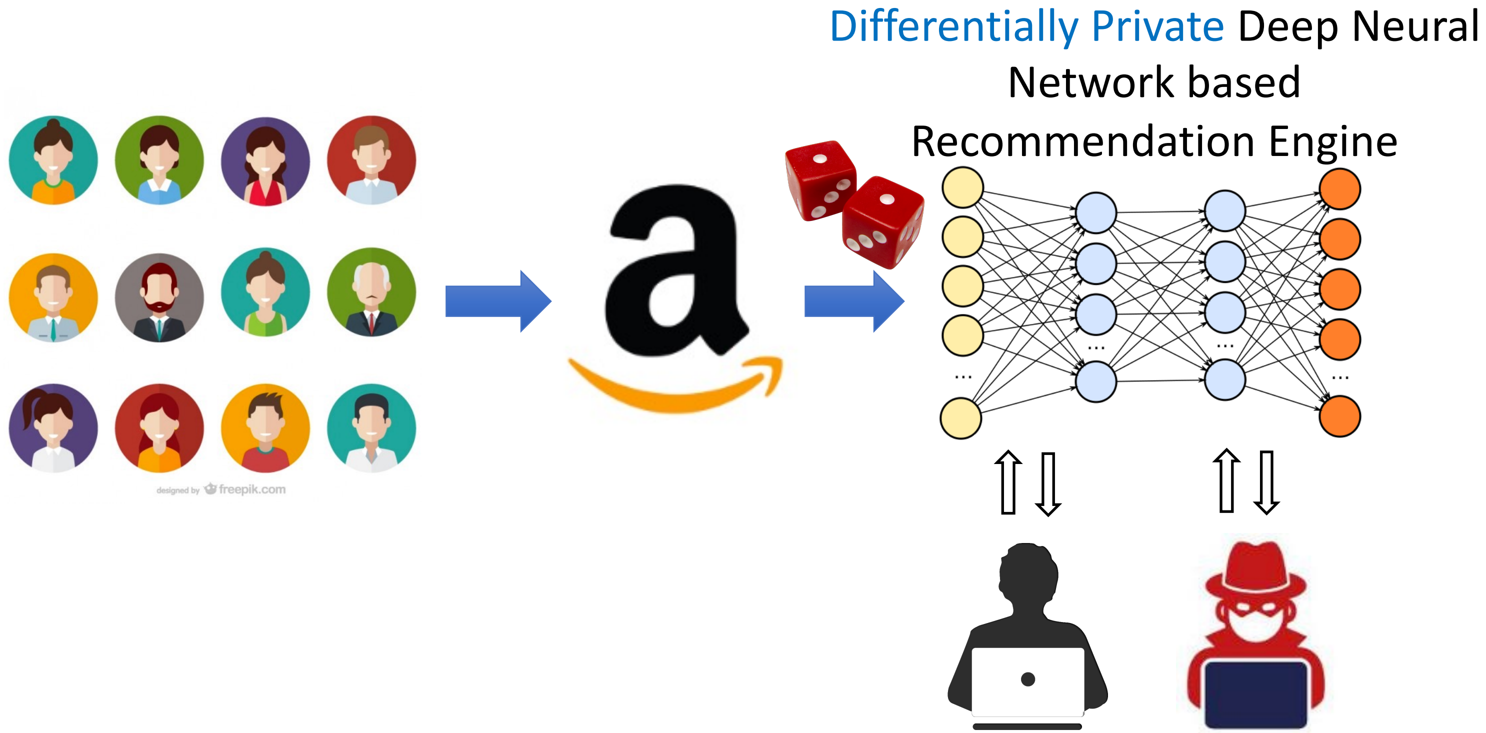


Example: Recommender System

Deep Neural Network based
Recommendation Engine



Example: Recommender System



“If your recommendation engine is private, then an adversary can’t infer whether a particular user was present”

A closely related setting: Federated Learning

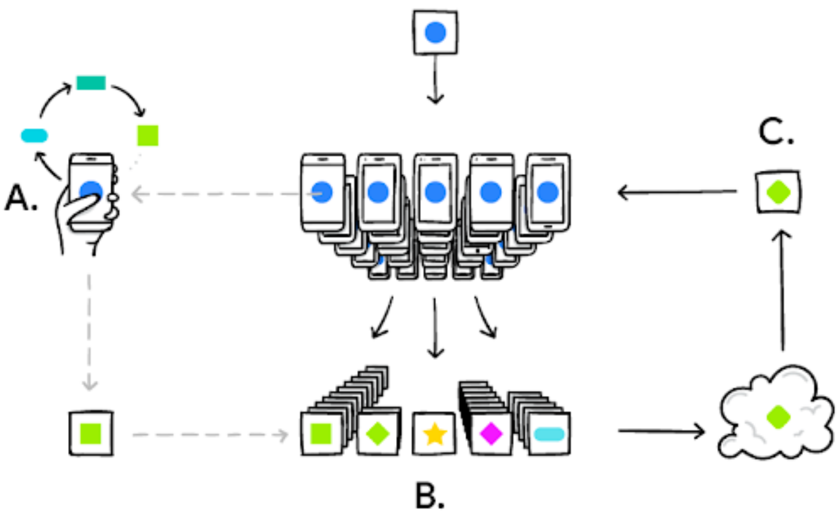


Illustration extracted from McMahan and Ramage (2017)

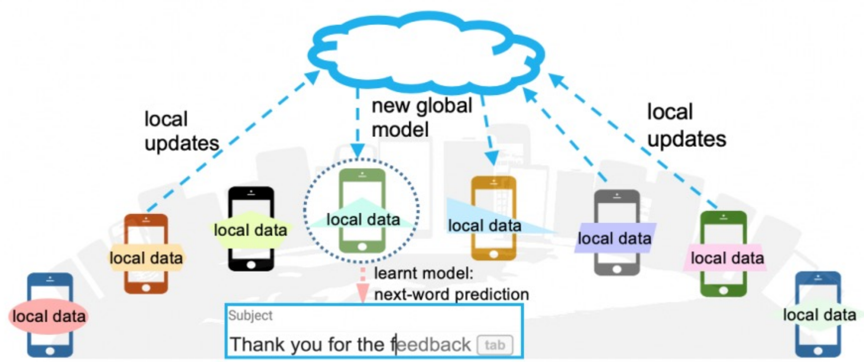
Additional considerations

- Communication cost
- Size of the model
- Rounds of adaptivity

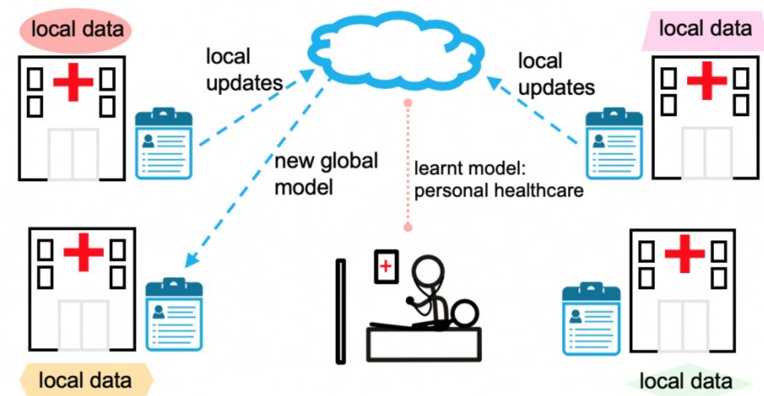
MPC ensures the security of distributed computation.
DP eliminates the risk from the output itself.

Two regimes of Federated Learning with DP

Regime 1 requires agent level DP



Regime 2 requires instance-level DP



(illustration taken from Tian Li et al, 2019)

***Need flexible tools for algorithm design.**

***Need tight privacy accounting to standardize empirical benchmarking of DP methods.**

Zhu, Y., Yu, X., Tsai, Y. H., Pittaluga, F., Faraki, M., & W. (2020). Voting-based Approaches For Differentially Private Federated Learning. *arXiv preprint arXiv:2010.04851*.

Notations and problem setup

- Data space
- Hypothesis space / model space
- Loss functions
- Learning algorithm

Example 1: Linear / Logistic regression

- Data space
- Hypothesis space / model space
- Loss functions
- Learning algorithm

Example 2: PAC learning / binary classification

- Data space

- Hypothesis space / model space

- Loss functions

- Learning algorithm

Example 3: k-means clustering

- Data space

- Hypothesis space / model space

- Loss functions

- Learning algorithm

Example 4: Recommender System / Matrix factorization

- Data space
- Hypothesis space / model space
- Loss functions
- Learning algorithm

What do we know about machine learning methods theoretically?

- Learnability
- Sample complexity of learning
- Key component: Uniform convergence / VC-theory

The fundamental problems in learning with differential privacy

- Private learnability
  - What problems are learnable with DP

- Sample complexity of private learning
  - Among those that are privately learnable, what is the number of samples needed to learn

- Efficient algorithms?

PAC learning with finite hypothesis space

- Standard statistical learning via Hoeffding's inequality and a union bound

Generic private optimization algorithm via exponential mechanism?

- Recap: utility of the EM

Theorem 3.11. Fixing a database x, let $\mathcal{R}_{\mathrm{OPT}} = \{r \in \mathcal{R} : u(x,r) = \mathrm{OPT}_u(x)\}$ denote the set of elements in $\mathcal{R}$ which attain utility score $\mathrm{OPT}_u(x)$. Then:

$$\Pr\left[u(\mathcal{M}_E(x,u,\mathcal{R})) \le \mathrm{OPT}_u(x) - \frac{2\Delta u}{\varepsilon}\left(\ln\left(\frac{|\mathcal{R}|}{|\mathcal{R}_{\mathrm{OPT}}|}\right) + t\right)\right] \le e^{-t}$$

- Let's apply this to the PAC learning problem with finite hypothesis space

What happens with continuous hypothesis space (satisfying VC-dim is finite)?

- Short answer: No DP algorithms can learn the VC class.
- Example: Learning a threshold function.

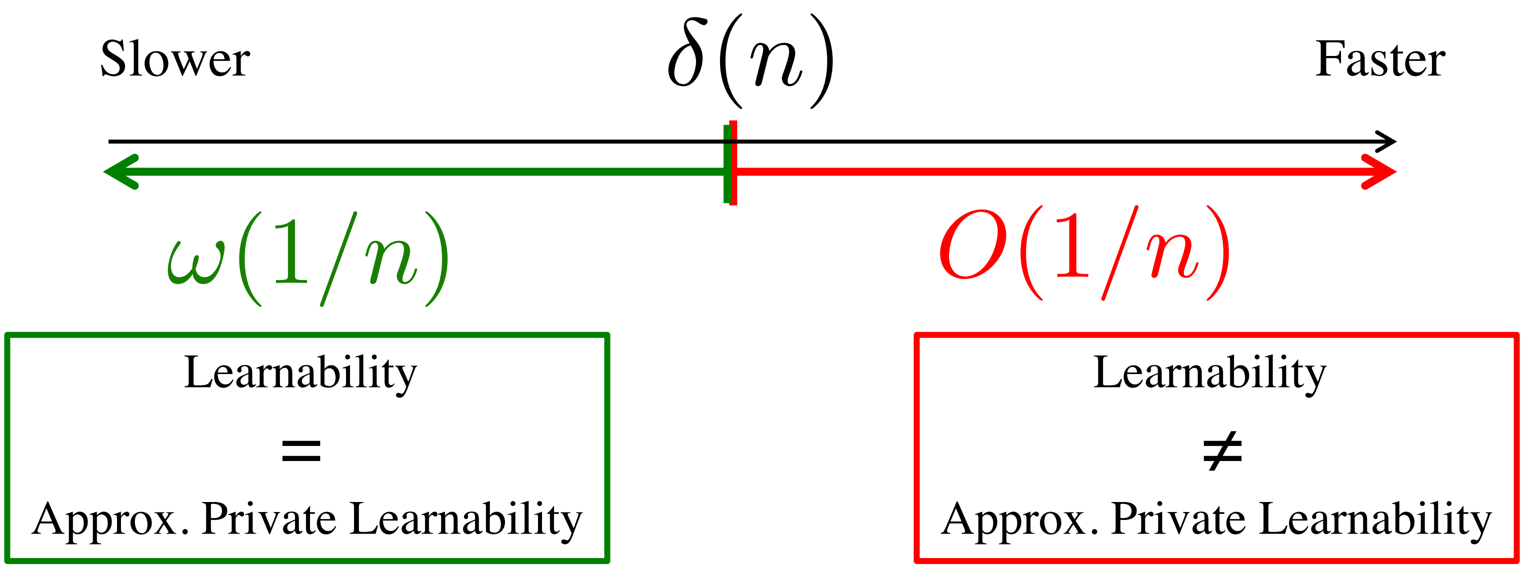
Any pure-DP algorithms will fail in learning threshold functions.

(Beimel et al. 2013) (Chaudhuri and Hsu. 2015)

Formalizing the lower bound.

(Beimel et al. 2013) (Chaudhuri and Hsu. 2015)

Is this an issue of pure-DP being too restrictive? Can we learn a VC class under approximate DP?



- Bun et. al. "Differentially private release and learning of threshold functions." FOCS'15

What is the key problem?

- Statistical learning requires a very strong notion of learning:
  - No assumptions on data distributions
  - Uniformly consistent learning for all distributions

- Differential privacy says that even if the data distributions are disjoint, they still need to be somewhat similar.
  - The construction relies on an exponentially large set of distributions (each coverging to a point-mass).

Remedies to this problem

- Assume the loss function is Lipschitz
  - 0-1 loss doesn't work, but we can use surrogate losses, e.g., logistic loss , hinge losses.

- Assume some regularity conditions on the probability distributions of data
  - e.g., Bounded probability density.

- Then most problems are learnable by an exponential mechanism.

W., Lei, and Fienberg. "Learning with differential privacy: Stability, learnability and the sufficiency and necessity of ERM principle." *The Journal of Machine Learning Research* 17.1 (2016): 6353-6392.

Check point: Learning with differential privacy in theory

- Finite hypothesis class
  - Exponential mechanism gives asymptotically vanishing additional error.
  - Does not benefit from being realizable.
- Continuous hypothesis class (bounded VC-dim)
  - No DP algorithm gives consistent learning
  - A Packing lower bound
- However, there are weak assumptions we can add
  - Lipschitz loss functions
  - data-distributions with bounded density

Connections of the Exponential mechanism to Bayesian learning

- Bayesian philosophy
  - I have a prior belief
  - When I collect data, I update my belief

- Deriving the posterior using iid data:

W., Fienberg, & Smola. (2015). Privacy for free: Posterior sampling and stochastic gradient monte carlo. In *International Conference on Machine Learning*.

Connections of the Exponential mechanism to Bayesian learning

- Getting one sample from the posterior distribution is equivalent to exponential mechanism

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Algorithm 1 One-Posterior Sample (OPS) estimator

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input Data X, log-likelihood function $\ell(\cdot|\cdot)$ satisfying $\sup_{\boldsymbol{x},\boldsymbol{\theta}} \|\ell(\boldsymbol{x}|\boldsymbol{\theta})\| \leq B$ a prior $\pi(\cdot)$.
Privacy loss ϵ.
1. Set $\rho = \min\{1, \frac{\epsilon}{4B}\}$.
2. Re-define log-likelihood function and the prior $\ell'(\cdot|\cdot) := \rho\ell(\cdot|\cdot)$ and $\pi'(\cdot) := (\pi(\cdot))^{\rho}$.

output $\hat{\boldsymbol{\theta}} \sim P(\boldsymbol{\theta}|X) \propto \exp\left(\sum_{i=1}^{N} \ell'(\boldsymbol{\theta}|\boldsymbol{x}_i)\right) \pi'(\boldsymbol{\theta})$.

- Some classical results from statistics
  - Asymptotic normality of the Bayesian posterior
  - Bernstein-Von Mises Theorem.
- Utility of the OPS estimator

Proposition 9. *Under the same assumption as Proposition 8, if we set a different ϵ by rescaling the log-likelihood by a factor of $\frac{\epsilon}{4B}$, then the the One-Posterior sample estimator obeys*

$$\sqrt{n}(\hat{\boldsymbol{\theta}} - \boldsymbol{\theta}_0) \overset{weakly}{\longrightarrow} \mathcal{N}\left(0, (1 + \frac{4B}{\epsilon})\mathbb{I}^{-1}\right),$$

in other word, the estimator has an ARE of $(1 + \frac{4B}{\epsilon})$.

Pros and cons of OPS

- Pros:
  - Generically applicable
  - Strong learning bounds under weak assumptions
  - No need to change existing learning workflow

- Cons:
  - It makes the distribution more diffused.
  - Can get only one sample (hard to do inference)
  - Requires bounded (clipped) loss functions
  - Computationally inefficient in general

Improved analysis of exponential mechanism with strong convexity

- Assume $\pi(\theta) = e^{-r(\theta)}$ where $r(\theta)$ is μ-strongly convex, i.e., the prior is strongly log-concave.

- Assume that the loss-function is Lipschitz

$$|-\log p(x|\theta) + \log p(x|\theta')| \leq L\|\theta - \theta'\|$$

- Then $\hat{\theta} \sim P(\theta|x_1, ..., x_n) \propto e^{(-\tau \sum_{\log p(x_i|\theta)} - r(\theta))}$

  obeys $(\epsilon, \delta) - \mathrm{DP}$ if $\tau = O(\frac{\epsilon\sqrt{\mu}}{L\sqrt{\log(1/\delta)}})$

Minami, K., Arai, H., Sato, I., & Nakagawa, H. (2016). Differential privacy without sensitivity. In *Advances in Neural Information Processing Systems* (pp. 956-964)

Idea of the proof for the improved analysis of EM

- The privacy loss random variable is

- Apply the tail bound lemma

- The strong log-concavity + Lipschitz assumption ensures that $\hat{\theta}$ satisfies a "Log-Sobolev Inequality"
  - Which ensures a subgaussian-like tail bound for all Lipschitz functions of $\hat{\theta}$
  - And a bound on the KL-divergence.

Minami, K., Arai, H., Sato, I., & Nakagawa, H. (2016). Differential privacy without sensitivity. In *Advances in Neural Information Processing Systems* (pp. 956-964)

Reiterate the main points

- Bayesian learning
  - Just a scaling of posterior sampling
  - For bounded log-likelihood functions, exponential mechanism is a consistent learner

- Boundedness is not needed if we use a strong prior
  - A tighter analysis of the exponential mechanism
  - Use a prior to ensure that PLRV is concentrated

Next lecture

- Convex empirical risk minimization
- Objective perturbation