Communication-Privacy-Accuracy Tradeoffs under Distributed DP for FL

University of California Santa Barbara

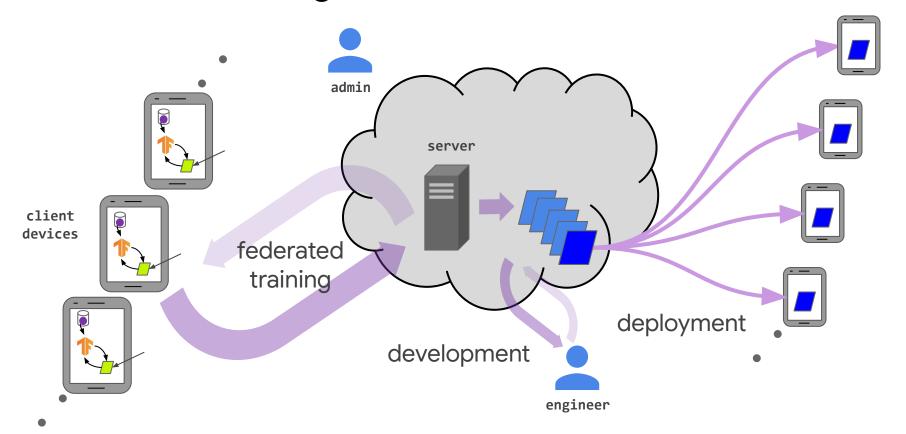


Peter Kairouz Research Scientist | Google

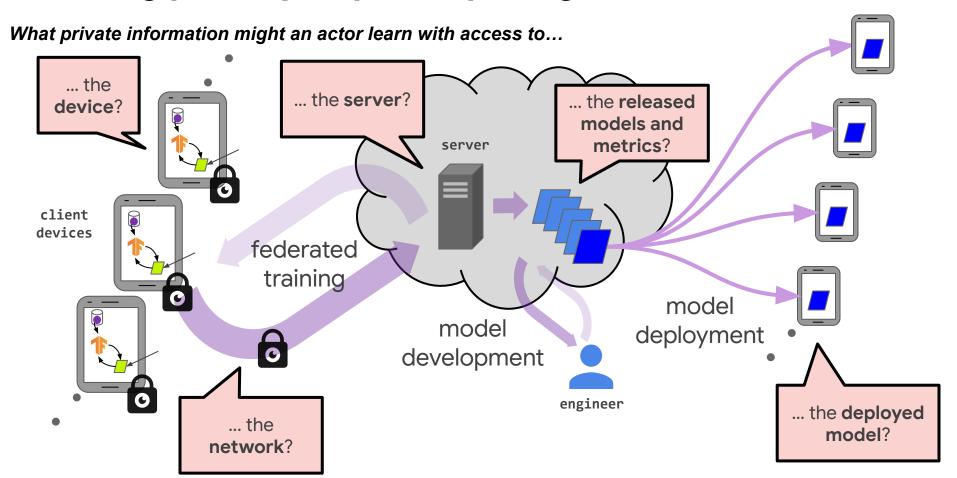
Twitter: @KairouzPeter

Presenting joint work with Ken Liu, Thomas Steinke, Naman Agarwal, and others

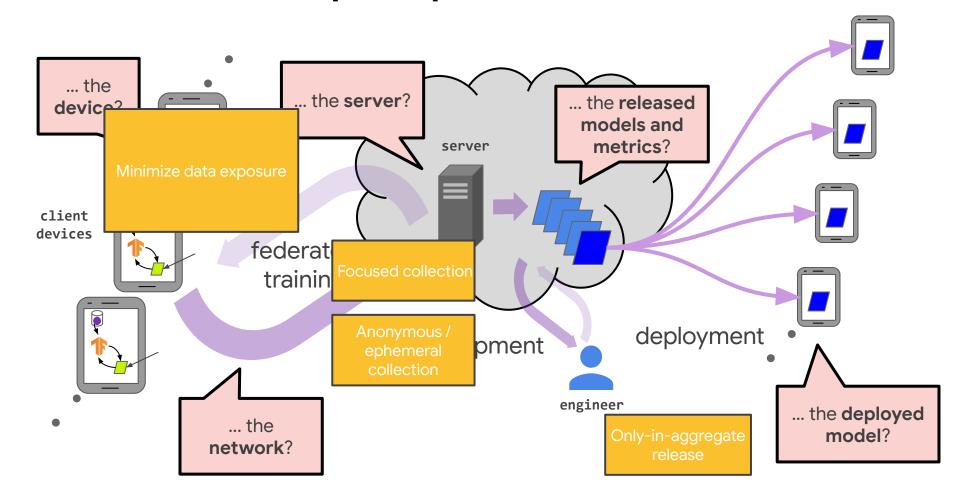
Federated Learning



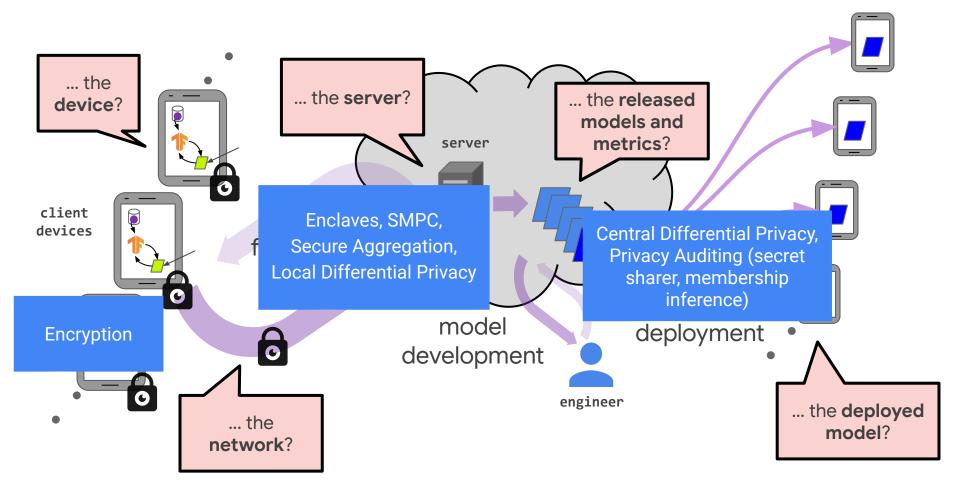
Ensuring privacy of participating users



Data minimization principles for FL

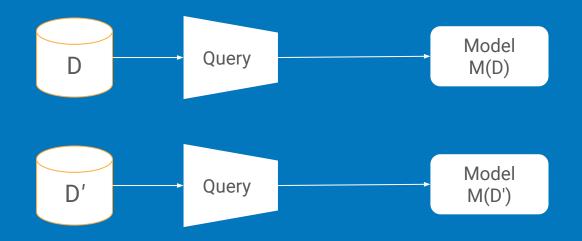


Complementary privacy technologies



Differential Privacy for FL

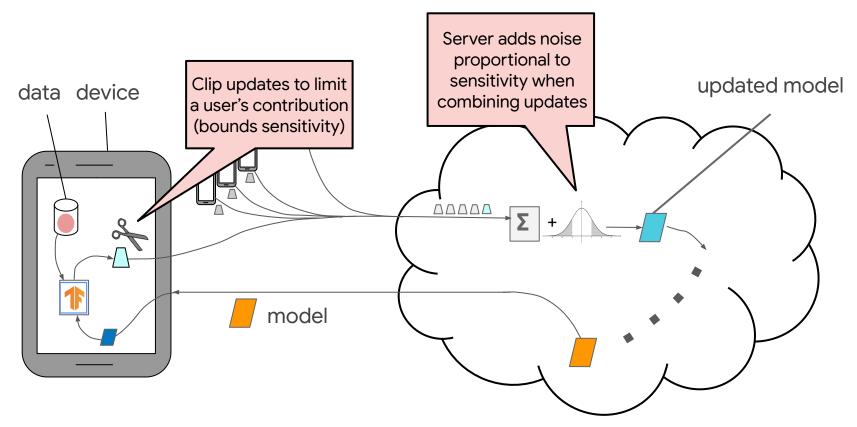
Differential Privacy



 (ε, δ) -Differential Privacy: The distribution of the output M(D) (a trained model) on database (training dataset) D is **nearly the same** as M(D') for all adjacent databases D and D'

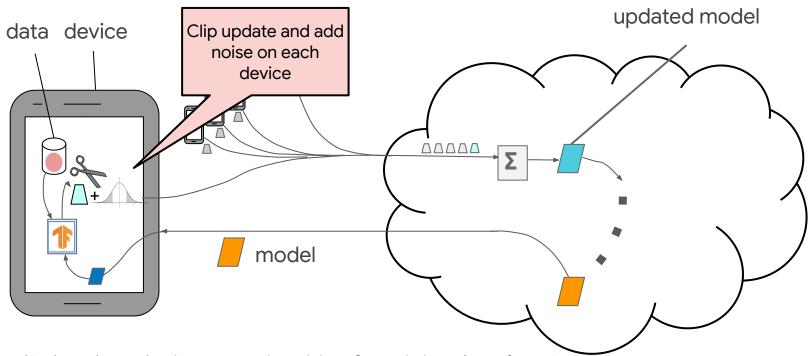
 $\forall S$: $\Pr[M(D) \in S] \le \exp(\varepsilon) \cdot \Pr[M(D') \in S] + \delta$

Centrally differentially private federated learning



H. B. McMahan, et al. Learning Differentially Private Recurrent Language Models. ICLR 2018

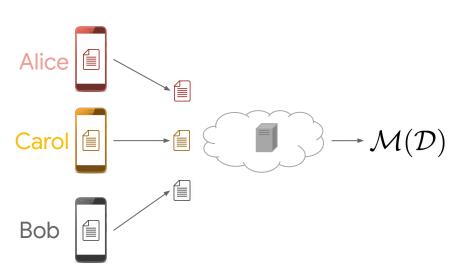
Locally differentially private federated training



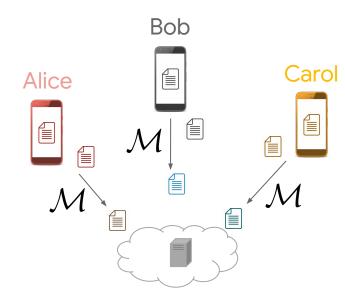
Evfimievski, Alexandre, et al. **Privacy preserving mining of association rules.** Information Systems 2004 Warner, Stanley L. **Randomized response: A survey technique for eliminating evasive answer bias.** JASA 1965

Can we combine the best of both worlds?

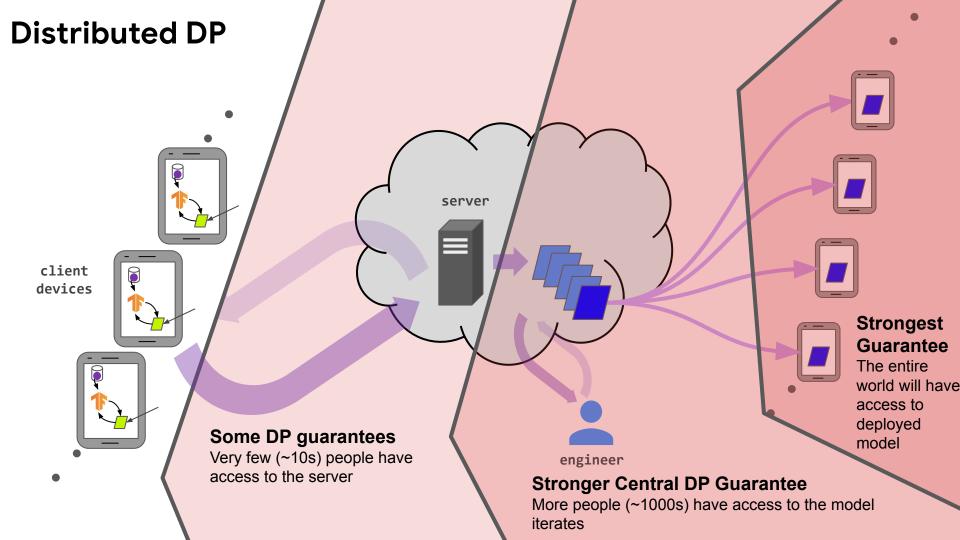
Distributed Differential Privacy



Central DP: full trust in service provider
Higher utility at reasonable privacy levels

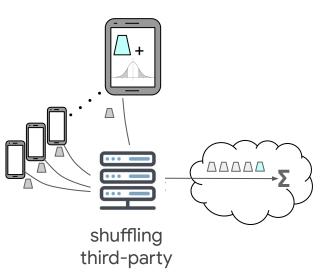


Local DP: weaker trust assumptions
Utility often suffers



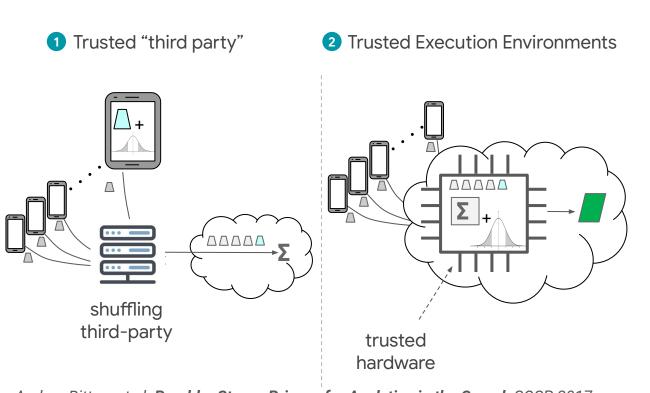
Distributing trust for private aggregation

1 Trusted "third party"



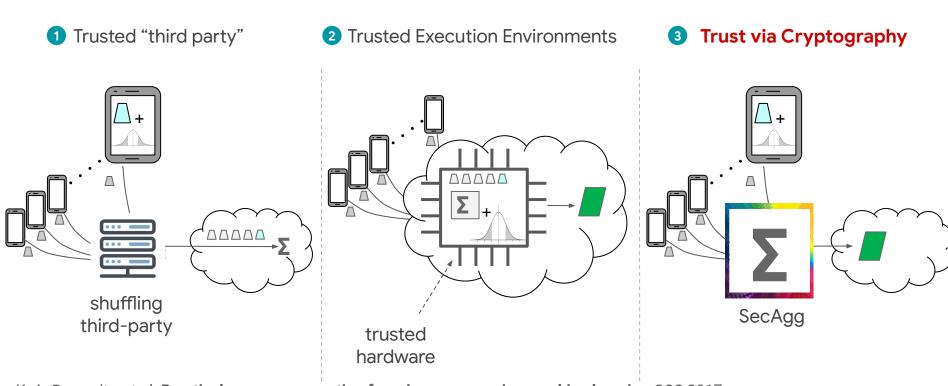
Andrea Bittau, et al. **Prochlo: Strong Privacy for Analytics in the Crowd**. SOSP 2017 Úlfar Erlingsson, et al. **Amplification by Shuffling: From Local to Central Differential Privacy via Anonymity.** SODA 2019

Distributing trust for private aggregation



Andrea Bittau, et al. **Prochlo: Strong Privacy for Analytics in the Crowd**. SOSP 2017 Úlfar Erlingsson, et al. **Amplification by Shuffling: From Local to Central Differential Privacy via Anonymity.** SODA 2019

Distributing trust for private aggregation

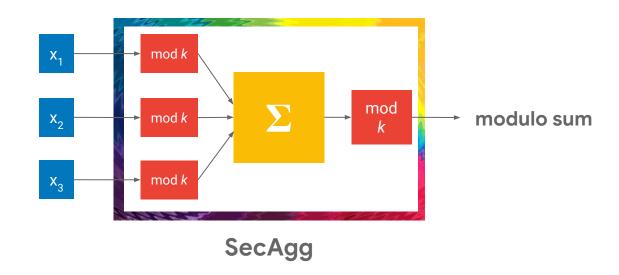


K. A. Bonawitz, et al. **Practical secure aggregation for privacy-preserving machine learning** CCS 2017 J. Bell, et al. **Secure Single-Server Vector Aggregation with (Poly) Logarithmic Overhead** CCS 2020

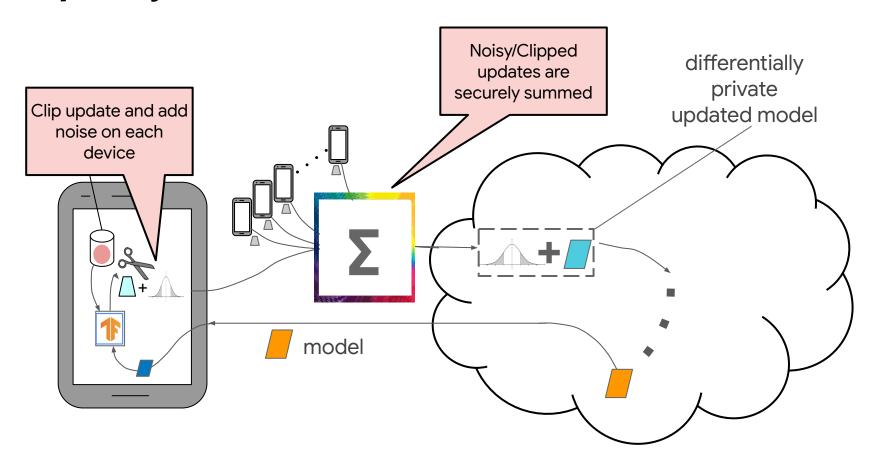
Secure Aggregation allows a server to obtain the sum of high-dimensional vectors of client-held data in a way that ensures (<u>cryptographically</u>) that the server learns *just the sum*, and *no individual data whatsoever* *.

^{*} even if some users are malicious (and collude with the server), and some drop out.

SecAgg: a closed box that performs integer modulo sums



Why not just add continuous Gaussian noise?



- SecAgg operates on a finite group with integer modulo arithmetic
 - Need clever data discretization methods that do not inflate the sensitivity
 - Cannot use continuous mechanisms

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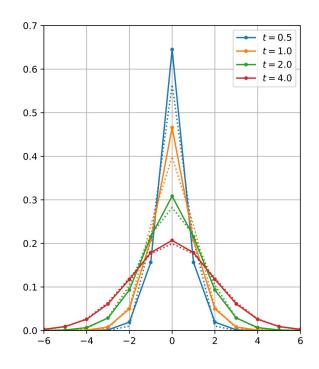
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- Discrete mechanisms with finite tails* do not satisfy Rényi or concentrated DP
 - o Avoids catastrophic privacy failures and allows for tight privacy accounting

*For example, the multi-dimensional binomial mechanism (Agarwal, et al. cpSGD: Communication-efficient and differentially-private distributed SGD. NeurIPS 2018.) does not achieve Rényi DP

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- Need mechanisms that can be sampled from exactly and efficiently

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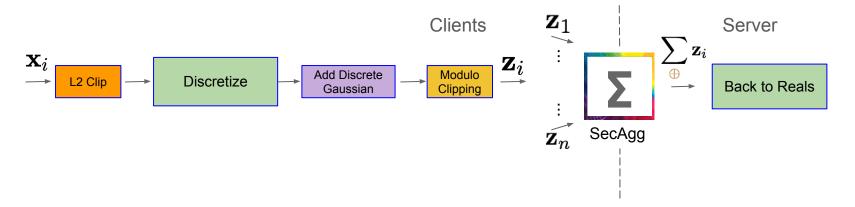
The discrete Gaussian mechanism



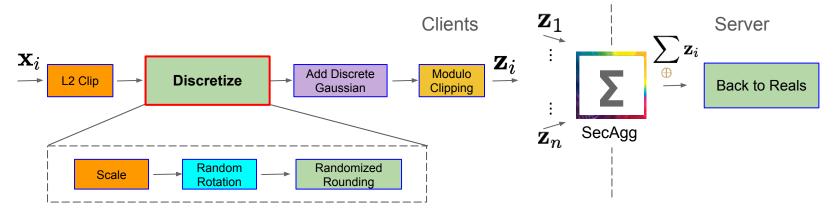
$$\mathbb{P}_{X \leftarrow \mathcal{N}_{\mathbb{Z}}(\mu, t^2)}[X = n] = \frac{e^{-(n-\mu)^2/2t^2}}{\sum_{k \in \mathbb{Z}} e^{-(k-\mu)^2/2t^2}}$$

- Discrete Gaussian: discrete analog of continuous Gaussian (≠ rounding Gaussian to nearest ints)
 - Essentially **the same privacy-accuracy trade-off** as continuous Gaussian*
 - zCDP / Rényi DP for tight compositions in learning contexts
- **Problem:** sums of discrete Gaussians ≠ discrete Gaussians

The Distributed Discrete Gaussian Mechanism

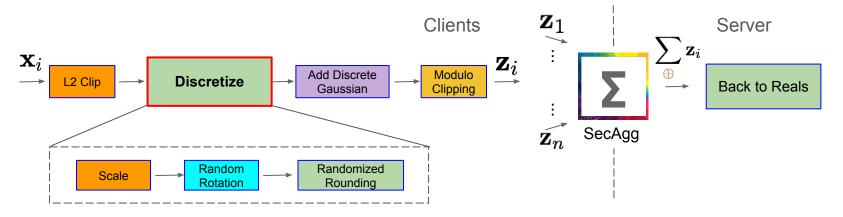


Data Quantization



- **Scaling:** stretching the signal → reduces quantization error
- Random rotation: "flatten" concentrated coordinates → controls the L-inf norm
- Randomized rounding: values stochastically rounded to integers (unbiased)

Data Quantization



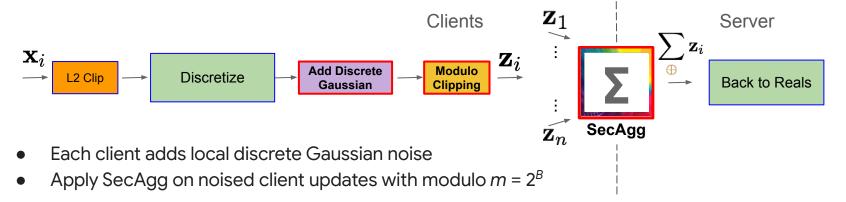
- **Scaling:** stretching the signal → reduces quantization error
- Random rotation: "flatten" concentrated coordinates → controls the L-inf norm
- Randomized rounding: values stochastically rounded to integers (unbiased)
- We can probabilistically bound the L2 norm growth from rounding (helps reduce DP noise):

$$\begin{array}{|c|c|c|c|c|} \hline \textbf{Proposition 22} & \text{(Properties of Randomized Rounding). } Let \ \beta \in [0,1), \ \gamma > 0, \ and \ x \in \mathbb{R}^d. \\ & \Delta_2^2 := \min \left\{ \begin{aligned} \|x\|_2^2 + \frac{1}{4}\gamma^2 d + \sqrt{2\log(1/\beta)} \cdot \gamma \cdot \left(\|x\|_2 + \frac{1}{2}\gamma\sqrt{d}\right), \\ & \left(\|x\|_2 + \gamma\sqrt{d}\right)^2 \end{aligned} \right\} \end{aligned}$$

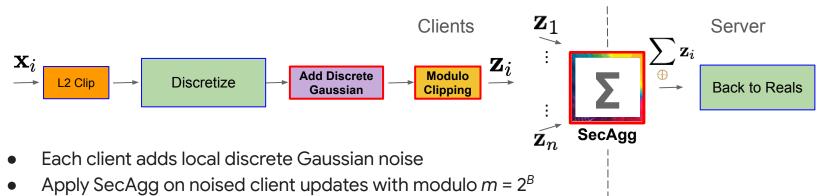
d: client vector dim

γ: rounding granularity;inverse scaling factorβ: rounding bias

Local Noising & (Secure) Sums of Discrete Gaussians



Local Noising & (Secure) Sums of Discrete Gaussians



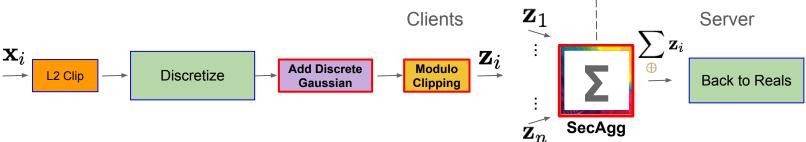
 While sums of discrete Gaussians ≠ discrete Gaussian, we show that they are extremely close:

Theorem 11 (Convolution of two Discrete Gaussians). Let $\sigma, \tau \geq \frac{1}{2}$. Let $X \leftarrow \mathcal{N}_{\mathbb{Z}}(0, \sigma^2)$ and $Y \leftarrow \mathcal{N}_{\mathbb{Z}}(0, \tau^2)$ be independent. Let Z = X + Y. Let $W \leftarrow \mathcal{N}_{\mathbb{Z}}(0, \sigma^2 + \tau^2)$. Then

$$\mathrm{D}_{\pm\infty}(Z\|W) = \sup_{z\in\mathbb{Z}} \left|\logigg(rac{\mathbb{P}[Z=z]}{\mathbb{P}[W=z]}igg)
ight| \leq 5\cdot e^{-2\pi^2/\left(1/\sigma^2+1/ au^2
ight)}.$$

- Exponentially small with larger variance; $\leq 10^{-12}$ if $\sigma_1^2 = \sigma_2^2 = 3$.
 - Noise is added on quantized client values, so σ^2 is scaled and this is even smaller

Local Noising & (Secure) Sums of Discrete Gaussians



- Each client adds local discrete Gaussian noise
- Apply SecAgg on noised client updates with modulo $m = 2^{B}$
- While sums of discrete Gaussians ≠ discrete Gaussian, we show that they are extremely close:

Theorem 11 (Convolution of two Discrete Gaussians). Let $\sigma, \tau \geq \frac{1}{2}$. Let $X \leftarrow \mathcal{N}_{\mathbb{Z}}(0, \sigma^2)$ and $Y \leftarrow \mathcal{N}_{\mathbb{Z}}(0, \tau^2)$ be independent. Let Z = X + Y. Let $W \leftarrow \mathcal{N}_{\mathbb{Z}}(0, \sigma^2 + \tau^2)$. Then

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ight)}.$$

- Weak dependence on d (number of model params);
- 1st term same as central Gaussian/DGaussian

Main Privacy Guarantees

with *n* clients:

$$au := 10 \cdot \sum_{k=1}^{n-1} e^{-2\pi^2 rac{\sigma^2}{\gamma^2} \cdot rac{k}{k+1}} \ \int \sqrt{rac{\Delta_2^2}{n\sigma^2} + rac{1}{2} au d} \, \Big)$$

$$:= \min \left\{ egin{array}{l} \sqrt{rac{1}{n\sigma^2} + rac{1}{2}} au \ rac{\Delta_2}{\sqrt{n}\sigma} + au \sqrt{d} \end{array}
ight.$$

Stack Overflow Next Word Prediction



- Next word prediction for question/answer sentences on StackOverflow.com with LSTMs
- $\sim 10^9$ sentences grouped by the N = 342477 SO users/clients
- Fig. 1: DDGauss matches continuous Gaussian as long as the bit-width B is sufficient
- Fig. 2: DDGauss scales (1000 clients per round) and works in low-noise (utility-first) settings

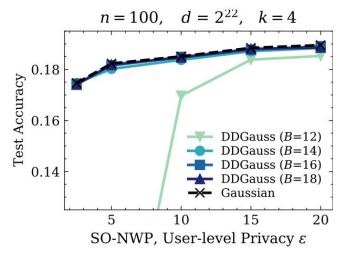


Fig. 1: Test acc with different ε and B (n = 100).

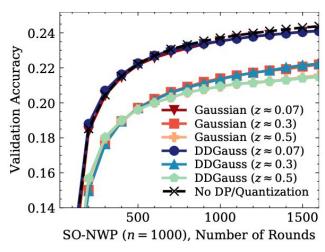
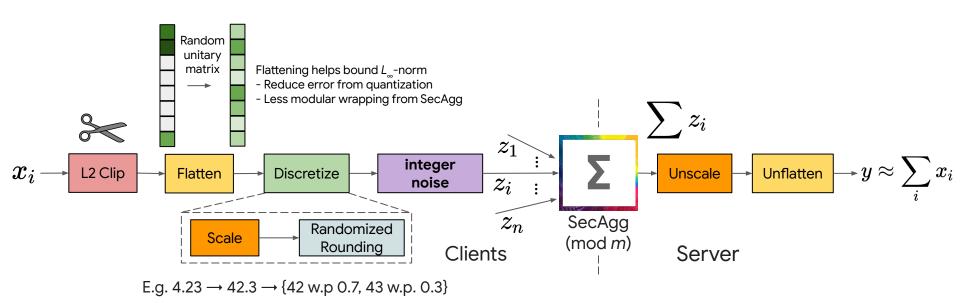


Fig. 2: Val acc with with n = 1000 clients, B = 18. z: approximate noise multiplier aligned on ε .

Code: https://github.com/google-research/federated/tree/master/distributed_dp

Our end-to-end solution



(Symmetric) Skellam Distribution

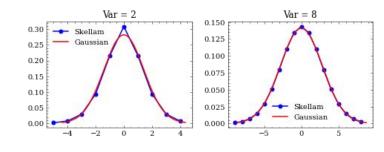
• Difference of two independent Poisson RVs. With mean Δ and variance μ ,

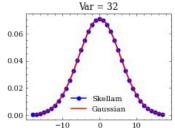
 $I_k(z)$: modified Bessel function of the first kind

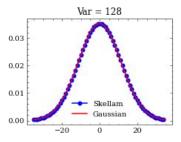
$$X_i \sim \mathrm{Sk}_{\Delta_i,\mu} \;\; ext{with} \;\; P(X_i = k) = e^{-\mu} I_{k-\Delta_i}(\mu)$$

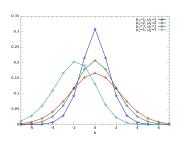
- Closed under summation: easily switch between central DP & distributed DP (central vs local noise)
- Easy to sample: `np.random.poisson`
- Skellam gets closer to Gaussian as variance increases and we scale the output appropriately
- **Skellam Mechanism**: for an integer-valued query f(D),

$$\mathrm{Sk}_{0,\mu}(f(D)) = f(D) + Z ext{ where } Z \sim \mathrm{Sk}_{0,\mu}$$







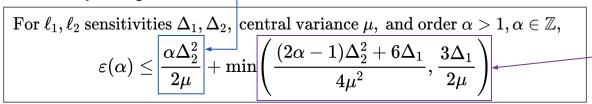


(Distributed) Skellam

 $X \sim \mathrm{Sk}_{\Delta,\mu}(X) riangleq e^{-\mu} I_{k-\Delta}(\mu)$

Main Rényi DP guarantee

Gaussian RDP



L1 bound (after quantization)

$$\Delta_1 \leq \Delta_2 \cdot \min\Bigl(\sqrt{d}, \Delta_2\Bigr)$$

2nd term goes to 0 with larger variance (higher privacy)

(Distributed) Skellam

 $X \sim \mathrm{Sk}_{\Delta,\mu}(X) riangleq e^{-\mu} I_{k-\Delta}(\mu)$

Main Rényi DP guarantee

Gaussian RDP

 $\varepsilon(\alpha) \leq \frac{\alpha \Delta_2^2}{2\mu} + \min\left(\frac{(2\alpha - 1)\Delta_2^2 + 6\Delta_1}{4\mu^2}, \frac{3\Delta_1}{2\mu}\right)$

 $\Delta_1 \leq \Delta_2 \cdot \min\Bigl(\sqrt{d}, \Delta_2\Bigr)$

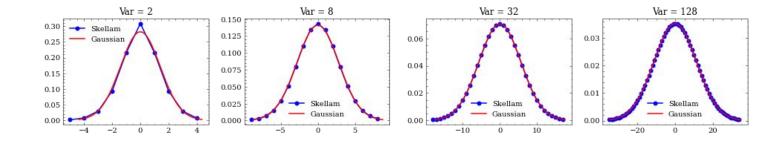
L1 bound (after quantization)

2nd term goes to 0 with larger variance (higher privacy or large scaling)

Effect of scaling (scale both noise stddev and sensitivity)

Corollary 4.1 (Scaled Skellam Mechanism). With a scaling factor $s \in \mathbb{R}$, the multi-dimensional Skellam Mechanism is (α, ε) -RDP with

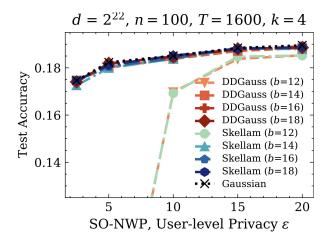
$$arepsilon(lpha) \leq rac{lpha\Delta_2^2}{2\mu} + \min\!\left(rac{(2lpha-1)\Delta_2^2}{4s^2\mu^2} + rac{3\Delta_1}{2s^3\mu^2}, rac{3\Delta_1}{2s\mu}
ight)$$

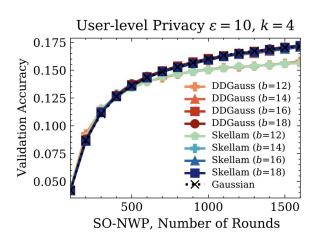


Stack Overflow Next Word Prediction



- Next word prediction for questions/answers sentences on StackOverflow.com with LSTMs
- \sim 10° sentences grouped by N = 342477 users on Stack Overflow
- Left: Test acc across various privacy levels ε and bit-widths b
- **Right:** Validation acc across training rounds
- Skellam matches continuous Gaussian and distributed discrete Gaussian





Code: https://github.com/google-research/federated/tree/master/distributed_dp

Better communication efficiency?

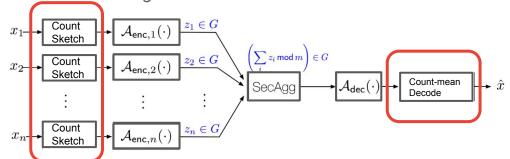
To achieve centralized error of $O\left(\frac{d}{n^2\varepsilon^2}\right)$ each client must transmit $\Omega\left(d\log\left(\frac{n^2\varepsilon^2}{d}\right)\right) = \tilde{\Omega}\left(d\right)$ its.

Better communication efficiency?

To achieve centralized error of
$$\left(O\left(\frac{d}{n^2\varepsilon^2}\right)\right)$$
 each client must transmit $\Omega\left(d\log\left(\frac{n^2\varepsilon^2}{d}\right)\right) = \tilde{\Omega}\left(d\right)$ its.

In the worst-case, each client **cannot** transmit less than the entire gradient!

- But, gradients may be near-sparse! Is their sum?
- We can leverage this structure to compress each x_i !



• We will use a count-mean sketch: **efficient** and **linear** dimensionality reduction

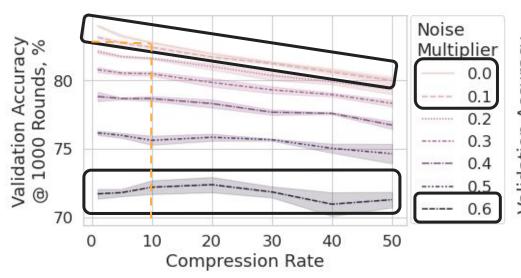
Experiments

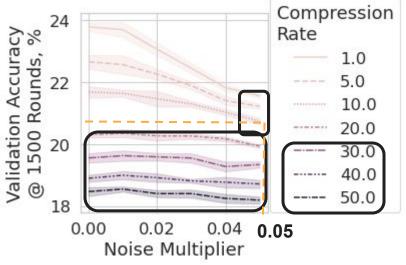


Noise Multiplier ↑ → ��
Accuracy ↑

Stack Overflow Next Word Prediction @ 100 Clients

Federated EMNIST-62 @ 100 Clients





Challenges & Opportunities

Open technical challenges in privacy

- Privacy is multifaceted
 - Need to better understand *privacy, communication, computation, accuracy, sparsity* tradeoffs
 - Tensions between privacy, robustness, and fairness are very interesting and remain underexplored – personalization may play an important role in easing the tensions
 - Cryptographic techniques will play a critical role in strengthening privacy

Differential privacy provides an incredibly useful tool

- But it often comes at a "hit" in accuracy
- If we have to pay, we'd usually rather pay with more computation (not privacy or accuracy)
- How to choose epsilon remains (and perhaps will always be) an open question
- Output Description

 How to make sense of large-ish epsilons?
- Model auditing techniques for measuring privacy loss (memorization) are complimentary

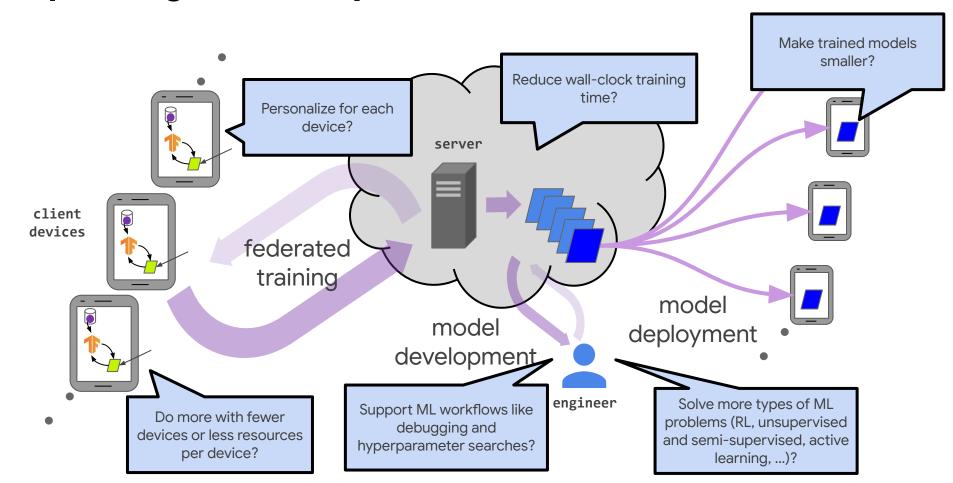
Privacy budgeting and management systems are not available

- Can scientists apply complex and repeated learning tasks on the same or similar datasets?
- O How do we efficiently track and quantify the privacy loss of a complex system?

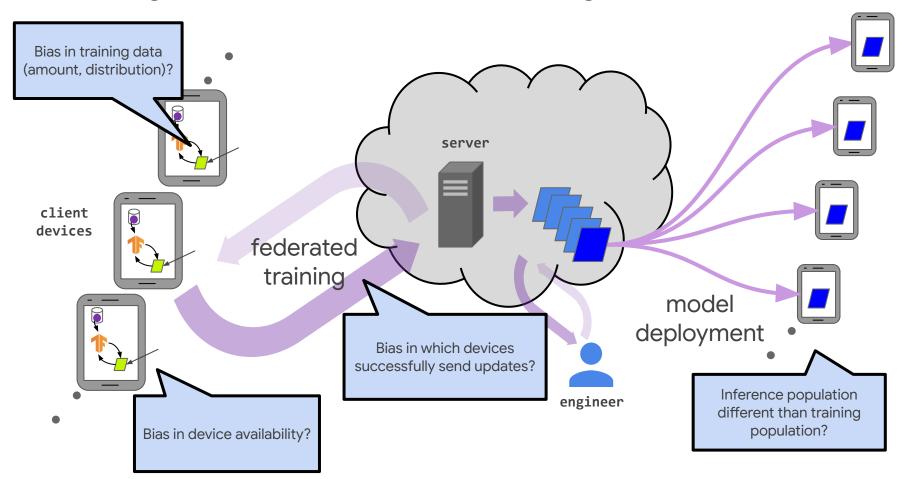
Public data is largely underutilized

- Public data will play a key role in improving privacy-accuracy tradeoffs
- How do we optimally combine public and private datasets during training?

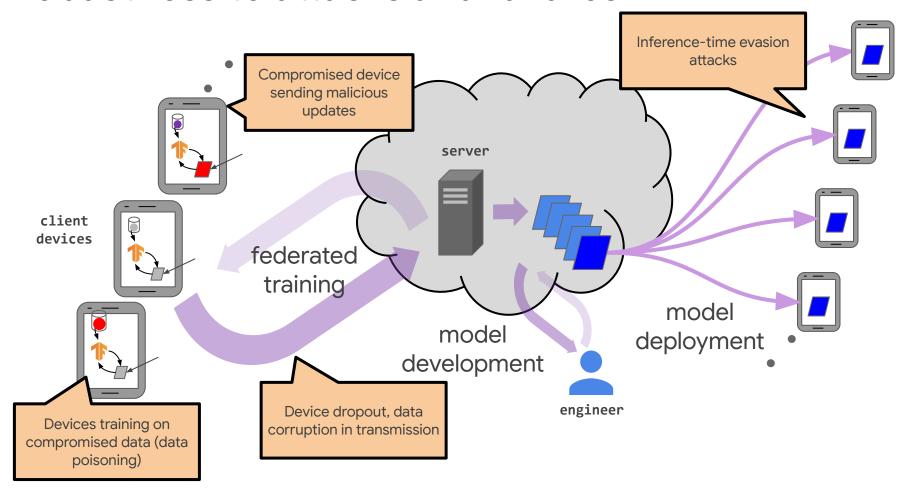
Improving efficiency and effectiveness



Ensuring fairness and addressing sources of bias •



Robustness to attacks and failures



Advances and Open Problems in Federated Learning

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 ¹⁴Rutgers University, ¹⁵Stanford University, ¹⁶University of California Berkeley,
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Abstract

Federated learning (FL) is a machine learning setting where many clients (e.g. mobile devices or whole organizations) collaboratively train a model under the orchestration of a central server (e.g. service provider), while keeping the training data decentralized. FL embodies the principles of focused data collection and minimization, and can mitigate many of the systemic privacy risks and costs resulting from traditional, centralized machine learning and data science approaches. Motivated by the explosive growth in FL research, this paper discusses recent advances and presents an extensive collection of open problems and challenges.

Advances and Open Problems in FL

59 authors from 25 top institutions

arxiv.org/abs/1912.04977

Foundations and Trends in Machine Learning



A Field Guide to Federated Optimization

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Abstract

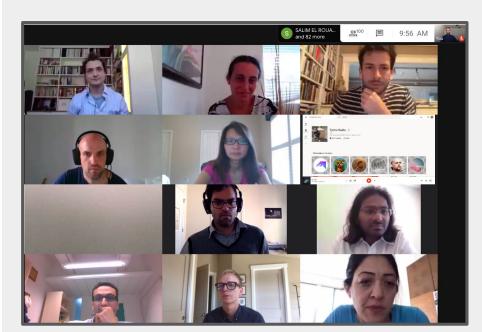
Federated learning and analytics are a distributed approach for collaboratively learning models (or statistics) from decentralized data, motivated by and designed for privacy protection. The distributed learning process can be formulated as solving federated optimization problems, which emphasize communication efficiency, data heterogeneity, compatibility with privacy and system requirements, and other constraints that are not primary considerations in other problem settings. This paper provides recommendations and guidelines on formulating, designing, evaluating and analyzing federated optimization algorithms through concrete examples and practical implementation, with a focus on conducting effective simulations to infer real-world performance. The goal of this work is not to survey the current literature, but to inspire researchers and practitioners to design federated learning algorithms that can be used in various practical applications.

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Tensorflow Federated Implementation



Thank you for your time!

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