Efficient Computation of Robust Low-Rank Matrix Approximations in the Presence of Missing Data using the L_1 Norm

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Outline

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Introduction: Problem formulation

- Low-rank matrix approximation in the presence of missing data.

$$\min_{U,V} ||\hat{W} \odot (Y - UV)||,$$

$Y \in \mathbb{R}^{m \times n}$ $U \in \mathbb{R}^{m \times r}$ $V \in \mathbb{R}^{r \times n}$ $\hat{W} \in \mathbb{R}^{m \times n}$

Introduction: Background

- Many applications such as SfM, shape from varying illumination (photometric stereo).
- Simple SVD (ML minimization of L2 Norm) is not applicable due to missing data.
- L2 norm is sensitive to outliers.
- L1 norm is used to reduce sensitivity to outliers.

Introduction: Advantage of L1 Nnorm

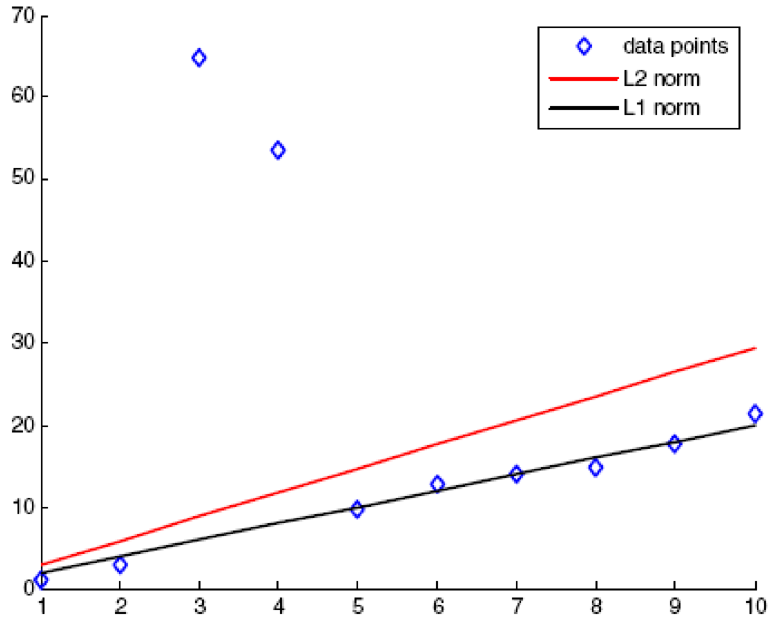


Figure 1. *Fit a line to 10 given data points. The two data points on upper-left are outliers.*

Extracted from [13]

Introduction: Challenges of L1

- Non-smooth (hence non-differentiable)
- Non-convex due to rank constraint and missing data. (Is this related to adoption of L1 norm?)
- More computational demanding than L2 (we'll see)

Key contribution of this paper

- Extends Wiberg algoritm to L1 norm
- Proposed algorithm is more efficient than other algorithms that deal with same problem.
- Address non-smoothness and computational requirement using LP.

Previous Works

[5] Buchanan, a quantitative survey

[16] Okatani 岡谷, Reintroduce Wiberg to Computer Vision (more to come)

[13] Ke&Kanade, ALP and AQP (state-of-art as claimed, explained in next slide)

[9] De la Torre&Black a robust version of PCA based on Huber distance.

[6] Chandraker&Kriegman, certain combinatorial optimization method to reach global optimal.

Previous Works: Ke & Kanade

- Ke&Kanade, Robust L1 norm factorization in the presence of outliers and missing data by alternative convex programming, CVPR2005
- Alternative Convex Programming

$$\mathtt{V}^{(t)} = \arg\min_{\mathtt{V}} \|\mathtt{M} - \mathtt{U}^{(t-1)}\mathtt{V}^{\top}\|_{L_1} \tag{8a}$$

$$\mathtt{U}^{(t)} = \arg\min_{\mathtt{U}} \|\mathtt{M} - \mathtt{U}\mathtt{V}^{(t)\top}\|_{L_1} \tag{8b}$$

Previous Works: Ke & Kanade

- Alternated Linear Programming:

$$E(\mathtt{V}) = \|\mathtt{M} - \mathtt{U}^{(t-1)}\mathtt{V}^\top\|_{L_1} = \sum_{j=1}^{n} \|\mathbf{m}_j - \mathtt{U}^{(t-1)}\mathbf{v}_j\|_1 \quad (9)$$

where $\mathbf{m}_j$ is the j-th column of $\mathtt{M}$, $\mathbf{v}_j$ is the j-th column of $\mathtt{V}^\top$. The problem of Eq. (8a) is therefore decomposed into *n independent small* sub-problems, each one optimizing $\mathbf{v}_j$:

$$\mathbf{v}_j = \arg\min_{\mathbf{x}} \|\mathtt{U}^{(t-1)}\mathbf{x} - \mathbf{m}_j\|_1 \quad (10)$$

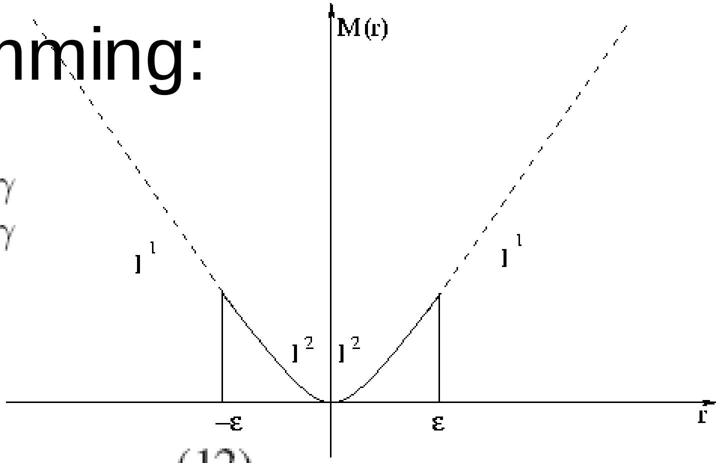
The *global* optimal solution of Eq. (10) can be found by the following linear program (LP):

$$\min_{\mathbf{x},\mathbf{t}} \mathbf{1}^\top\mathbf{t}$$

$$s.t. \; -\mathbf{t} \leq \mathtt{U}^{(t-1)}\mathbf{x} - \mathbf{m}_j \leq \mathbf{t} \quad (11)$$

Previous Works : Ke & Kanade

- Alternated Quadratic Programming:



Huber Norm: $\rho(e) = \begin{cases} \frac{1}{2}e^2, & \text{if } |e| \leq \gamma \\ \gamma|e| - \frac{1}{2}\gamma^2, & \text{if } |e| > \gamma \end{cases}$

$$\mathbf{v}_j = \arg\min_{\mathbf{x}} \rho(\mathrm{U}^{(t-1)}\mathbf{x} - \mathbf{m}_j) \qquad (12)$$

Since Huber M-estimator is a differentiable convex function, Eq. (12) can be converted to a convex quadratic programming (QP) problem whose *global* minimum can be computed efficiently [17]:

$$\min_{\mathbf{x},\mathbf{z},\mathbf{t}} \frac{1}{2}\|\mathbf{z}\|_2^2 + \gamma \mathbf{1}^\top \mathbf{t}$$
$$s.t. - \mathbf{t} \leq \mathrm{U}^{(t-1)}\mathbf{x} - \mathbf{m}_j - \mathbf{z} \leq \mathbf{t} \qquad (13)$$

Previous Works : Ke & Kanade

Remarks:

1. Handle missing data by dropping constraints in LP and QP formulation.

2. Result in convex sub problem, but solution might not be good for the original problem.

Notations:

- Small case means “vec” operator.
- W=diag(Ŵ), so Wy means the masked vec(Y).
- $\otimes$: Kronecker Product

$$\mathbf{A} \otimes \mathbf{B} = \begin{bmatrix} a_{11}B & \cdots & a_{1n}B \\ \vdots & \ddots & \vdots \\ a_{m1}B & \cdots & a_{mn}B \end{bmatrix}.$$

Property of Kronecker Product

$$(\mathbf{B}^\top \otimes \mathbf{A})\,\mathrm{vec}(\mathbf{X}) = \mathrm{vec}(\mathbf{AXB}) = \mathrm{vec}(\mathbf{C}).$$

- Assume V is fixed, U is unknown, $\mathrm{vec}(UV)=\mathrm{vec}(IUV)=(V' \otimes I)\mathrm{vec}(U)$;
- Assume U is fixed and V is unknown, $\mathrm{vec}(UV)=\mathrm{vec}(UVI)=(I \otimes U)\mathrm{vec}(V)$;

Alternated least square (ALS)

- So $\min_{U,V} ||\hat{W} \odot (Y - UV)||,$ is equivalent to:

$$\min_{v} ||Wy - W(I_n \otimes U)v||_2^2, \quad =||Wy\text{-}G(U)v||_2^2$$

$$\min_{u} ||Wy - W(V^T \otimes I_m)u||_2^2, \quad = ||Wy\text{-}F(V)u||_2^2$$

$f = Fu - Wy = Gv - Wy;$

$1/2 f^T f = \Phi(U,V)$

Take partial derivative of Φ and assign to 0, we get:

$u = (F^T F)^{-1} F^T (Wy); \ v = (G^T G)^{-1} G^T (Wy);$

Gauss-Newton Method

- Let $x=[u^T\ v^T]$, $\Phi(U,V) = 1/2f^Tf$
- Newton's method: To attain $\partial\Phi/\partial x=0$ using the step δ, satisfying:

$$\partial^2\Phi/\partial x^2\ \Delta x + \partial\Phi/\partial x = 0 \qquad (*)$$

- Calculate the first and second derivative:

$$\partial\Phi/\partial x = f^T\partial f/\partial x$$

$$\partial^2\Phi/\partial x^2 = (\partial f/\partial x)^T\partial f/\partial x + f^T\ \partial^2 f/\partial x^2 \approx (\partial f/\partial x)^T\partial f/\partial x$$

- Note that $\partial f/\partial x = [\partial f/\partial u\ \partial f/\partial v] = [F\ G]$

Equation (*) now becomes a Ax=b problem, which can be solved by least square.

Wiberg Algorithm

- Fixing part of the parameters AND apply the idea of Newton's method with Gauss-Newton Assumption.
- By doing so (fixing V for example), the algorithm becomes more efficient.
- Let $\Phi(U,V) = \psi(U,V(U)) = 1/2g^Tg$

  where g(u)=f(u,v(u)) is a function of u only.
- Now try to use Newton's method:

$$\partial^2 \psi / \partial u^2 \, \Delta u + \partial \psi / \partial u = 0$$

We need to know the first and second derivative of ψ w.r.t. u.

Wiberg Algorithm

- Again, using Gauss-Newton Approx, we have:

  $\partial \psi / \partial u = g^T \partial g / \partial u$

  $\partial^2 \psi / \partial u^2 = (\partial g/\partial u)^T \partial g/\partial u + g^T \partial^2 g/\partial u^2 \approx (\partial g/\partial u)^T \partial g/\partial u$

- Using chain rule:

  $\partial g/\partial u = \partial f/\partial u + \partial f/\partial v \, \partial v/\partial u = F+G(\partial v/\partial u)$

- At optimal (u,v): $\partial \Phi/\partial u = 0$ AND $\partial \Phi/\partial v = 0$ (KKT)

  $\partial \Phi/\partial u = \partial \psi / \partial u$ will be enforce to 0 by Gauss-Newton.

  $\partial \Phi/\partial v=0$ is irrelevant to u, so:

  $\partial^2 \Phi/(\partial v \partial u)= \partial (f^T \partial f/\partial v)/ \partial u = \partial (f^T G)/ \partial u$

  $= (\partial f/\partial v * \partial v/\partial u+ \partial f/\partial u)^T G + f^T (\partial G)/ \partial u \approx 0$

Wiberg Algorithm

- $(\partial f/\partial v * \partial v/\partial u + \partial f/\partial u)^T G = 0$

  Take Transpose both side:

  $$G^T(G\, \partial v/\partial u + F) = 0$$

- $\partial v/\partial u = -(G^T G)^{-1} G^T F$
- Substitute into $\partial g/\partial u = F + G(\partial v/\partial u) = (I - G(G^T G)^{-1} G^T)F$

  Let $Q_G = I - G(G^T G)^{-1} G^T$

- We obtained the step Δu for Gauss-Newton update in:

  $$\text{Minimize } \| -Q_G W y + Q_G F \Delta u \|$$

  This is a least square problem. As derived in Okatani Paper [16].

Insufficient Rank of $Q_G F$

- Okatani proved that this $Q_G F$ have dimension (m-r)r instead of full rank mr.
- So further constraints that $\| \Delta u \|$ is minimized should be used to uniquely determine a Δu.
- Yet, in this Eriksson paper, it is not mentioned. (corresponds to the Insufficient Rank of Jacobian)

Connection to Linearized Model

Consider

$$\text{Minimize } \| -Q_G W y + Q_G F \Delta u \|$$

Substitute in the value of $Q_G = I - G(G^T G)^{-1} G^T$

We have

$$\text{Minimize} \| -Wy + G(G^T G)^{-1} G^T Wy + Q_G F \Delta u \|$$

Recall that $G(G^T G)^{-1} G^T Wy = Gv^*(u)$ and $\partial g/\partial u = Q_G F$

We may define function $g' = g + Wy = Gv^*(u)$,

Since Wy is const. so $\partial g'/\partial u = \partial g/\partial u$

Connection to Linearized Model

- Now it becomes

  Minimize $\| -Wy + g'(u_k) + \partial g'/\partial u \, \Delta u \|$

- This corresponds to Equation (8) in the paper and $J_k = \partial g'/\partial u|_{u=u_k} = Q_G F$.

  (Btw there's a missing term in Equation (8))

- We have showed that Gauss-Newton update is equivalent to such linearized model.

Proposed algorithm: Wiberg L1

- So using the same “Linearized Model” argument, the L2 Wiberg model can be extended to L1.

$$\min_{U,V} ||\hat{W} \odot (Y - UV)||_1 .$$

Detour: Linear Programming

- Canonical form of an LP

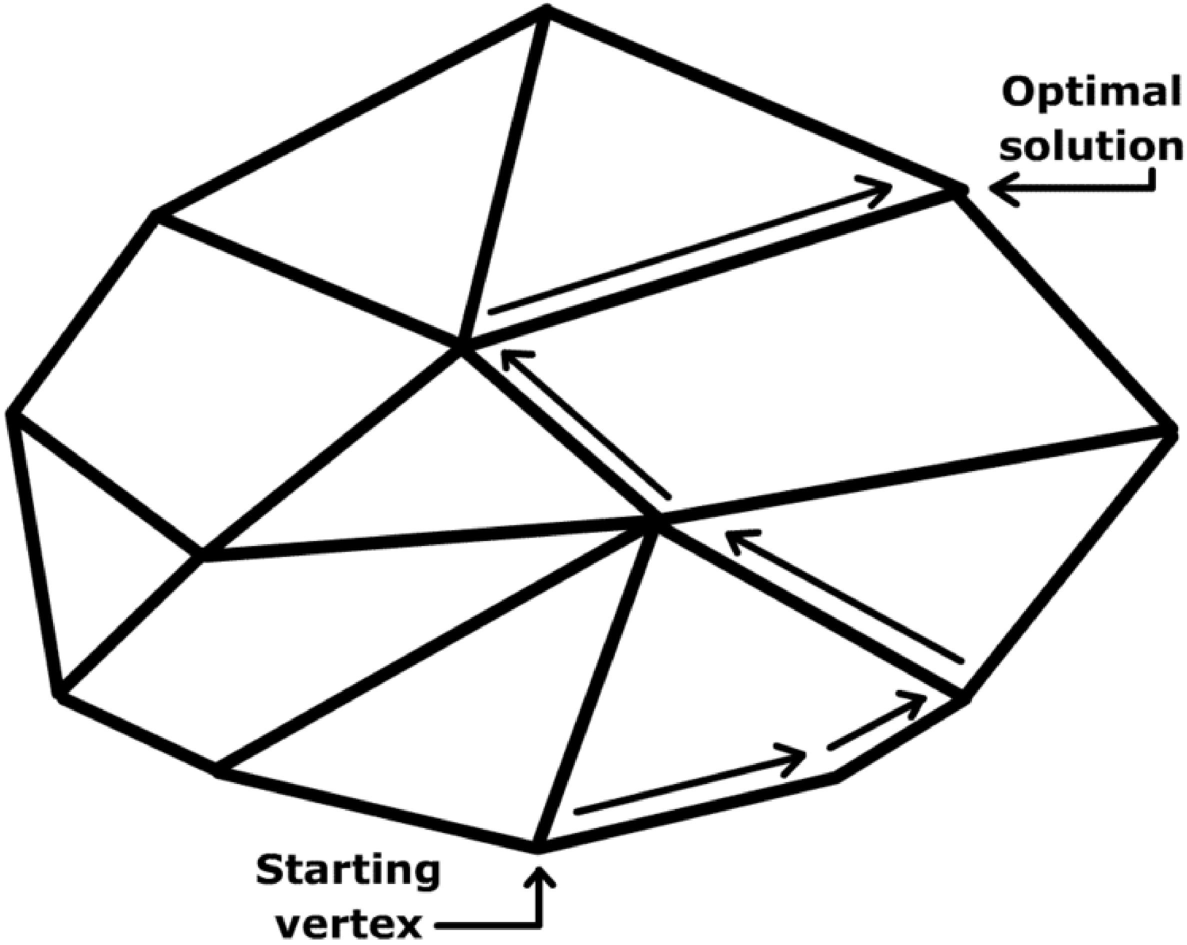
$$\begin{aligned} \min_{x \in \mathbb{R}^n} \quad & c^T x \\ \text{s.t.} \quad & Ax = b \\ & x \geq 0 \end{aligned}$$

- w.o.l.g, A (m*n) is of full rank m.
- No inequality constraints because we can add slack variables.

Basic Solution and Non-basic Solution

- Reorder A to get A=[B N] where B is m*m non-singular.
- $x=[x_B \ x_N]$ and $x_N = 0$, then x is b.s.
- If $x_B >= 0$, x is called basic feasible solution(b.f.s.), if $c^T x$ is optimal, then optimal basic solution.
- **Fundamental Thm of LP**: If an LP is feasible, then exists a optimal b.f.s.

Simplex Method



Simplex Method

- Pivot operation to go from one vertex to another until optimal (in finite steps).
- Each vertex represents a set of b.f.s., pivot changes columns between B and N and find a new b.f.s.
- Can be done efficiently using any commercial solver.

Sensitivity Analysis of LP

- How does optimal solution change if we change constraints (**A** and **b**) or objective function (**c**) locally?
- Thm 3.2:

$$\frac{\partial x_B^*}{\partial B} = -(x_B^*)^T \otimes B^{-1} \quad (13)$$

$$\frac{\partial x_B^*}{\partial N} = 0 \quad (14)$$

$$\frac{\partial x_B^*}{\partial b} = B^{-1} \quad (15)$$

$$\frac{\partial x_N^*}{\partial A} = \frac{\partial x_N^*}{\partial b} = 0. \quad (16)$$

The use of sensitivity analysis in Wiberg L1

1. Fix $U=U_k$, calculate the optimal L1 Norm w.r.t. V using LP, and get $V_k=V^*$.

2. Based on the sensitivity analysis, calculate the gradient of V^* w.r.t. U at U_k, hence linearize $\Phi(U)$ at U_k and calculate the optimal L1 Norm w.r.t. a change step δ of U using LP again.

3. Then update U and proceed in a gradient descent manner.

Proposed Algorithm: Wiberg L1

$$v^*(U) = \arg\min_v ||Wy - W(I_n \otimes U)v||_1,$$

$$u^*(V) = \arg\min_u ||Wy - W(V^T \otimes I_m)u||_1,$$

- f= F(V)u-Wy= G(U)v-Wy

$$\min_U \; f(U) = ||Wy - WUV^*(U)||_1 =$$

$$= ||Wy - \phi_1(U)||_1.$$

- Φ_1(u) = vec(WUV*(U))

  =F(V)u=G(U)v=G(u)v*(u)=F(v*(u))u

LP to find v* given any U

- Given a U_k, we can solve for v* that minimize the L1 norm using the following LP. (Simplex will do.)

$$\min_{v^+, v^-, t, s} \quad \begin{bmatrix} 0 & 0 & 1^T & 0 \end{bmatrix} \begin{bmatrix} v^+ \\ v^- \\ t \\ s \end{bmatrix} \tag{33}$$

$$\text{s.t.} \quad \underbrace{\begin{bmatrix} -G(U) & G(U) & -I & I \\ G(U) & -G(U) & -I & \end{bmatrix}}_{A(U)} \begin{bmatrix} v^+ \\ v^- \\ t \\ s \end{bmatrix} = \underbrace{\begin{bmatrix} -Wy \\ Wy \end{bmatrix}}_{b} \tag{34}$$

$$v^+, v^-, t, s \geq 0 \tag{35}$$

$$v^+, v^- \in \mathbb{R}^{rn}, \;\; t \in \mathbb{R}^{mn}, \;\; s \in \mathbb{R}^{2mn}. \tag{36}$$

Given v*, linearize the model and find the optimal Δu

- In order to do that, we must calculate J(U)

  $\partial\Phi_1/\partial u = \partial\Phi_1/\partial u + \partial\Phi_1/\partial v^* \partial v/\partial u$

  $$= \quad J(U) = F(V) + G(U)\frac{\partial v^*}{\partial U}.$$

- F and G is constant or known,

$$\frac{\partial v^*}{\partial U} = \frac{\partial v^{*+}}{\partial U} - \frac{\partial v^{*-}}{\partial U}$$

Given v*, linearize the model and find the optimal Δu

$$\frac{\partial z^*}{\partial U} = \begin{bmatrix} \frac{\partial v^{*+}}{\partial U} \\ \frac{\partial v^{*-}}{\partial U} \\ \frac{\partial t^*}{\partial U} \\ \frac{\partial s^*}{\partial U} \end{bmatrix} = \frac{\partial z^*}{\partial B}\frac{\partial B}{\partial G}\frac{\partial G}{\partial U}$$

$$\frac{\partial G}{\partial U} = (I_{nr} \otimes W)(I_n \otimes T_{r,n} \otimes I_m)(\mathrm{vec}(I_n) \otimes I_{mr}) \quad (37)$$

$$\begin{aligned} \frac{\partial B}{\partial G} &= \frac{\partial (AQ)}{\partial G} = (Q^T \otimes I_{2mn})\frac{\partial A}{\partial G} = \\ &= (Q^T \otimes I_{2mn})\left[\begin{array}{cc} \frac{\partial}{\partial G}\left(\begin{smallmatrix} -G & G \\ G & -G \end{smallmatrix}\right) & 0 \end{array}\right] = \\ &= (Q^T \otimes I_{2mn})\left[\begin{array}{cc} \frac{\partial\left(\left[\begin{smallmatrix} -1 & 1 \\ 1 & -1 \end{smallmatrix}\right] \otimes G\right)}{\partial G} & 0 \end{array}\right] \end{aligned} \quad (38)$$

$$\frac{\partial z^*}{\partial B} = Q\frac{\partial z_B^*}{\partial B} = -Q\left((z_B^*)^T \otimes B^{-1}\right) \quad (39)$$

Very high dimension even for small data matrix.
Even sparse representation doesn't work.

Minimizing L1 norm over Δu

- To find the optimal Δu, we again want to minimize the L1 norm.

$$\min_{\delta} ||Wy - J(U)(\delta - u)||_1$$

- Again, there's a typo, a term is missing here at (41)(42) in the paper. Also, the $J(u_k)u$ term makes no sense at all. Correct formulation is:

$$\text{min. } ||Wy - \Phi_1(u_k) - J(u_k)(\Delta u)||$$

LP to find Δu and minimize L1 norm

- Note the trust region defined here.
- Only within a small step, the linearity assumption is true.

$$\min_{\delta,t} \quad \begin{bmatrix} 0 & 1^T \end{bmatrix} \begin{bmatrix} \delta \\ t \end{bmatrix} \tag{43}$$

$$\text{s.t.} \quad \begin{bmatrix} -J(U)-I \\ J(U)-I \end{bmatrix} \begin{bmatrix} \delta \\ t \end{bmatrix} = \begin{bmatrix} -(Wy-W\text{vec}(UV^*)) \\ Wy-W\text{vec}(UV^*) \end{bmatrix} \tag{44}$$

$$||\delta||_1 \leq \mu \tag{45}$$

$$\delta \in \mathbb{R}^{mr}, \quad t \in \mathbb{R}^{mn}. \tag{46}$$

- Again, this is solved by Simplex method.

Wiberg L1 Summary

Algorithm 1: L_1-Wiberg Algorithm

input : U_0, $1 > \eta_2 > \eta_1 > 0$ and $c > 1$

$k = 0$;

repeat

    Compute the Jacobian of $\phi_1 = J(U_k)$ using (37)-(40) ;

    Solve the subproblem (43)-(46) to obtain δ_k^* ;

    Let $gain = \dfrac{f(U_k) - f(U_k + \delta^*)}{\tilde{f}(U_k) - \tilde{f}(U_k + \delta^*)}$;

    if $gain \geq \epsilon$ **then**

        $U_{k+1} = U_k + \delta^*$;

    end

    if $gain < \eta_1$ **then**

        $\mu = \eta_1 ||\delta^*||_1$

    end

    if $gain > \eta_2$ **then**

        $\mu = c\mu$

    end

    $k = k + 1$;

until *convergence* ;

Experiments

- The paper presents comparison between proposed algorithm and ALP & AQP presented in [13] over synthetic data.
- There is also an application in SfM using the Dinosaur Sequence and compare to Wiberg L2.
- Results are pretty presentable in terms accuracy of recovery and speed.

Synthetic Data Matrix

- m = 7; n = 12; r = 3; (very small!)
- 20% Random Distributed Missing Entries
- 10% Random Distributed Sparse outlier noise Uni(-5,5)
- Tested over 100 data matrices to obtain a statistic.

Histogram: Frequency vs. Error

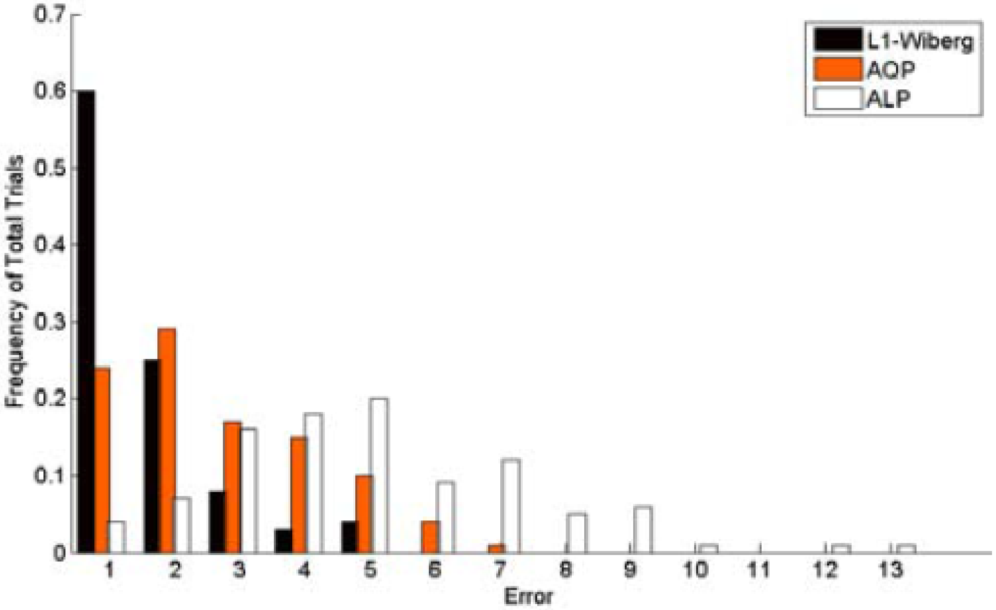


Figure 1. A histogram representing the frequency of different magnitudes of error in the estimate generated by each of the methods. [Frequency vs. Error]

Speed of convergence: Residual vs. Iteration

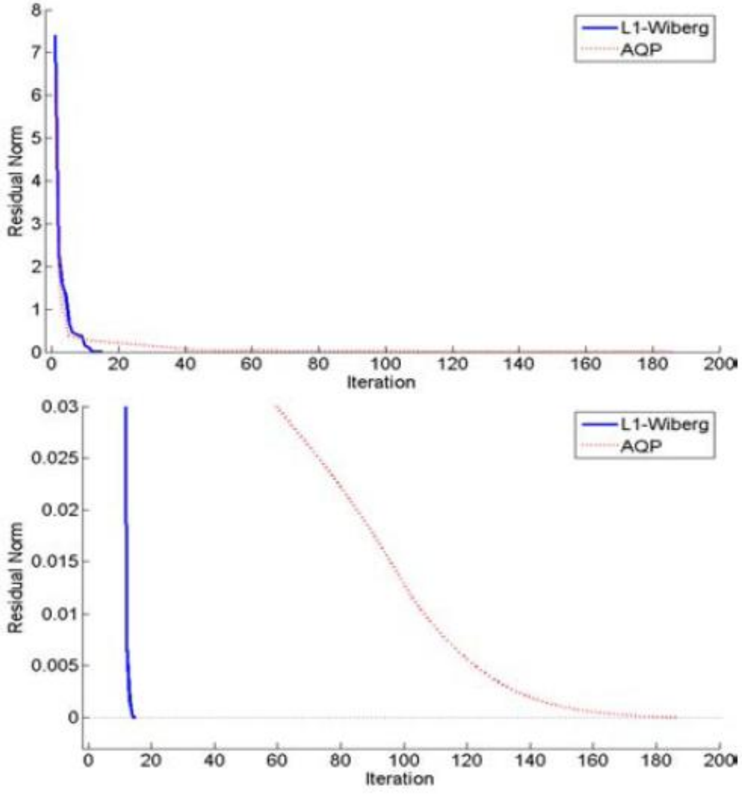


Figure 2. Plots showing the norm of the residual at each iteration of two randomly generated tests for both the L_1 Wiberg and alternated quadratic programming algorithms. [Residual norm vs. Iteration]

Speed of convergence: log error vs. Iteration

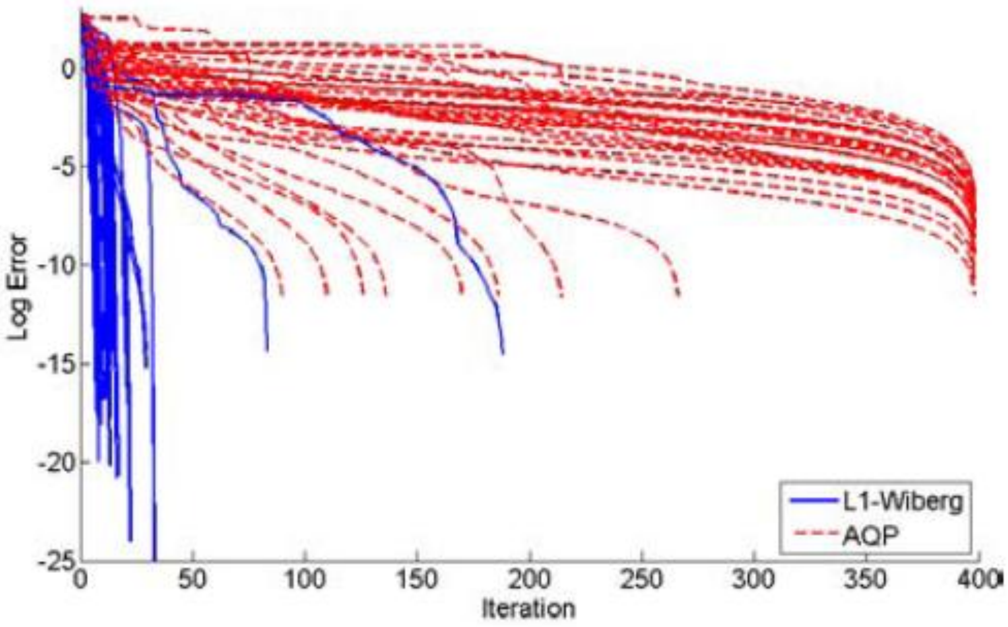


Figure 3. A plot showing the convergence rate of the alternated quadratic programming and L_1-Wiberg algorithms over 100 trials. The error are presented on a logarithmic scale. [Log Error vs. Iteration]

Table of Results

Algorithm	Alt. LP [13]	Alt. QP [13]	Wiberg $L_1(Alg.1)$
Error (L_1)	4.60	2.29	**1.01**
Execution Time (sec)	0.16	93.57	1.51
# Iterations	4.72	177.64*	21.77
# LP/QP solved	9.44	355.28*	24.13
Time per LP/QP	0.016	0.264*	0.061

* The alternated QP algorithm was terminated after 200 iterations and 400 solved quadratic programs.

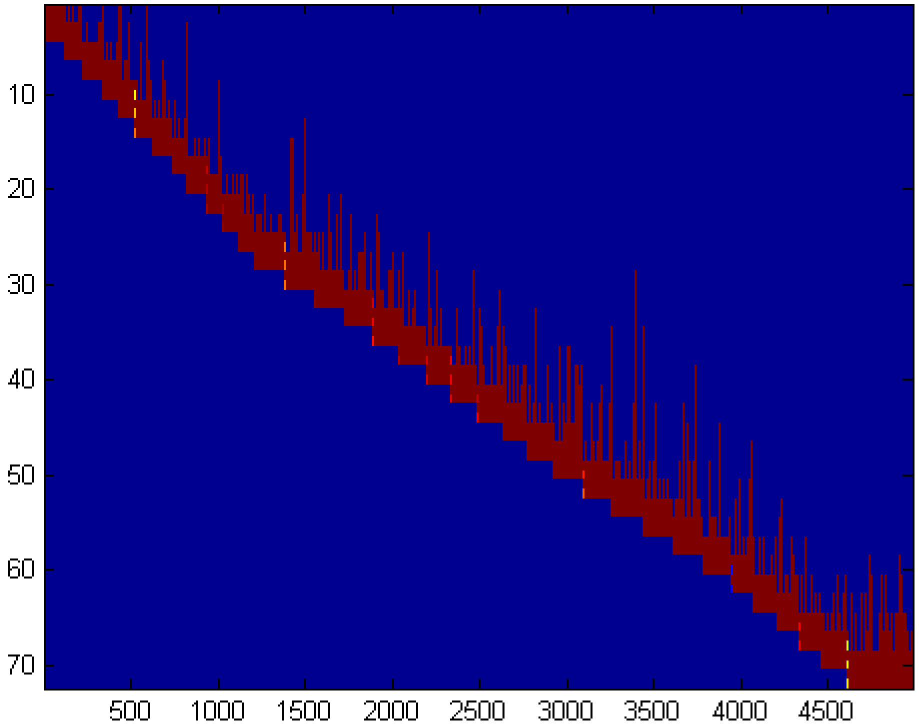
Table 1. *The averaged results from running* 100 *synthetic experiments.*

- ALP has poor results in terms of error, though speed is almost real time.
- AQP has long running time and moderate performance.
- Wiberg outperforms others in average error, and is considerably fast.

Structure from Motion

- 319 features are tracked over 36 views
- Uni[-50,50] outliers are artificially added into the 10% of tracked points.
- Unknown sample rate

Data matrix of Dinosour Sequence



Generated using the data provided by Oxford Visual Geometry Group: There are 4983 feature tracks instead of 319.
http://www.robots.ox.ac.uk/~vgg/data1.html

Comparison of L1 and L2 Wiberg completion results

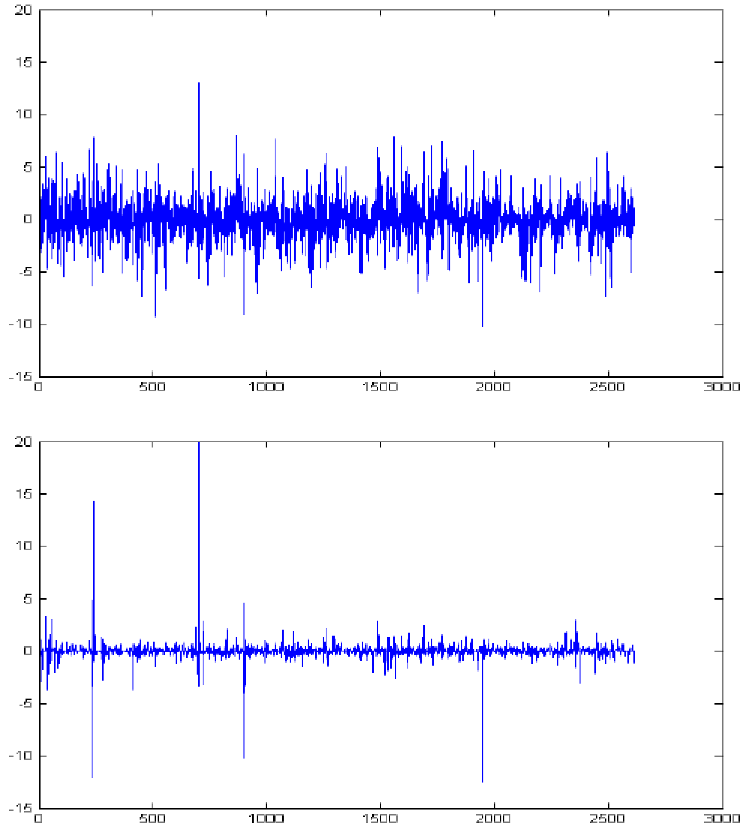


Figure 4. Resulting residuals using the standard Wiberg algorithm (top), and our proposed L_1-Wiberg algorithm (bottom).

Result table and Reconstructed point cloud

Algorithm	Alt. QP [13]	Wiberg L_2 [20]	Wiberg L_1 (Alg. 1)
RMS Error of Inliers	-	2.029	0.862
Execution Time	>4 hrs	3 min 2 sec	17 min 44 sec

Table 2. *Results from the dinosaur sequence with 10% outliers.*

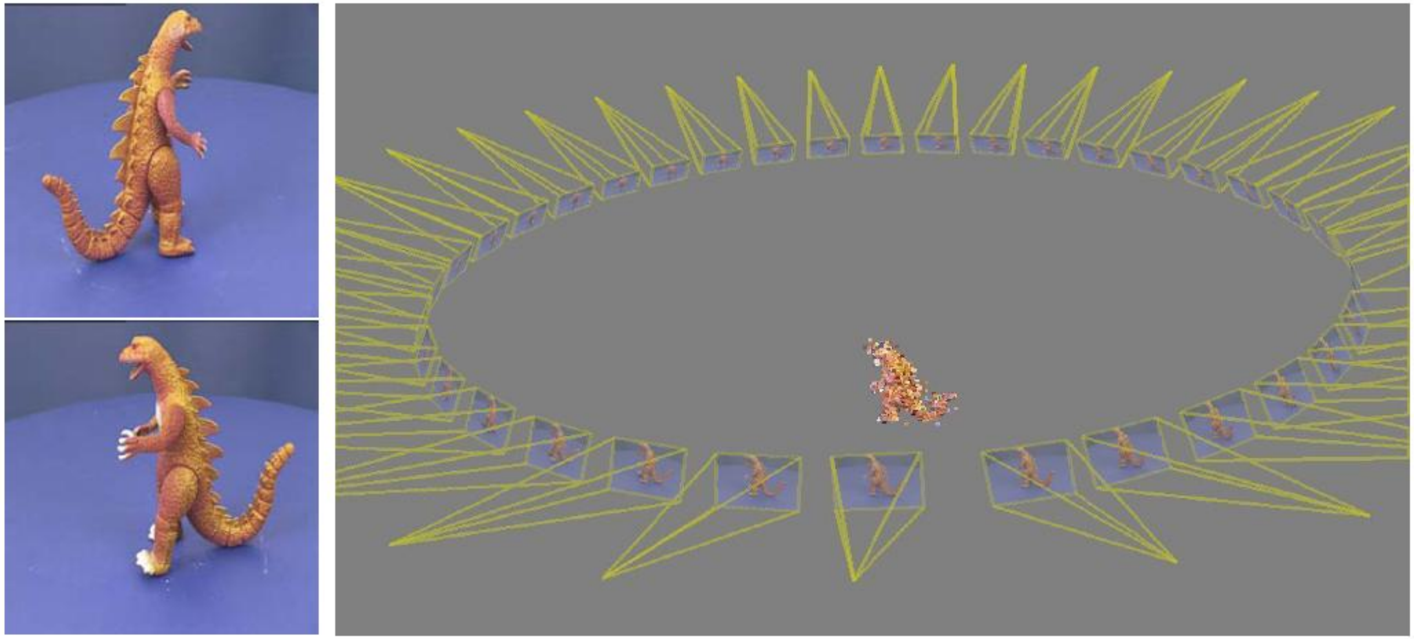


Figure 5. *Images from the dinosaur sequence, and the resulting reconstruction using our proposed method.*

Critique on this SfM attempt

- Too little information is provided how they did this SfM. What’s the sample rate? The factorization to U and V is not unique as mentioned earlier.
- The code provided doesn’t work for 72*319 at all. It breaks down at 20*70…
- What residual is used in Figure 4 is not clear L1 or L2.

Comparison with Rosper

- Rosper: Lee Choon Meng's explicit rank constraint low-rank optimizer

$$\min \frac{1}{2}\|\mathcal{A}(W) + \mathcal{A}(E) - \mathcal{A}(\widehat{W})\|_F^2 + \mu\|E\|_{2,1}$$
$$s.t.\ rank(W) \leqslant r$$

- Solving by Proximal Gradient (no A due to non-convexity)
- Optimize W and E concurrently

Compared algorithms

- **Wiberg L1:** unoptimized version of code provided by Anders Eriksson. Slow and not scalable. (Not working for matrices as small as 20*80)
- **Original Wiberg (L2):** provided by Okatani, citation [16]. Features checking if a data matrix is well-posed for the algorithm. (If $Q_F G$ is of its maximum rank (n-r)r)
- These two algorithms both result in non-unique U and V. In order to compare, we only compare W=U*V.

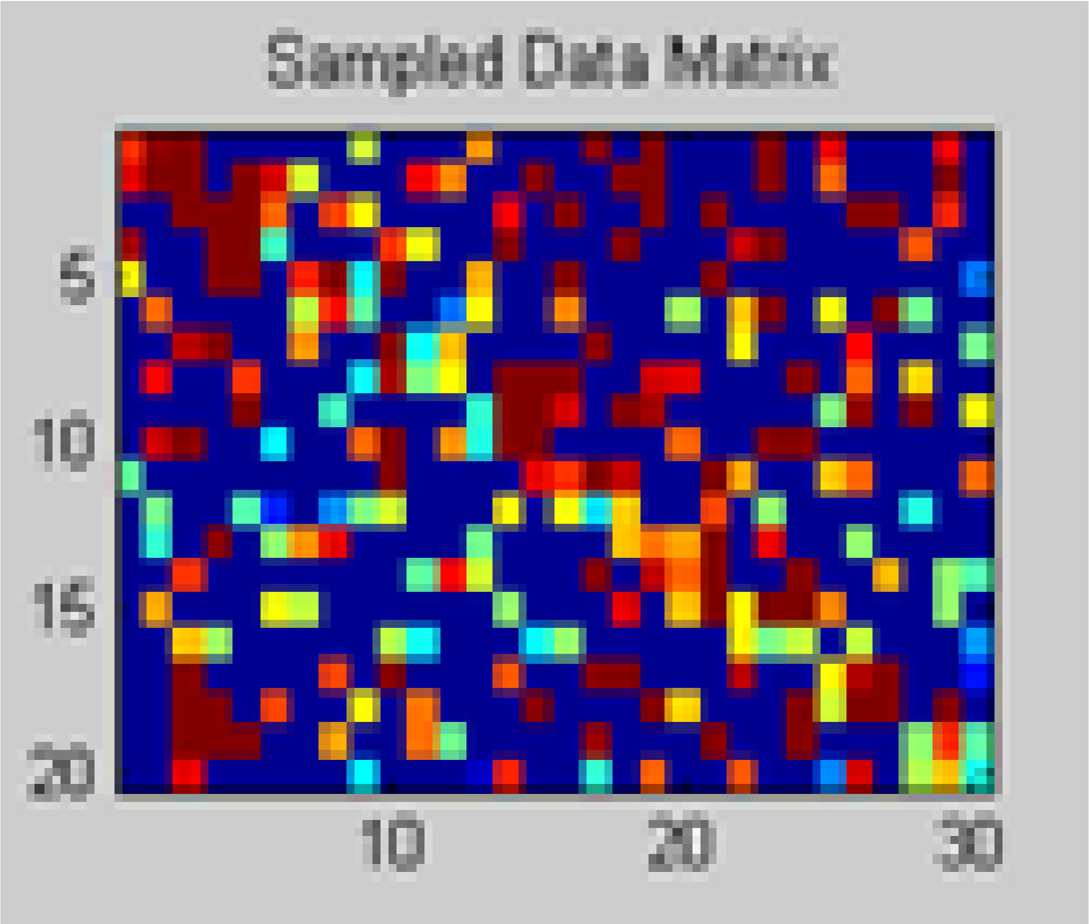
Difference in Error Model

- Wiberg L1 minimizes L1 Norm, which handles sparse outliers in data matrix.
- Wiberg L2 minimizes L2 Norm, which in theory is optimal (MLE) if the noise is Gaussian distributed.
- Rosper models noise more explicitly. The first term handles dense Gaussian noise while the second term handles sparse outliers.

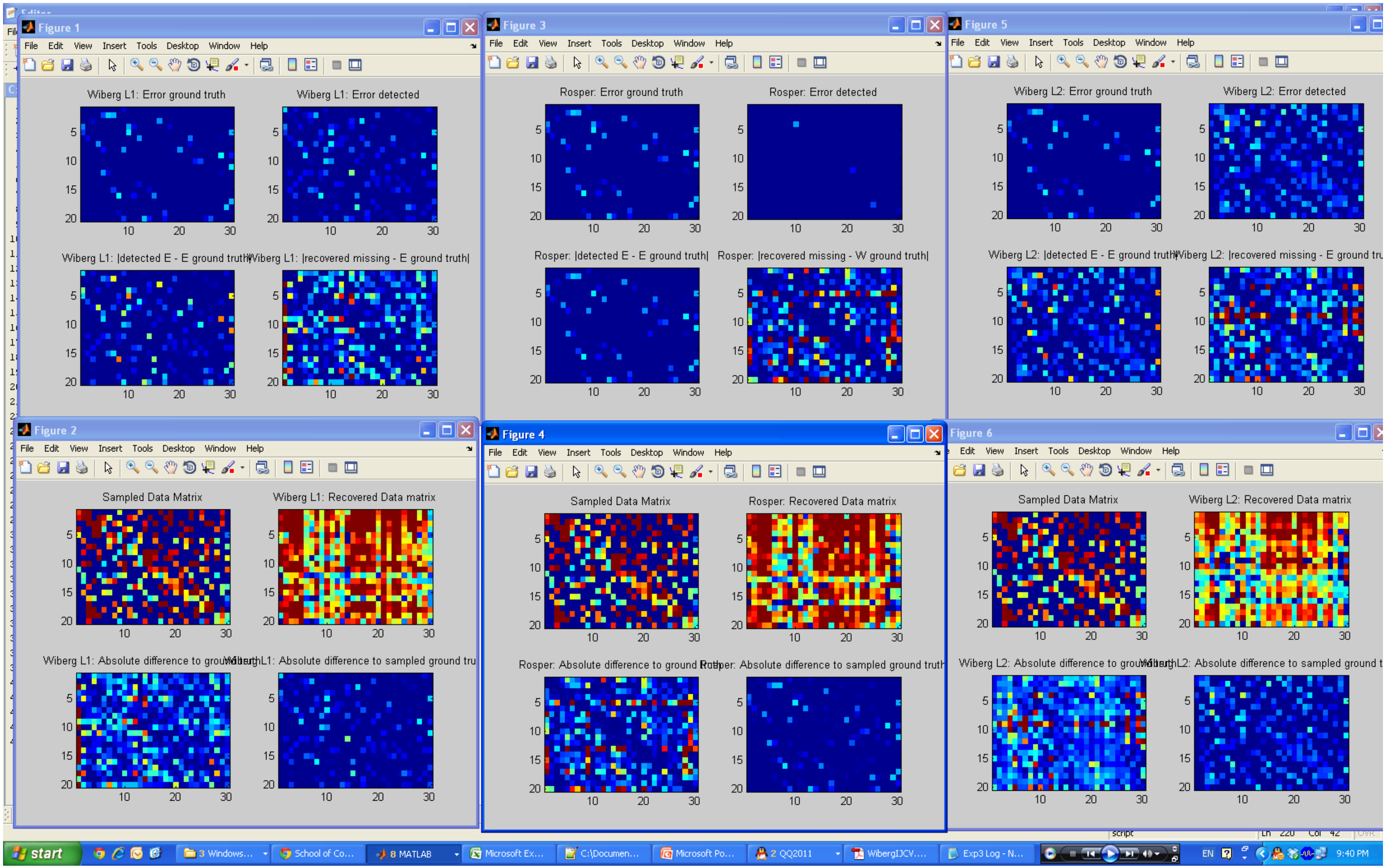
Experiment design

- Data matrix dimension: 20*30
- Each element of U and V fall within [0,1]
- Sample rate: ~40%
- Uniformly sampled
- Noise of various scale:
  - Uniform noise to represent sparse outlier
  - Gaussian noise to represent dense noise

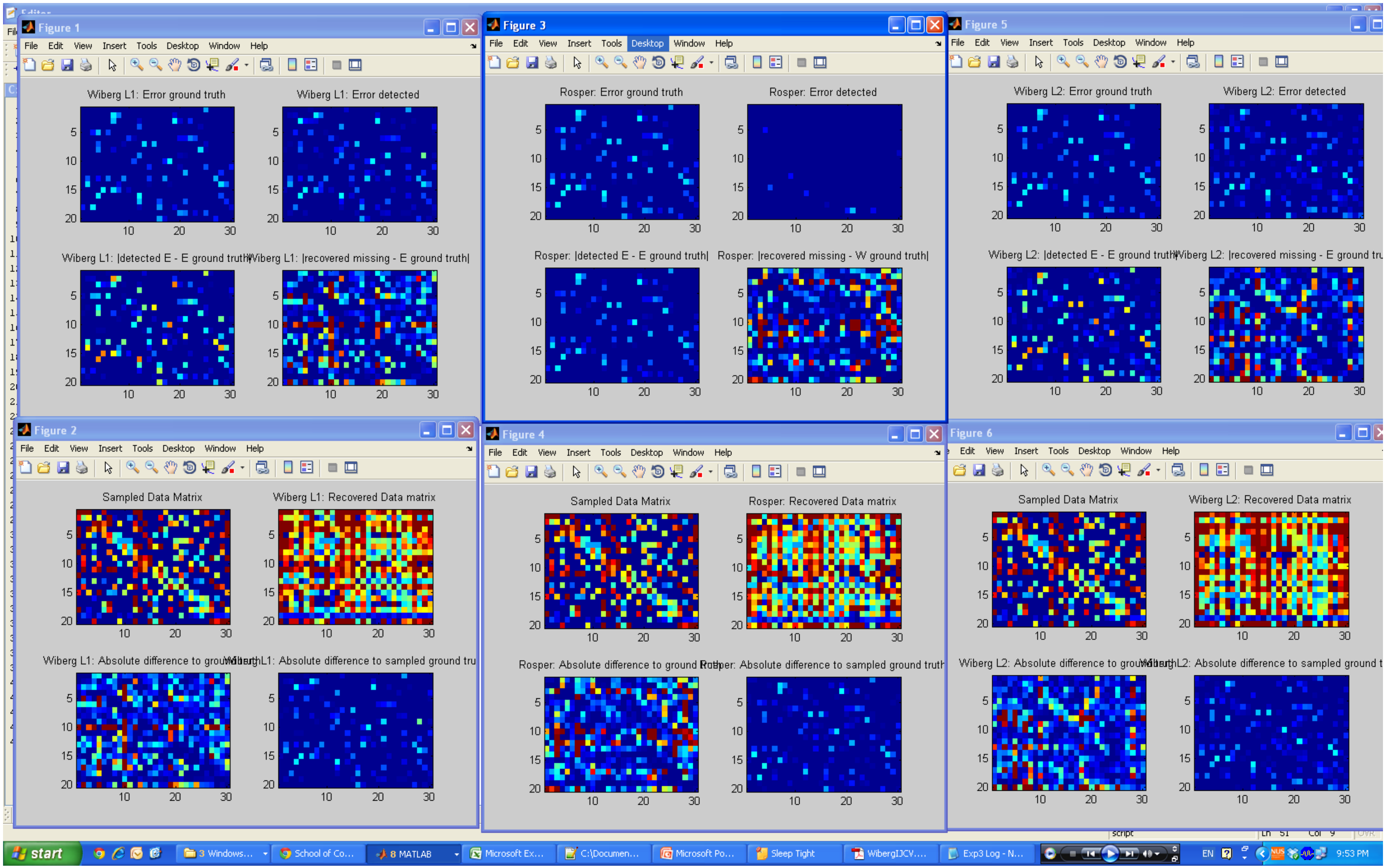
An illustration of the sampled data matrix



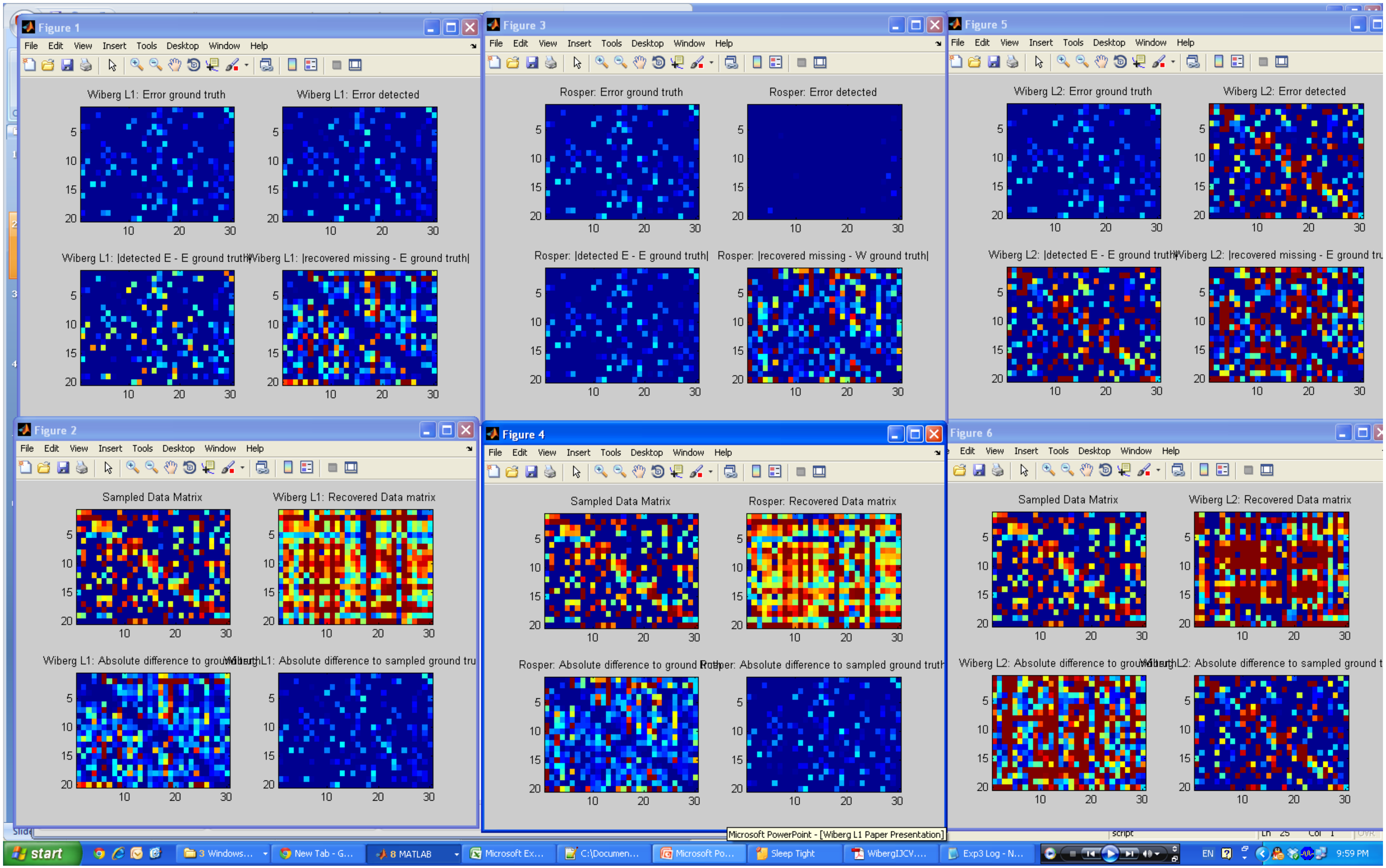
Experiment 3: 0.3 Error



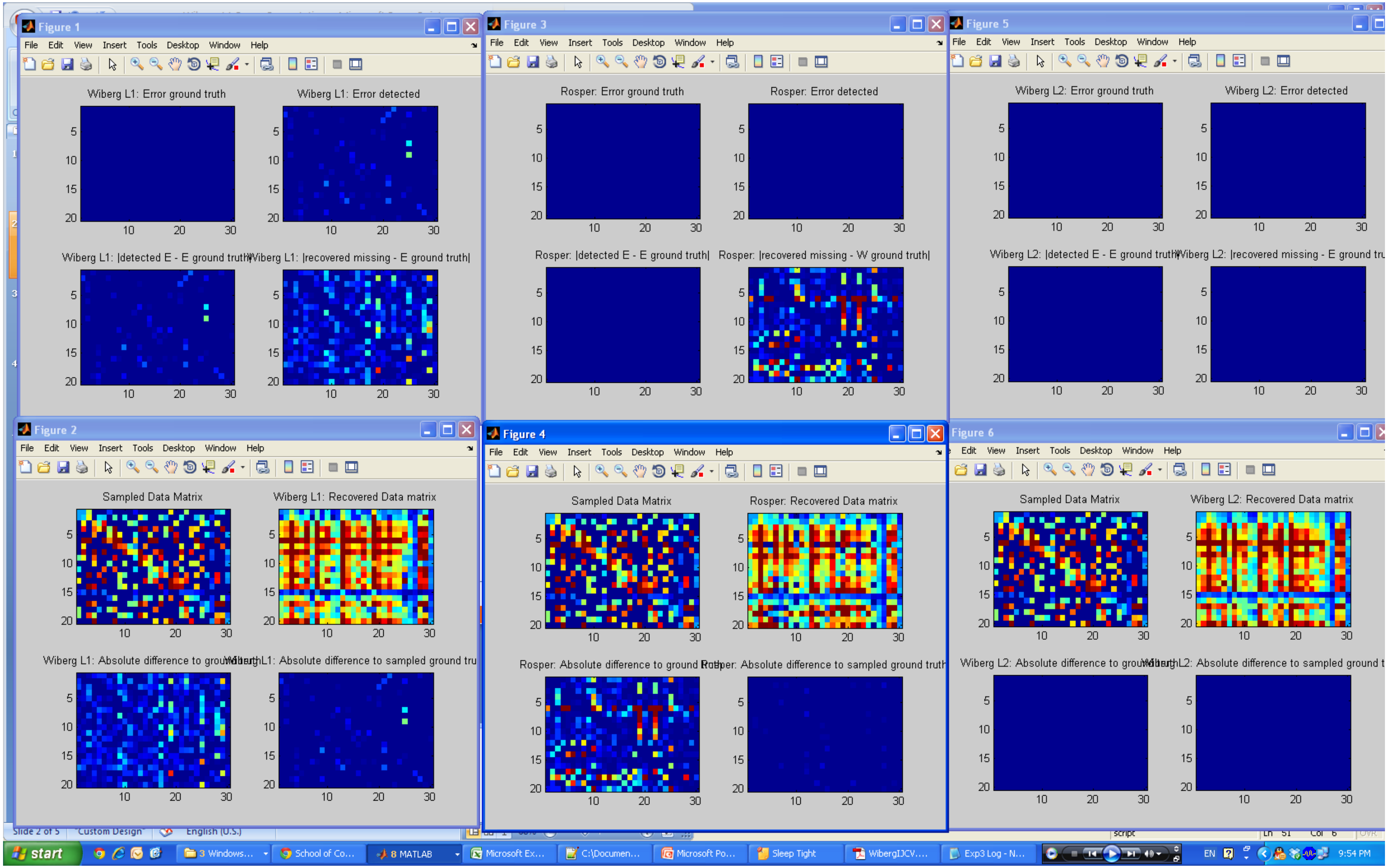
Experiment 5: 0.5 Error



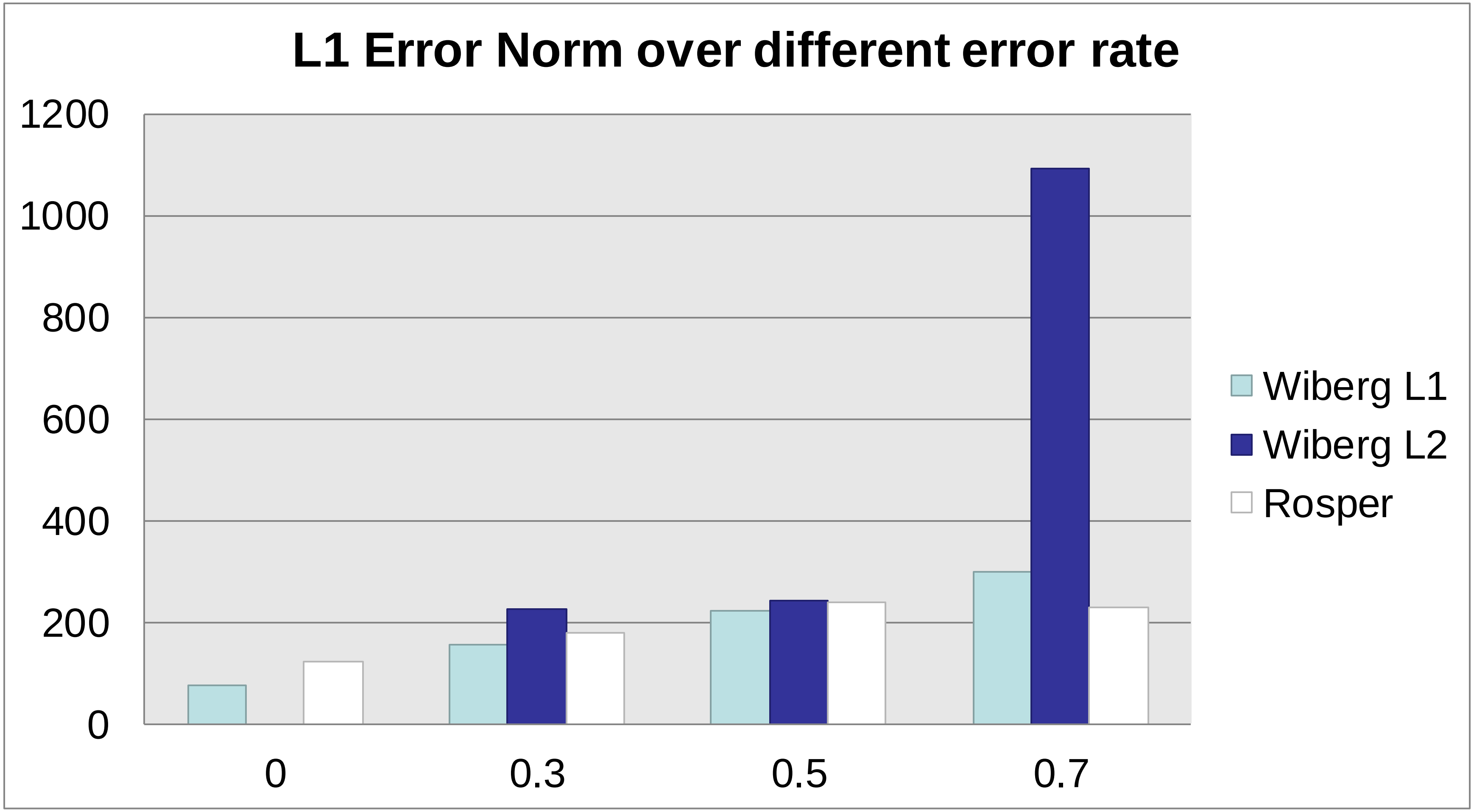
Experiment 6: 0.7 Error



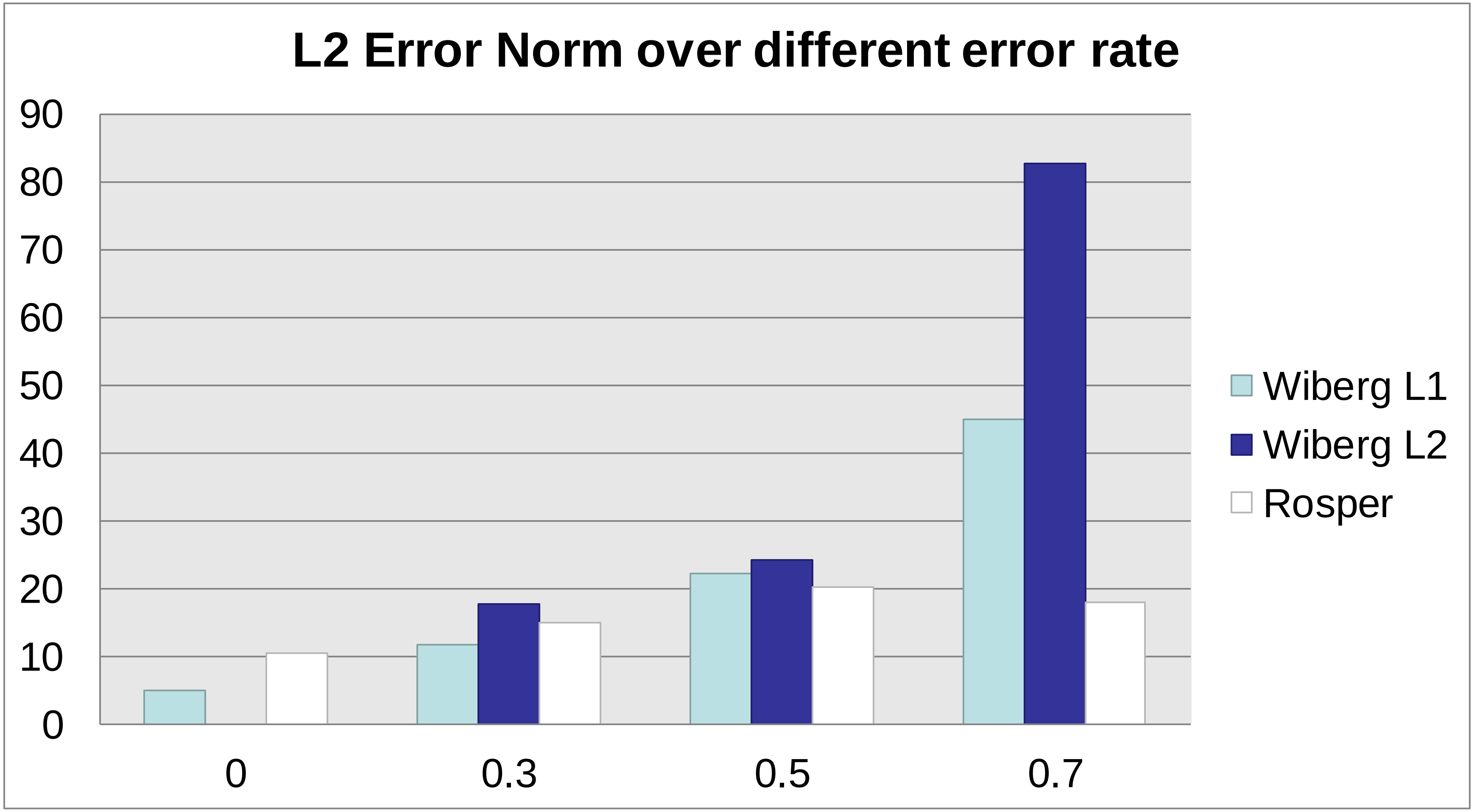
Experiment 4: 0 Error



Different error rate



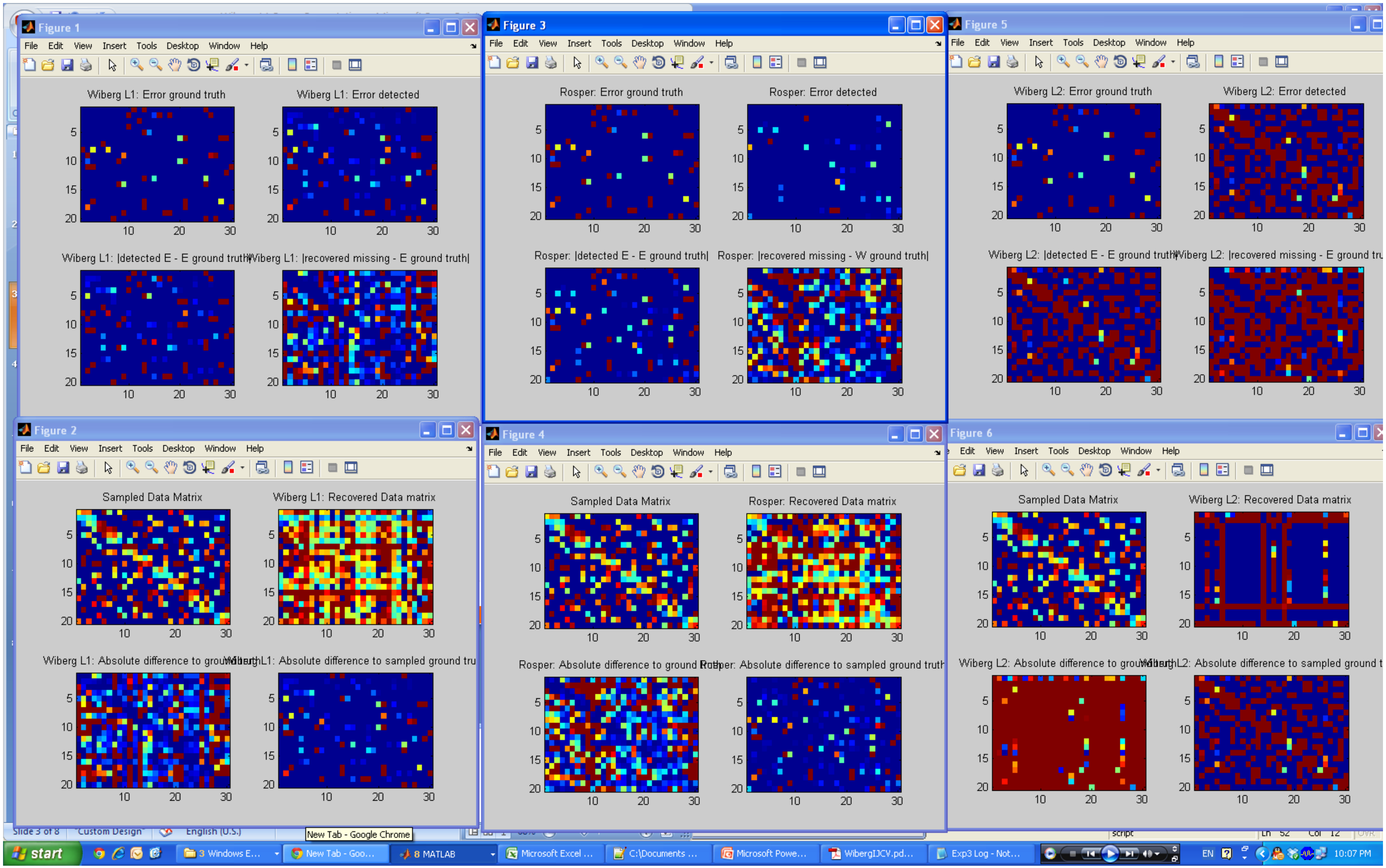
Different error rate



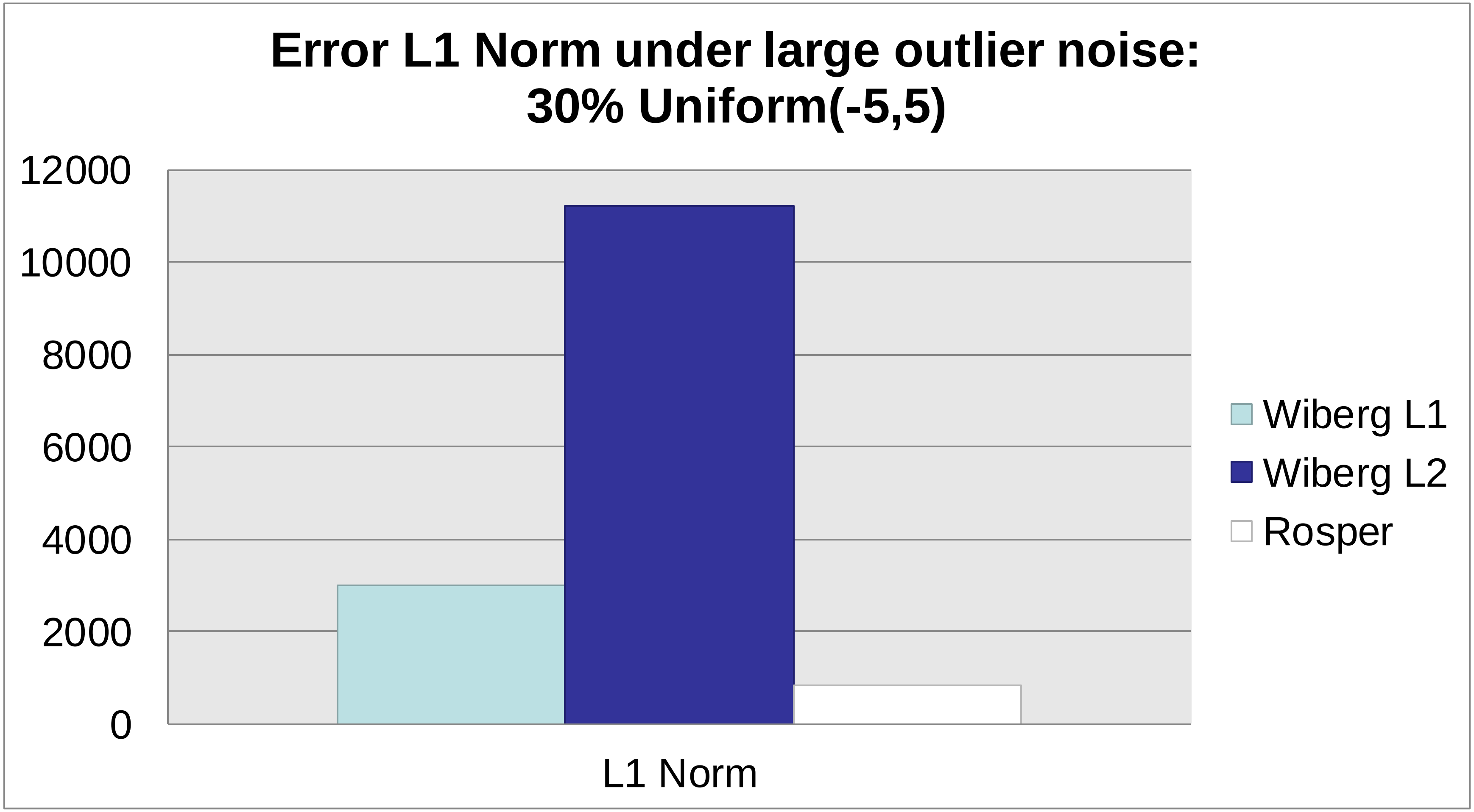
Result for error free experiment

```
EXP 4:
Summary of Problem:
Number of frames = 10
Size of matrix: 20 x 30
Sampling rate = 0.40
Error Rate: 0.00
Error Amp: 1


Summary of Wiberg L1
Wiberg L1: Time elapsed = 0 hours : 0 minutes : 24 seconds
Wiberg L1: The residual through the 8 iterations are: 9.01
Wiberg L1: The L1 norm of error is 74.8112
Wiberg L1: The L2 norm of error is 4.9362
Wiberg L1: The L1 norm of sampled error is 9.0133
Wiberg L1: The L2 norm of sampled error is 1.3045
Wiberg L1: |detected E - E ground truth| = 9.0133
Wiberg L1: |recovered missing - W ground truth| = 65.7979


Summary of Wiberg L2
Wiberg L2: Time elapsed = 0 hours : 0 minutes : 2 seconds
Wiberg L2: The L1 norm of error is 0.0000
Wiberg L2: The L2 norm of error is 0.0000
Wiberg L2: The L1 norm of sampled error is 0.0000
Wiberg L2: The L2 norm of sampled error is 0.0000
Wiberg L2: |detected E - E ground truth| = 0.0000
Wiberg L2: |recovered missing - W ground truth| = 0.0000


Summary of Rosper
Rosper: Time elapsed = 0 hours : 0 minutes : 5 seconds
Rosper: The L1 norm of error is 121.1264
Rosper: The L2 norm of error is 10.3511
Rosper: The L1 norm of sampled error is 4.6059
Rosper: The L2 norm of sampled error is 0.4392
Rosper: |detected E - E ground truth| = 0.0316
Rosper: |recovered missing - E ground truth| = 116.5205
```

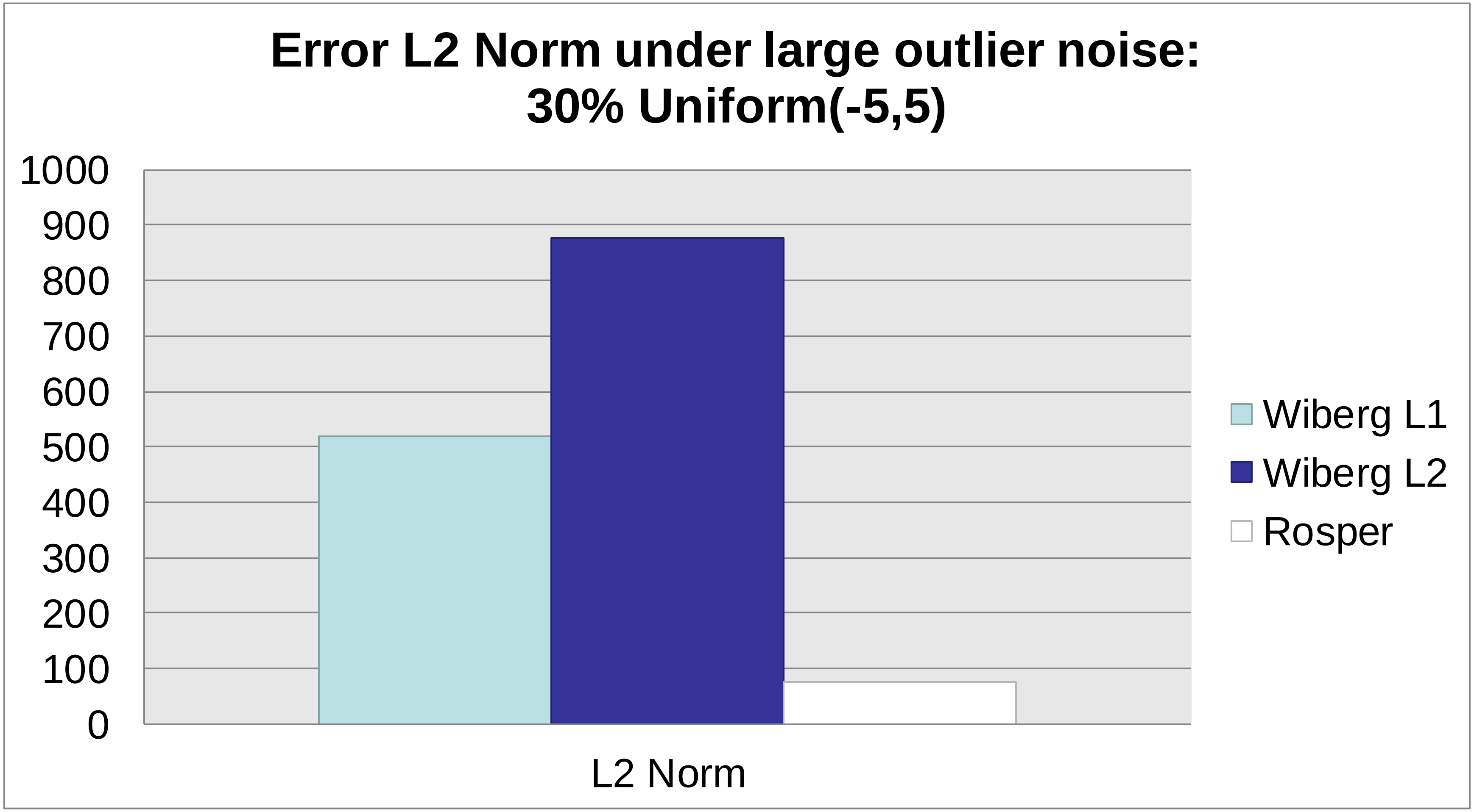
Experiment 7: 30% Large Outlier!



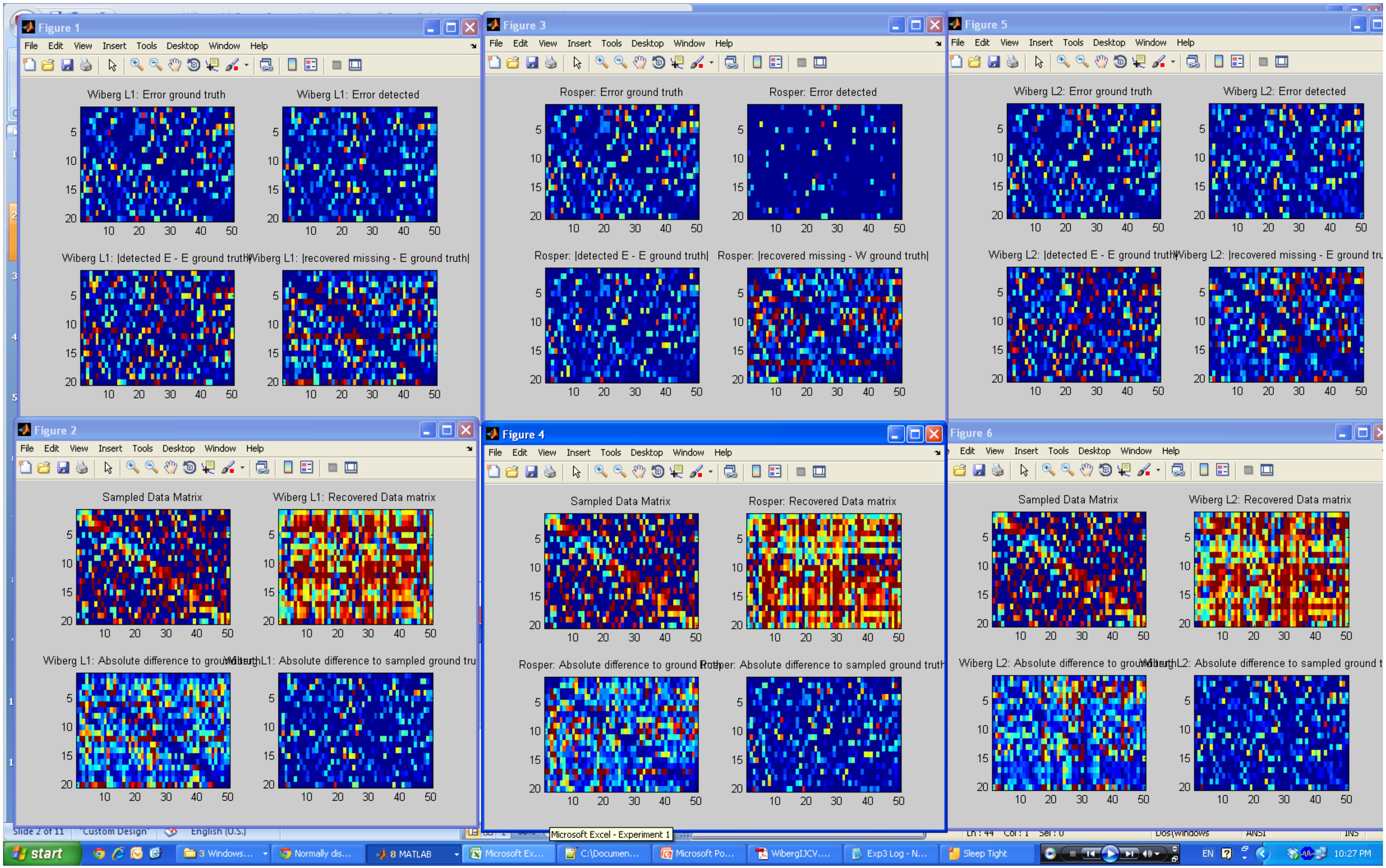
Large Sparse Outlier



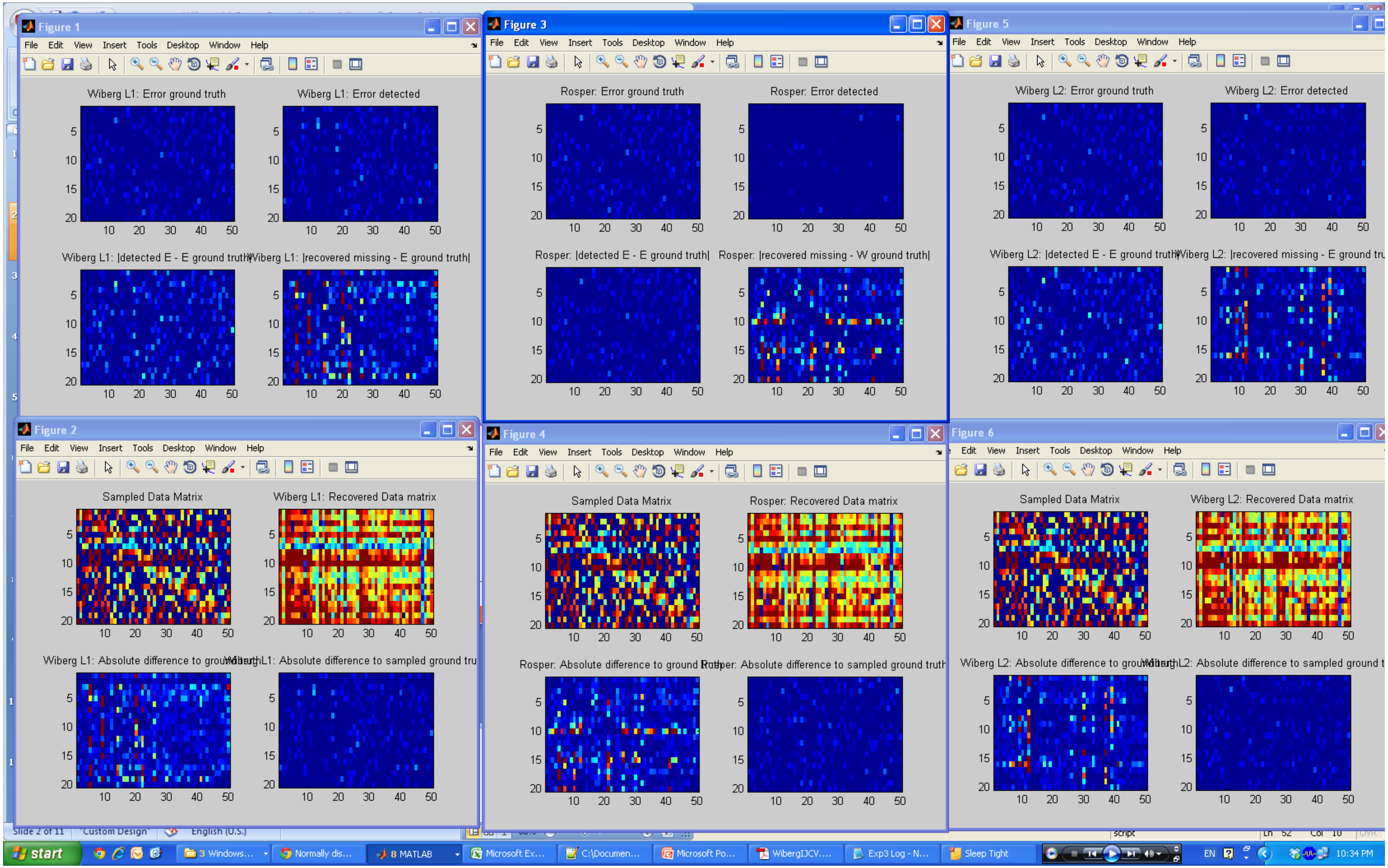
Large Sparse Outlier



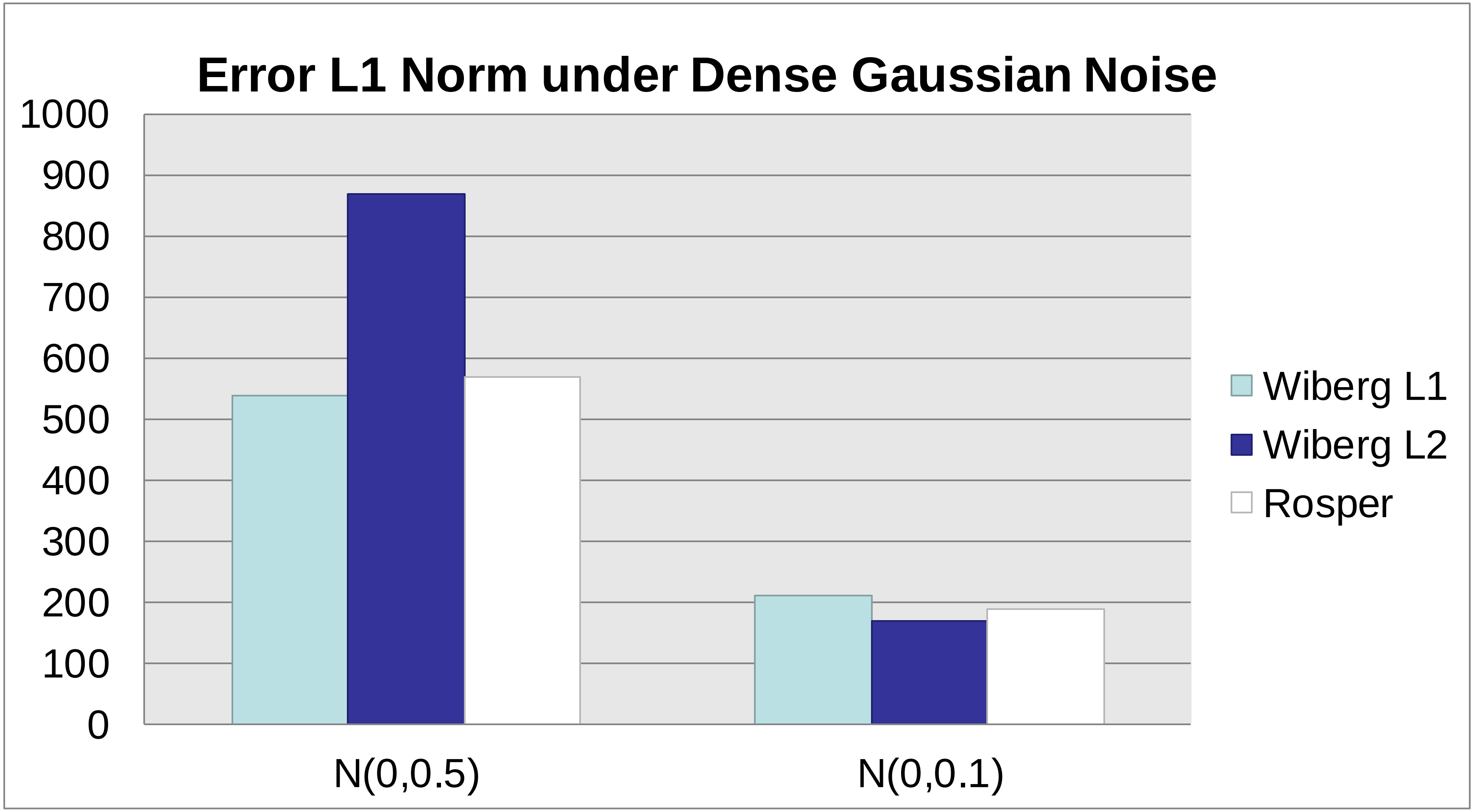
Experiment 9:Dense N(0,0.5)



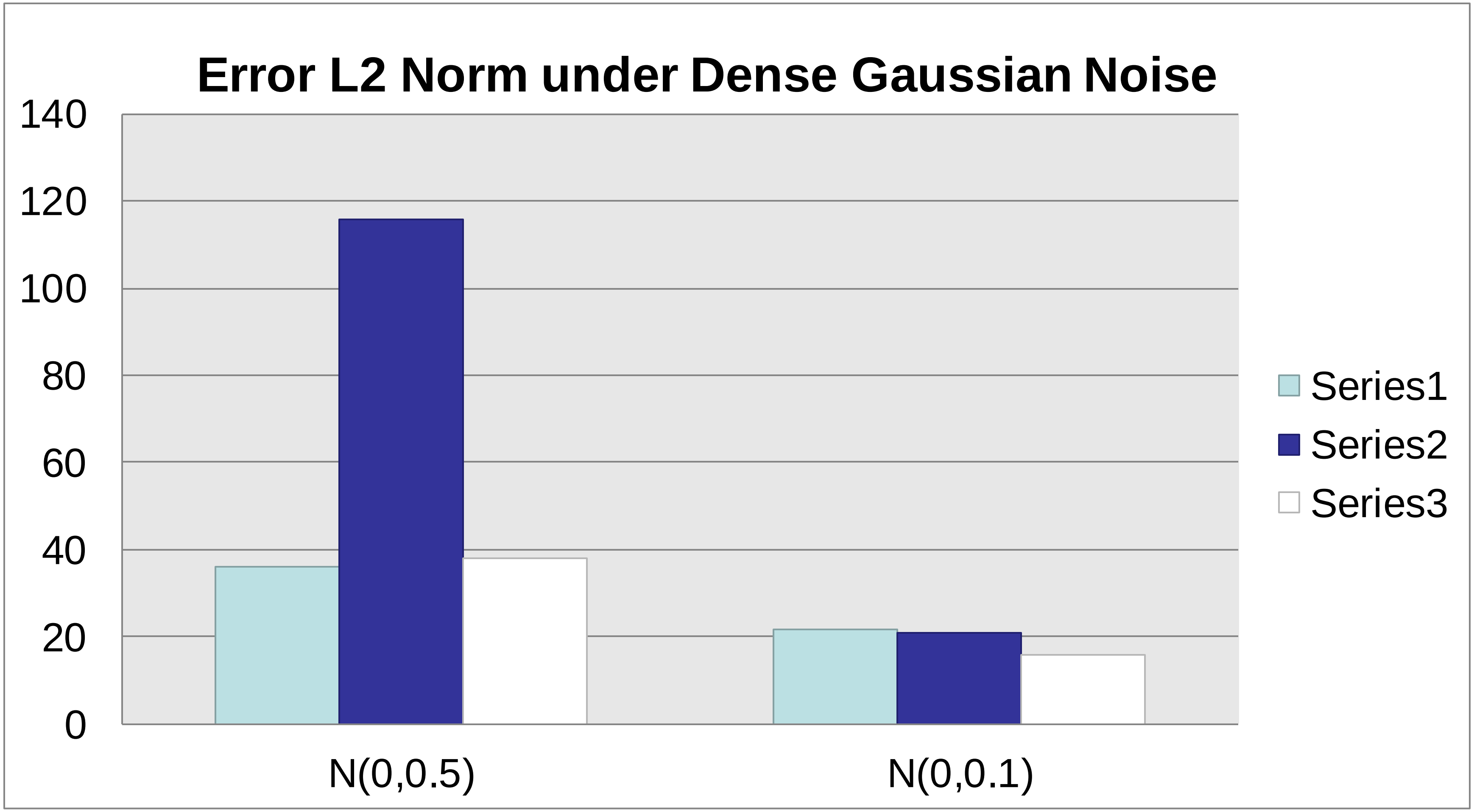
Experiment 10:Dense N(0,0.1)



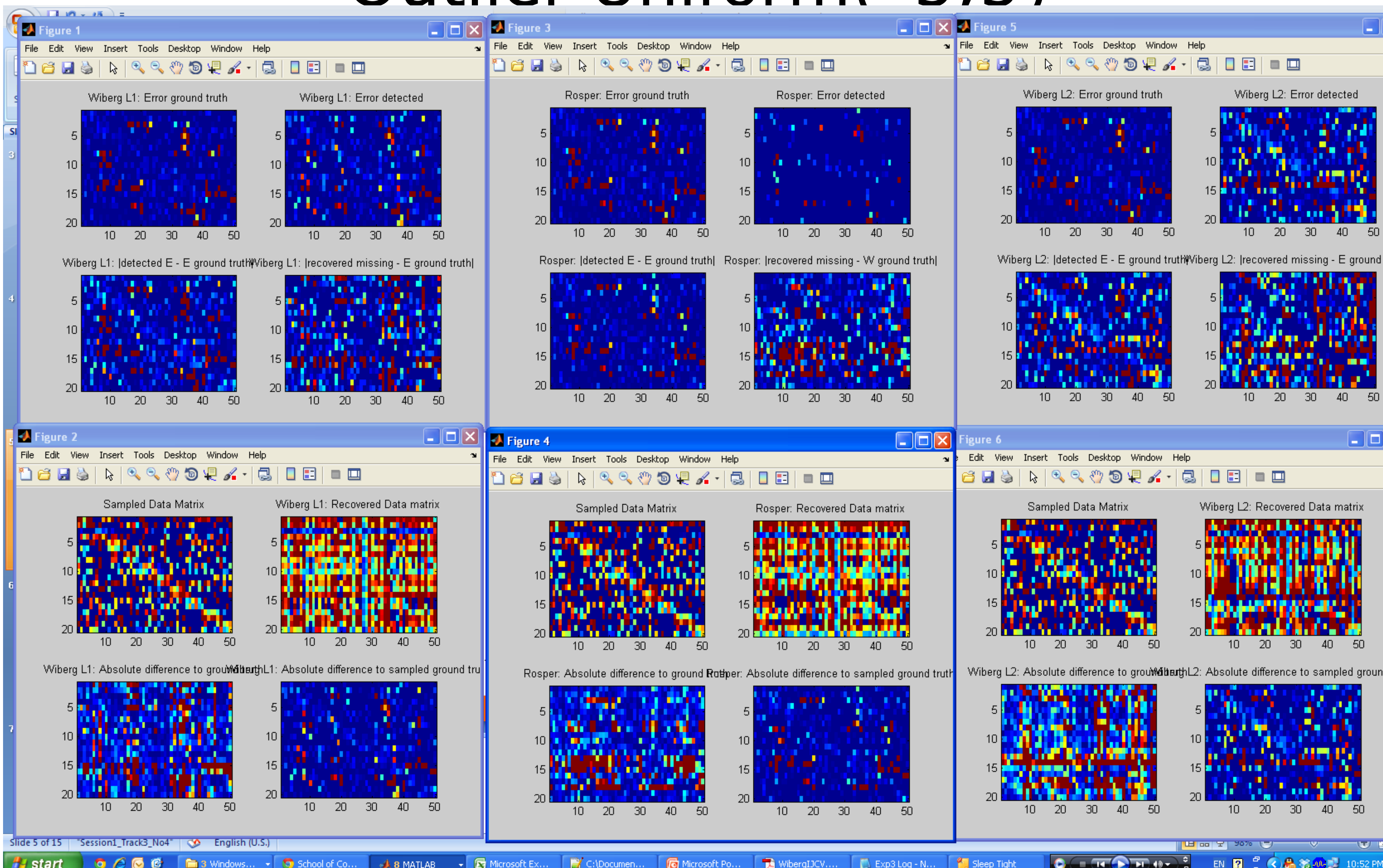
Dense Gaussian Noise



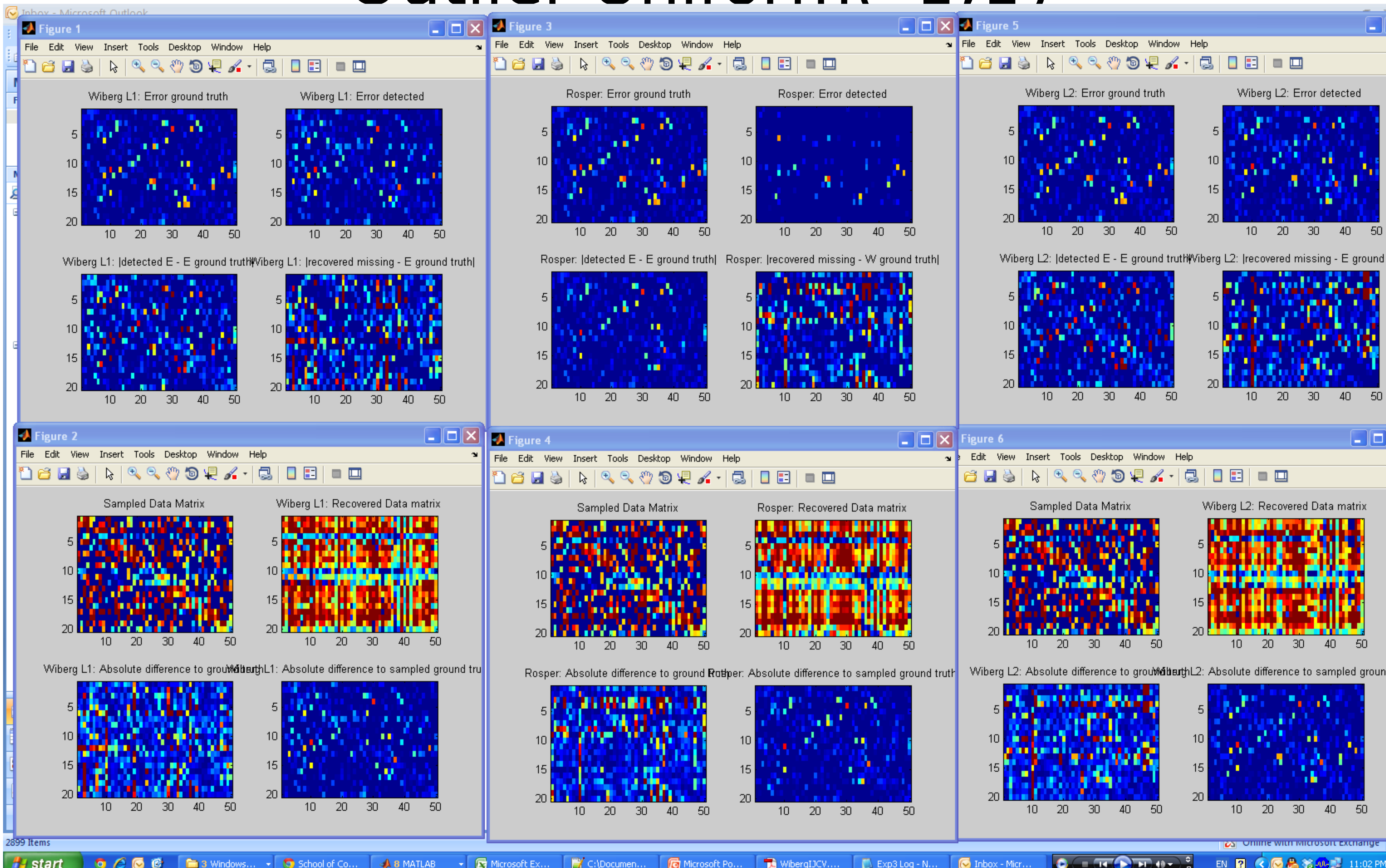
Dense Gaussian Noise



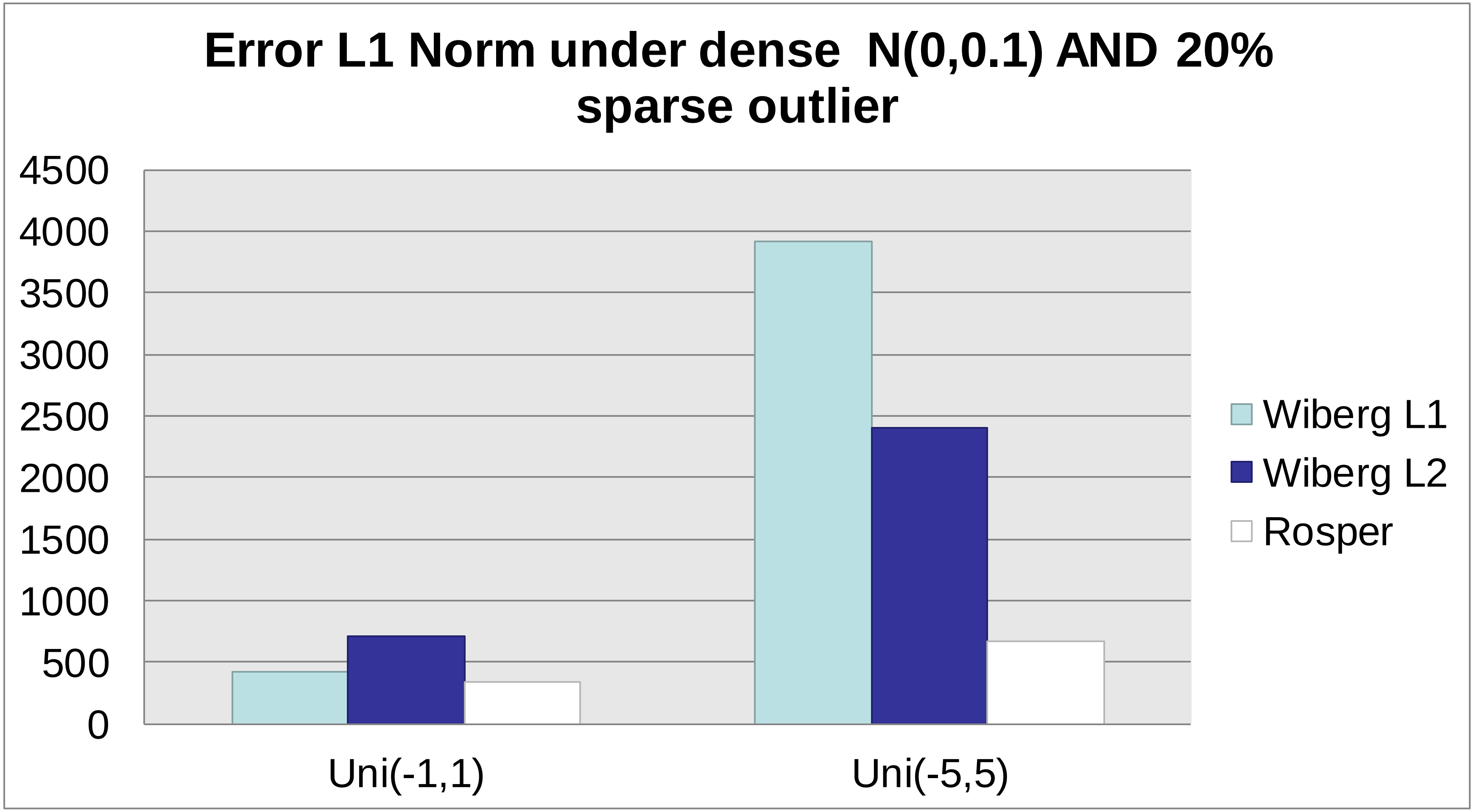
Experiment 11:Dense N(0,0.1)+0.2* Outlier Uniform(-5,5)



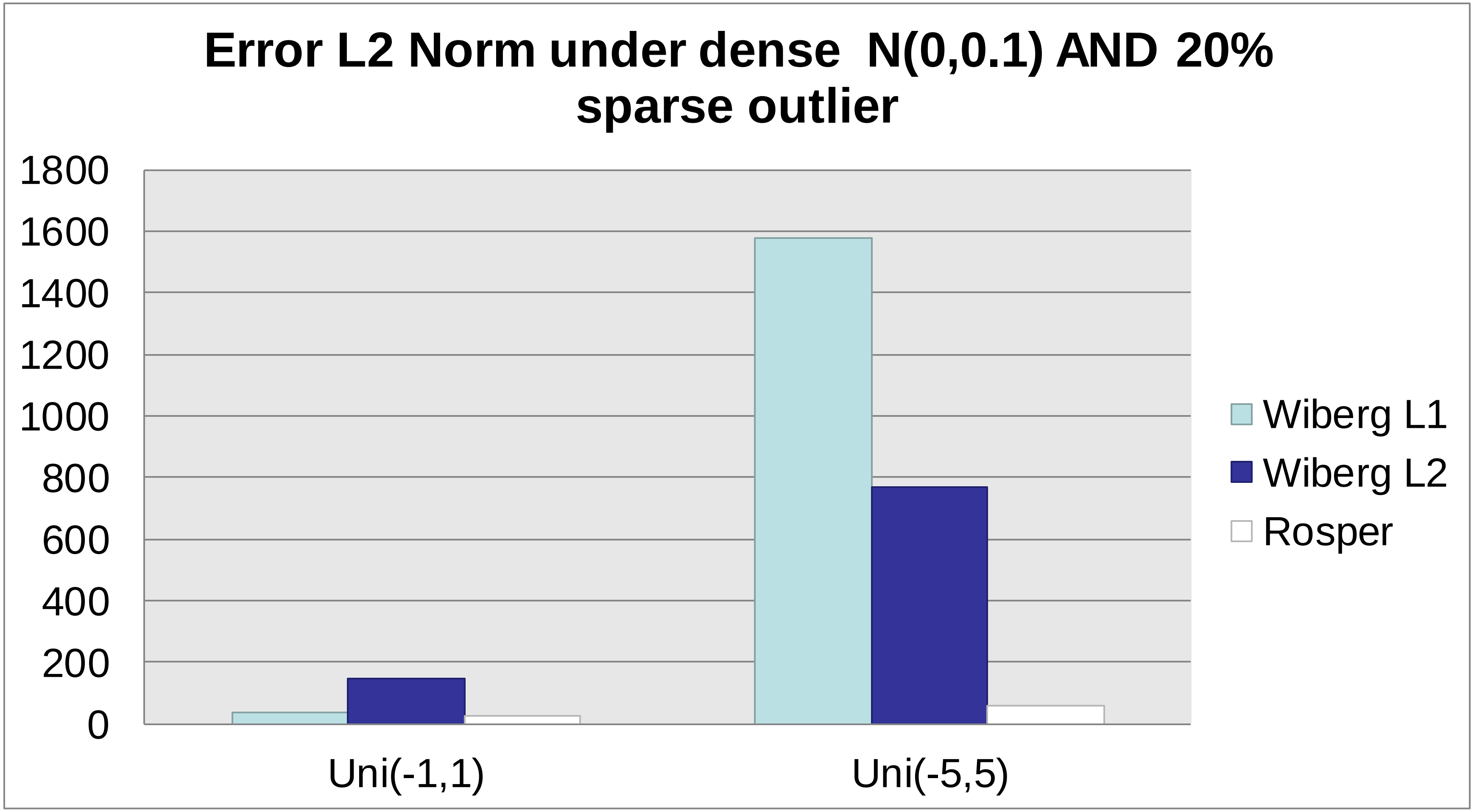
Experiment 12:Dense N(0,0.1)+0.2* Outlier Uniform(-1,1)



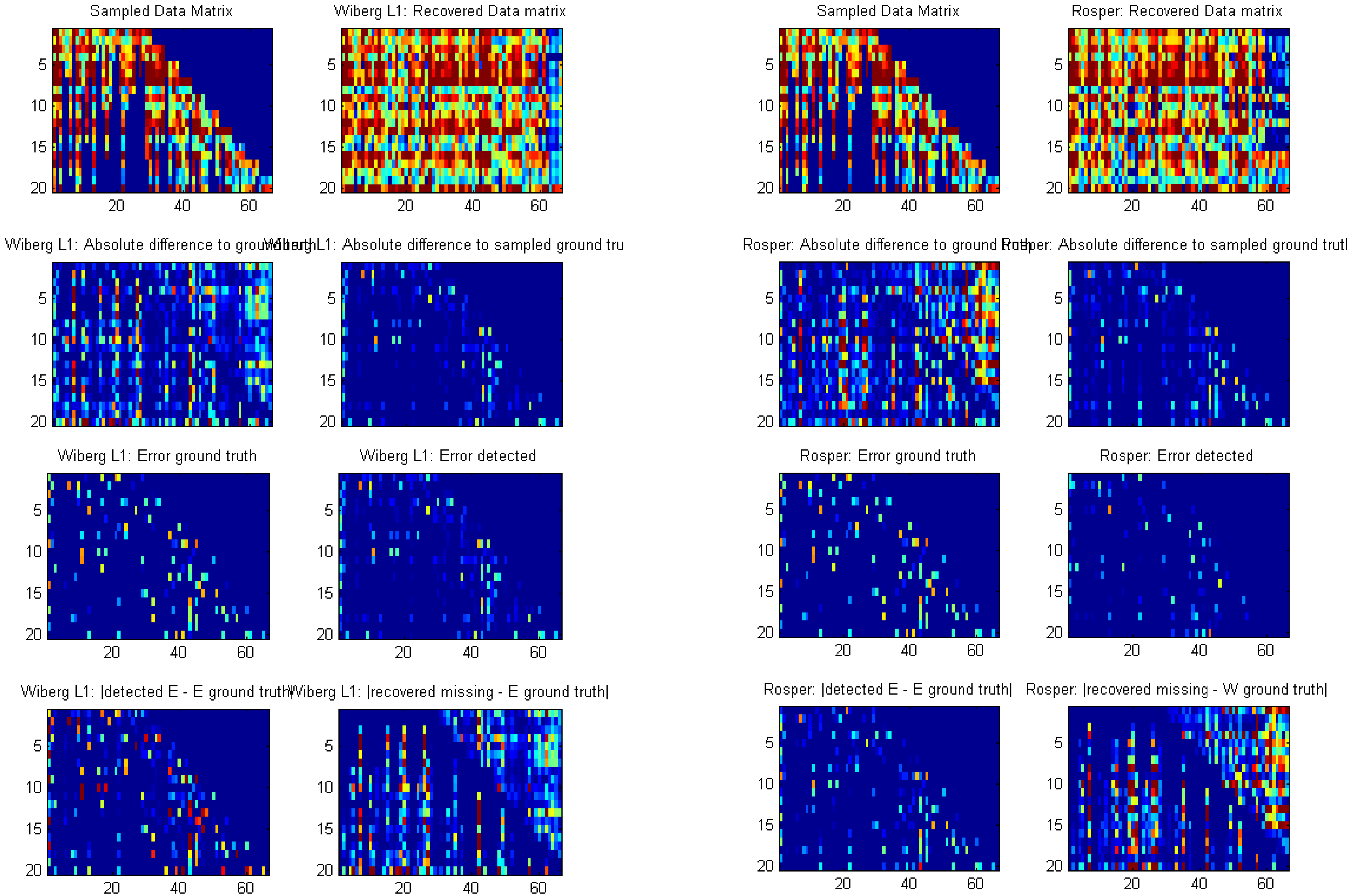
Dense Gaussian Noise + Sparse Outlier



Dense Gaussian Noise + Sparse Outlier



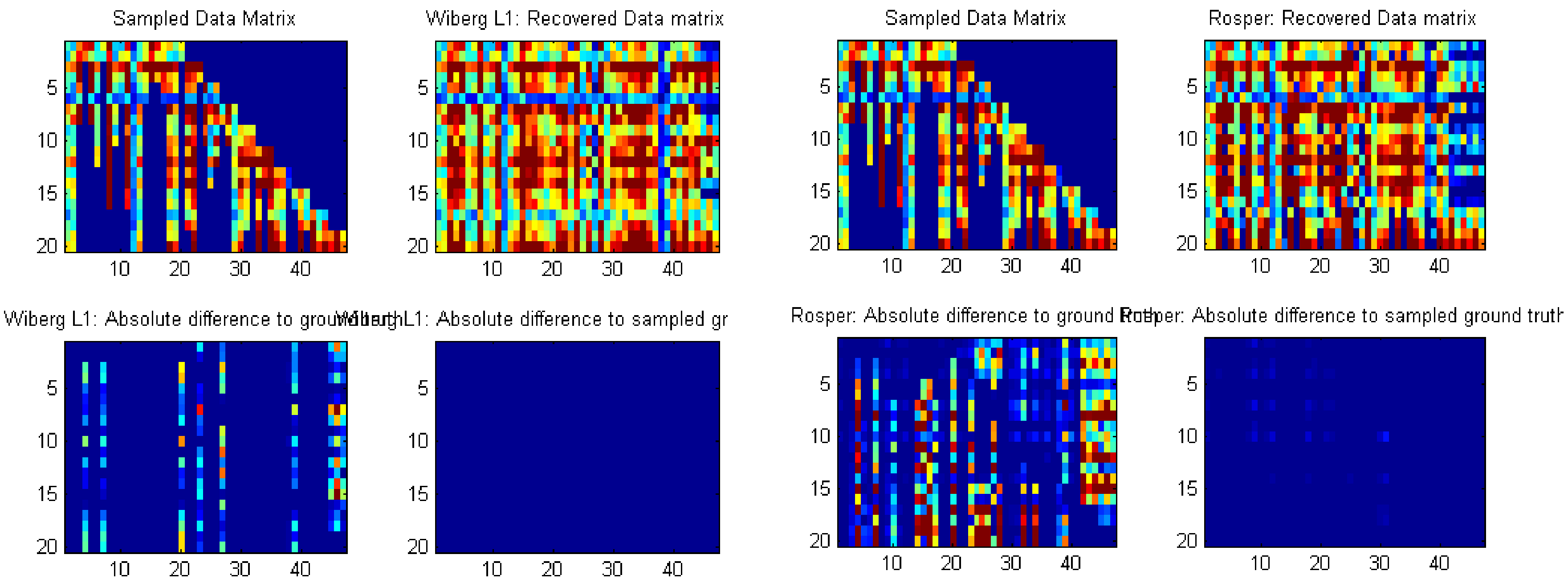
Diagonal band shaped data matrix



Diagonal band shaped data matrix

EXP1: 20% Uniform Error **0.45 Sample Rate** **[m n r]=[20 66 4]**	**Wiberg L1**	**Rosper**
# of iterations	17	2027
Running Time	6 min 36 sec	9 sec
|Recovered E - E_ground_truth|	106.72	52.55
|Recovered Missing - W_ground_truth|	253.35	397.26
L1 Norm: |Wgnd - Wr|	304.35	464.8
L2 Norm: |Wgnd - Wr|	16.38	25.53
L1 Norm: |mask(Wgnd - Wr)|	51	67.54
L2 Norm: |mask(Wgnd - Wr)|	4.55	5.2

Diagonal band shaped data matrix



Banded Results table

EXP2: 0 Uniform Error **0.43 Sample Rate** **[m n r]=[20 56 4]**	**Wiberg L1**	**Rosper**
# of iterations	35	5675
Running Time	6 min 59 sec	23 sec
|Recovered E - E_ground_truth|	<0.01	0.015
|Recovered Missing - W_ground_truth|	61.6	315.6
L1 Norm: |Wgnd - Wr|	61.61	322.02
L2 Norm: |Wgnd - Wr|	6.22	20.87
L1 Norm: |mask(Wgnd - Wr)|	<0.01	6.46
L2 Norm: |mask(Wgnd - Wr)|	<0.01	0.56

Conclusion

- L1 norm is very good in terms of detecting outliers
- Wiberg L1 outperforms ALP and AQP in terms of smaller residual.
- The methods of alternatively optimizing multiple variables are likely to be faster than optimizing all variables together.

Conclusion

- Though claimed to be “efficient”, Wiberg L1 is slow and not scalable. Wiberg L2 however is a fast and reliable algorithm for many possible applications.
- Current state of factorization methods/robust matrix completion still not sufficient for robust application in most computer vision applications.

Conclusion

- Explicitly model noises in Rosper will have positive effects on results if the noise model is correct. Yet, it can be difficult tuning the weighting.
- In general, Rosper is similar to Wiberg L1 in performance, and is much faster.

Future works

- Deal with missing entries in rPCA?
- Generating the same 100 small random matrix and run the full test for Rosper and compare.

- Jacob, Martinez
- Spectrally optimal Factorization of Incomplete Matrix