Summarizing Answer Graphs Induced by Keyword Queries

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Abstract

Keyword search has been popularly used to query graph data. Due to the lack of structure support, a keyword query might generate an excessive number of matches, referred to as "answer graphs", that could include different relationships among keywords. An ignored yet important task is to group and summarize answer graphs that share similar structures and contents for better query interpretation and result understanding. This paper studies the summarization problem for the answer graphs induced by a keyword query Q. (1) A notion of *summary graph* is proposed to characterize the summarization of answer graphs. Given Q and a set of answer graphs $\mathcal{G}$, a summary graph preserves the relation of the keywords in Q by summarizing the paths connecting the keywords nodes in $\mathcal{G}$. (2) A quality metric of summary graphs, called *coverage ratio*, is developed to measure information loss of summarization. (3) Based on the metric, a set of summarization problems are formulated, which aim to find minimized summary graphs with certain coverage ratio. (a) We show that the complexity of these summarization problems ranges from PTIME to NP-complete. (b) We provide exact and heuristic summarization algorithms. (4) Using real-life and synthetic graphs, we experimentally verify the effectiveness and the efficiency of our techniques.

1. Introduction

Keyword queries have been widely used for querying graph data, such as information networks, knowledge graphs, and social networks [36]. A keyword query Q is a set of keywords $\{k_1, \ldots, k_n\}$. The evaluation of Q over graphs is to extract data related with the keywords in Q [5, 36].

Various methods were developed to process keyword queries. In practice, these methods typically generate a set of graphs $\mathcal{G}$ *induced by* Q. Generally speaking, (a) the keywords in Q correspond to a set of nodes in these graphs, and (b) a path connecting two nodes related with keywords k_1, k_2 in Q suggests how the keywords are connected, *i.e.,* the relationship between the keyword pair (k_1, k_2). We refer to these graphs as *answer graphs* induced by Q. For example, (1) a host of work on keyword querying [12, 13, 16, 17, 20, 36] defines the query results as answer graphs; (2) keyword query interpretation [3, 34] transforms a keyword query into graph structured queries via the answer graphs extracted for the keyword; (3) result summarization [15, 22] generates answer graphs as *e.g.,* "snippets" for keyword query results.

Nevertheless, keyword queries usually generate a great number of answer graphs (as intermediate or final results) that are too many to inspect, due to the sheer volume of data. This calls for effective techniques to summarize answer graphs with representative structures and contents. Better still, the summarization of answer graphs can be further used for a range of important keyword search applications. We briefly describe several key applications as follows.

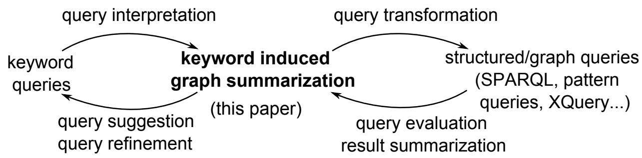


Figure 1: Keyword induced graph summarization – bridging keyword query and graph query

Enhance Search with Structure. It is known that there is an usability-expressivity tradeoff between keyword query and graph query [32] (as illustrated in Fig. 1). For searching graph data, keyword queries are easy to formulate; however, they might be ambiguous due to the lack of structure support. In contrast, graph queries are more accurate and selective, but difficult to describe. Query interpretation targets the trade-off by constructing graph queries, *e.g.,* SPARQL [30], to find more accurate query results. Nevertheless, there may exist many interpretations as answer graphs for a single keyword query [8]. A summarization technique may generate a small set of summary graphs, and graph queries can be induced, or extracted from these summaries. That is, a user can first submit keyword queries and then pick up the desired graph queries, thus taking advantage of both keyword query and graph query.

Improve Result Understanding and Query Refinement. Due to query ambiguity and the sheer volume of data, keyword query evaluation often generates a large number of results [15, 19]. This calls for effective methods to summarize the query results, such that users may easily understand the results without checking them one by one. Moreover, users may inspect the summary to come up with better queries that are *e.g.,* less ambiguous, by checking the connection of the keywords reflected in the summary. Based on the summarization result, efficient query refinement and suggestion techniques [23, 29] may also be proposed.

Example 1: Consider a keyword query $Q = \{$ Jaguar, America, history $\}$ issued over a knowledge graph. Suppose there are three graphs G_1, G_2 and G_3 induced by the keywords in Q as *e.g.,* query results [16, 20], as shown in Fig. 2. Each node in an answer graph has a type, as well as its unique id. It is either (a) a keyword node marked with $'*'$ (*e.g.,*Jaguar XK*) which corresponds to a keyword (*e.g.,*Jaguar), or (b) a node connecting two keyword nodes.

Observe that for the same query, the induced graphs illustrate different relations among the same keywords. For example, G_1 suggests that "Jaguar" is a brand of cars with multiple offers in many cities of USA, while G_3 suggests that "Jaguar" is a kind of animals found in America. To find out the answers the users need, reasonable graph structured queries are required for more accurate searching [3]. To this end, one may construct a summarization over the answer graphs. Two summaries can be constructed as G_{s_1}

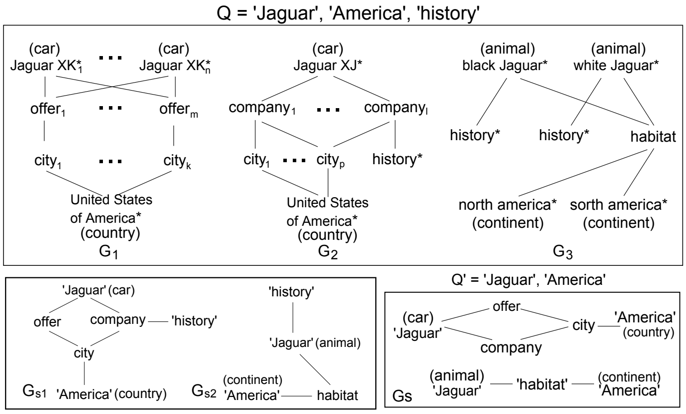


Figure 2: Keyword query over a knowledge graph

and G_{s_2}, which suggest two graph queries where "Jaguar" refers to a brand of car, and a kind of animal, respectively. Better still, by summarizing the relation between two keywords, more useful information can be provided to the users. For example, G_{s_1} suggests that users may search for "offers" and "company" of "Jaguar", as well as their locations.

Assume that the user wants to find out how "Jaguar" and "America" are related in the search results. This requires a summarization that only considers the connection between the nodes containing the keywords. Graph G_s depicts such a summarization: it shows that (1) "Jaguar" relates to "America" as a type of car produced and sold in cities of USA, or (2) it is a kind of animal living in the continents of America.

The above scenarios show the need of summarization techniques that preserve the connection for a set of keyword pairs. Moreover, in practice users often place a budget for the size of summarizations. This calls for quality metrics and techniques to measure and generate summarizations, constrained by the budgets. □

This example suggests that we summarize answer graphs $\mathcal{G}$ induced by a keyword query Q to help keyword query processing. We ask the following questions. (1) How to define a "query-aware" summarization of $\mathcal{G}$ in terms of Q? (2) How to characterize the quality of the summarization? (3) How to efficiently identify the summarization with high quality under a budget constraint?

Contributions. This paper investigates the above problems for summarizing keyword induced answer graphs.

(1) We formulate the concept of answer graphs for a keyword query Q (Section 2). To characterize the summarization for answer graphs, we propose a notion of *summary graph* (Section 2). Given Q and $\mathcal{G}$, a summary graph captures the relationship among the keywords from Q in $\mathcal{G}$.

(2) We introduce quality metrics for summary graphs (Section 3). One is defined as the size of a summary graph, and the other is based on *coverage ratio* α, which measures the number of keyword pairs a summary graph can *cover* by summarizing pairwise relationships in $\mathcal{G}$.

Based on the quality metrics, we introduce two *summarization* problems (Section 3). Given Q and $\mathcal{G}$, (a) the α-summarization problem is to find a minimum summary graph with a certain coverage ratio α; we consider 1-summarization problem as its special case where $\alpha = 1$; (b) the K summarization problem is to identify K summary graphs for $\mathcal{G}$, where each one summarizes a subset of answer graphs in $\mathcal{G}$. We show that the complexity of these problems ranges from PTIME to NP-complete. For the NP-hard problems, they are also hard to approximate.

(3) We propose exact and heuristic algorithms for the summarization problems. Specifically, (1) we show that for a given keyword query Q and $\mathcal{G}$, it is in quadratic time to find a minimum 1-summarization, by providing such an algorithm (Section 4); (2) we provide two heuristic algorithms for the α-summarization (Section 4) and k summarization problems (Section 5), respectively.

(4) We experimentally verify the effectiveness and efficiency of our summarization techniques using both synthetic data and real-life datasets. We find that our algorithms effectively summarize the answer graphs. For example, they generate summary graphs that cover every pair of keywords with size in average 24% of the answer graphs. They also scale well with the size of the answer graphs. These effectively support summarization over answer graphs.

Related Work. We categorize related work as follows.

Graph Data Summarization. There has been a host of work on general graph summarization techniques.

Graph summarization and minimization. [26,33,37] propose graph summarization to approximately describe the topology and content of graph data. These techniques are designed for summarizing an entire graphs, rather than for a set of graphs *w.r.t.* a keyword query. Indexing and summarization techniques are developed based on (1) bisimulation equivalence relation to preserve path information for every pair of nodes in a graph [25], and (2) relaxed bisimulation relation that preserves paths with length up to K [18, 25]. In addition, simulation based minimization [2] reduces a transition system based on simulation equivalence relation. In contrast, our work summarizes the paths that contain keywords. Moreover, we introduce quality metrics and algorithms to find summaries for specified keyword queries, which is not studied in the prior work mentioned above.

Relation discovery. Relation discovery is to extract the relations between keywords over a (single) graph [7, 16, 31]. [31] studies the problem to extract related information for a single entity from a knowledge graph. [7] considers extracting relationships for a pair of keywords. In contrast to these studies, we summarize relationships as a summary graph for a keyword query. In addition, users can place constraints such as size and coverage ratio to identify summaries with high quality, which are not addressed before.

Graph clustering. A number of graph clustering approaches have also been proposed to group similar graphs [1]. As remarked earlier, these techniques are not query-aware, and may not be directly applied for summarizing query results as graphs [21]. In contrast, we propose algorithms to (1) group answer graphs in terms of a set of keywords, and (2) find best summaries for each group.

Result Summarization. Result summarization over relational databases and XML are proposed to help users understand the query results. [15] generates summaries for XML results as trees, where a snippet is produced for each result tree. This may produce snippets with similar structures that should be grouped for better understanding [21]. To address this issue, [22] clusters the query results based on the classification of their search predicates. Our work differs in that (1) we generates summarizations as general graphs, (2) in contrast to result snippets, we study how to summarize answer graphs for keyword queries.

Application Scenarios. There have been a host of studies on processing keyword queries that generate answer graphs. Our work can be applied to these applications.

Keyword queries over graphs. Various methods are proposed for keyword search over graphs, which typically return graphs that contain all the keywords [36]. For example, an answer graph as a query result is represented by (1) subtrees for XML data [12,13], or (2) subgraphs of schema-free graphs [16,17,20]. The summarization techniques in our work can be applied in these applications as post-processing, to provide result summarizations [15].

Query interpretation. Keyword query interpretation transforms a keyword query into graph structured queries, *e.g.,* XPath queries [27], SPARQL queries [30], or a group of formal queries [34] (see [3] for a survey). The summary graphs proposed in this work can be used to suggest *e.g.,* formal queries for keyword queries, or graph queries themselves.

Query expansion. [23] considers generating suggested keyword queries from a set of clustered query results. [29] studies the keyword query expansion that extends the original queries with "surprising words" as additional search items. Neither considers structured expansions. Our work produces structural summaries that not only include keywords and their relationships, but also a set of highly related nodes and relations, which could provide good suggestions for query refinement (Section 6).

2. Answer Graphs and Summarizations

In this section, we formulate the concept of answer graphs induced by keyword queries, and their summarizations.

2.1 Keyword Induced Answer Graphs

Answer graphs. Given a keyword query Q as a set of keywords $\{k_1, \ldots, k_n\}$ [36], an *answer graph* induced by Q is a connected undirected graph $G = (V, E, L)$, where V is a node set, $E \subseteq V \times V$ is an edge set, and L is a labeling function which assigns, for each node v, a label $L(v)$ and a *unique* identity. In practice, the node labels may represent the type information in *e.g.,* RDF [16], or node attributes [37]. The node identity may represent a name, a property value, a *URI*, *e.g.,* "*dbpedia.org/resource/Jaguar*," and so on. Each node $v \in V$ is either a *keyword node* that corresponds to a keyword k in Q, or an *intermediate* node on a path between keyword nodes. We denote as v_k a keyword node of k. The keyword nodes and intermediate nodes are typically specified by the process that generates the answer graphs, *e.g.,* keyword query evaluation algorithms [36]. A path connecting two keyword nodes usually suggests a relation, or "connection pattern", as observed in *e.g.,* [7].

We shall use the following notations. (1) A path from keyword nodes v_k to v'_k is a nonempty *simple* node sequence $\{v_k, v_1 \ldots, v_n, v'_k\}$, where v_i ($i \in [1, n]$) are intermediate nodes. The label of a path ρ from v_k to v'_k, denoted as $L(\rho)$, is the concatenation of all the node labels on ρ. (2) The union of a set of answer graphs $G_i = (V_i, E_i, L_i)$ is a graph $G = (V, E, L)$, where $V = \bigcup V_i$, $E = \bigcup E_i$, and each node in V has a unique node id. (3) Given a set of answer graphs $\mathcal{G}$, we denote as $\mathsf{card}(\mathcal{G})$ the number of the answer graphs $\mathcal{G}$ contains, and $|\mathcal{G}|$ the total number of its nodes and edges. Note that an answer graph does not necessarily contain keyword nodes for all the keywords in Q, as common found in *e.g.,* keyword querying [36].

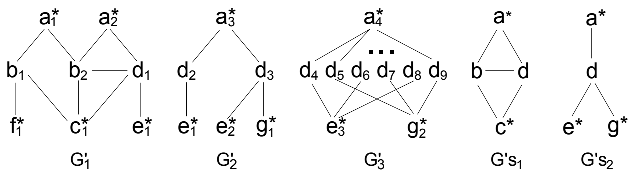


Figure 3: Answer graphs and summary graphs

Example 2: Fig. 2 illustrates a keyword query Q and a set of answer graphs $\mathcal{G} = \{G_1, G_2, G_3\}$ induced by Q. Each node in an answer graph has a label as its type (*e.g.,*car), and a unique string as its id (*e.g.,*Jaguar XK_1).

Consider the answer graph G_1. (a) The keyword nodes for the keyword Jaguar are Jaguar_{XK_i} ($i \in [1, n]$), and the node United States of America is a keyword node for America. (b) The nodes offer_i ($i \in [1, m]$) and city_j ($j \in [1, k]$) are the intermediate nodes connecting the keyword nodes of Jaguar and America. (c) A path from Jaguar to USA passing the nodes offer_1 and city_1 has a label {car,offer, city,country}. Note that (1) nodes with different labels (*e.g.,* Jaguar_{XK_1} labeled by "car" and black jaguar by "animal") may correspond to the same keyword (*e.g.,*Jaguar), and (2) a node (*e.g.,* city_1) may appear in different answer graphs (*e.g.,* G_1 and G_2). □

2.2 Answer Graph Summarization

Summary graph. A summary graph of $\mathcal{G}$ for Q is an undirected graph $G_s = (V_s, E_s, L_s)$, where V_s and E_s are the node and edge set, and L_s is a labeling function. Moreover, (1) each node $v_s \in V_s$ labeled with $L_s(v_s)$ represents a node set $[v_s]$ from $\mathcal{G}$, such that (a) $[v_s]$ is either a keyword node set, or an intermediate node set from $\mathcal{G}$, and (b) the nodes v in $[v_s]$ have the same label $L(v) = L_s(v_s)$. We say v_{s_k} is a *keyword node* for a keyword k, if $[v_{s_k}]$ is a set of keyword nodes of k; (2) For any path ρ_s between keyword nodes v_{s_1} and v_{s_2} of G_s, there exists a path ρ with the same label of ρ_s from v_1 to v_2 in the *union* of the answer graphs in $\mathcal{G}$, where $v_1 \in [v_{s_1}]$, $v_2 \in [v_{s_2}]$. Here the path label in G_s is similarly defined as its counterpart in an answer graph.

Hence, a summary graph G_s never introduces "false" paths by definition: if v_{s_1} and v_{s_2} are connected via a path ρ_s in G_s, it suggests that there is a path ρ of the same label connecting two keyword nodes in $[v_{s_1}]$ and $[v_{s_2}]$, respectively, in the union of the answer graphs. It might, however, "lose" information, *i.e.,* not all the labels of the paths connecting two keyword nodes are preserved in G_s.

Example 3: Consider Q and $\mathcal{G}$ from Fig. 2. One may verify that G_{s_1}, G_{s_2} and G_s are summary graphs of $\mathcal{G}$ for Q. Specifically, (1) the nodes Jaguar, history and America are three keyword nodes in G_{s_1}, and the rest nodes are intermediate ones; (2) G_{s_2} contains a keyword node Jaguar which corresponds to keyword nodes {black jaguar, white jaguar} of the same label animal in $\mathcal{G}$. (3) For any path connecting two keyword nodes (*e.g.,* {Jaguar, offer, city, America}) in G_{s_1}, there is a path with the same label in the union of G_1 and G_2 (*e.g.,* {Jaguar_{XK_1}, offer_1, city_1, United States of America}).

As another example, consider the answer graphs G'_1, G'_2 and G'_3 induced by a keyword query $Q' = \{$a, c, e, f, g$\}$ in Fig. 3. Each node a_i (marked with $*$ if it is a keyword node) in an answer graph has a label a and an id a_i, similarly for the rest nodes. One may verify the following. (1) Both G'_{s_1} and G'_{s_2} are summary graphs for the answer graph set $\{G'_1, G'_2\}$; while G'_{s_1} (resp. G'_{s_2}) only preserves the labels of the paths connecting keywords a and c (resp. a, e and g). (2) G'_{s_2} is not a summary graph for G'_3. Although it correctly suggests the relation between keywords (a, e) and (a, g), it contains a "false" path labeled (e, d, g), while there is no path in G'_3 with the same label between e_3 and g_2. □

Remarks. One can readily extend summary graphs to support directed, edge labeled answer graphs by incorporating edge directions and labels into the path label. We can also extend summary graphs for preserving path labels for each answer graph, instead of for the union of answer graphs, by reassigning node identification to answer graphs.

3. Quality Measurement

In order to measure the quality of summary graphs, we introduce two metrics based on information coverage and summarization conciseness, respectively. We then introduce a set of summarization problems. To simplify the discussion, we assume that the union of the answer graphs contains keyword nodes for each keyword in Q.

3.1 Coverage Measurement

It is recognized that a summarization should summarize as much information as possible, *i.e.,* to maximize the information coverage [11]. In this context, a summary graph should be able to capture the relationship among the query keywords as much as possible. To characterize the information coverage of a summary graph, we first present a notion of *keywords coverage*.

Keywords coverage. Given a keyword pair (k_i, k_j) (k_i, $k_j \in Q$ and $k_i \neq k_j$) and answer graphs $\mathcal{G}$ induced by Q, a summary graph G_s *covers* (k_i, k_j) if for any path ρ from keyword nodes v_{k_i} to v_{k_j} in the union of the answer graphs in $\mathcal{G}$, there is a path ρ_s in G_s from v_{s_i} to v_{s_j} with the same label of ρ, where $v_{k_i} \in [v_{s_i}]$, $v_{k_j} \in [v_{s_j}]$. Note that the coverage of a keyword pair is "symmetric" over undirected answer graphs. Given Q and $\mathcal{G}$, if G_s covers a keyword pair (k_i, k_j), it also covers (k_j, k_i).

Coverage ratio. Given a keyword query Q and $\mathcal{G}$, we define the *coverage ratio* α of a summary graph G_s of $\mathcal{G}$ as

$$\alpha = \frac{2 \cdot M}{|Q| \cdot (|Q| - 1)}$$

where M is the total number of the keyword pairs (k, k') covered by G_s. Note that there are in total $\frac{|Q||Q|-1}{2}$ pairs of keywords from Q. Thus, α measures the information coverage of G_s based on the coverage of the keywords.

We refer to as *α-summary graph* the summary graph for $\mathcal{G}$ induced by Q with coverage ratio α. The coverage ratio measurement favors a summary graph that covers more keyword pairs, *i.e.,* with larger α.

Example 4: Consider Q and $\mathcal{G}$ from Fig. 2. Treating G_{s_1} and G_{s_2} as a single graph G_{s_0}, one may verify that G_{s_0} is a 1-summary graph of $\mathcal{G}$ for Q. Indeed, for any keyword pair from Q (*e.g.,* (Jaguar, America)) and any path between the keyword nodes in $\mathcal{G}$, there exists a path of the same label in G_{s_0}.On the other hand, G_s is a $\frac{1}{3}$ summary graph for Q: it only covers the keyword pairs (Jaguar, America). Similarly, one may verify that G'_{s_1} (resp. G'_{s_2}) in Fig. 3 is a 0.1-summary graph (resp. 0.3-summary graph), for answer graphs $\{G'_1, G'_2, G'_3\}$ and $Q = \{a, c, e, f, g\}$. □

3.2 Conciseness Measurement

A summary graph should also be concise, without introducing too much detail of answer graphs. The measurement of conciseness for summarization is commonly used in information summarization [11, 31].

Summarization size. We define the size of a summary graph G_s, (denoted as $|G_s|$) as the total number of the nodes and edges it has. For example, the summary graph G_{s_1} and G_{s_2} (Fig. 2) are of size 12 and 7, respectively. The smaller a summary graph is, the more concise it is.

Putting the information coverage and conciseness measurements together, We say a summary graph G_s is a *minimum* α-summary graph, if for any other α-summary graph G'_s of $\mathcal{G}$ for Q, $|G_s| \leq |G'_s|$.

Remarks. The bisimulation relation [10] and graph summarization [26,33] also induce summarized graphs, by grouping similar nodes and edges together for an entire graph, rather than for specified keyword nodes. Moreover, (a) they may not necessarily generate concise summary graphs; and (b) their summary graphs may introduce "false" paths.

Example 5: The bisimulation relation [10] constraints the node equivalence via a recursively defined neighborhood label equivalence, which is too restrictive to generate concise summary graphs for keyword relations. For example, the nodes b_1 and b_2 cannot be represented by a single node as in G_{s_1} via bisimulation (Fig. 3), due to different neighborhood. One the other hand, error-tolerant [26] and structure-based graph summarization [33] may generate summary graphs with "false paths", such as G'_{s_2} for G'_3. To prevent this, additional auxiliary structures and parameters are required. In contrast in our work, a summary graph preserves path labels for keywords without any auxiliary structures. □

3.3 Summarization Problems

Based on the quality metrics, we next introduce two summarization problems for keyword induced answer graphs. These problems are to find summary graphs with high quality, in terms of information coverage and conciseness.

Minimum α-Summarization. Given a keyword query Q and its induced answer graphs $\mathcal{G}$, and a user-specified coverage ratio α, the *minimum α-summarization* problem, denoted as MSUM, is to find an α-summary graph of $\mathcal{G}$ with minimum size. Intuitively, the problem aims to find the smallest summary graph [31] which can cover the keyword pairs no less than user-specified coverage requirement.

The problem is, however, nontrivial.

Theorem 1: MSUM *is* NP*-complete (for decision version) and* APX*-hard (as an optimization problem).* □

The APX-hard class consists of all problems that cannot be approximated in polynomial time within arbitrary small approximation ratio [35]. We prove the complexity result and provide a heuristic algorithm for MSUM in Section 4.

Minimum 1*-summarization.* We also consider the problem of finding a summary graph that covers every pair of keywords (k_i, k_j) ($k_i, k_j \in Q$ and $i \neq j$) as concise as possible, *i.e.,* the *minimum* 1-*summarization problem* (denoted as PSUM). Note that PSUM is a special case of MSUM, by setting $\alpha = 1$. In contrast to MSUM, PSUM is in PTIME.

Theorem 2: *Given* Q *and* $\mathcal{G}$, PSUM *is in* $O(|Q|^2|\mathcal{G}| + |\mathcal{G}|^2)$ *time,* i.e., *it takes* $O(|Q|^2|\mathcal{G}| + |\mathcal{G}|^2)$ *time to find a minimum* 1-*summary graph, where* $|\mathcal{G}|$ *is the size of* $\mathcal{G}$. □

We will prove the above result in Section 4.

K **Summarization**. In practice, users may expect a set of summary graphs instead of a single one, where each summary graph captures the keyword relationships for a set of "similar" answer graphs in terms of path labels. Indeed, as observed in text summarization (*e.g.,* [11]), a summarization should be able to cluster a set of similar objects.

Given Q, $\mathcal{G}$, and an integer K, the K *summarization* problem (denoted as KSUM) is to find a summary graph set G_S, such that (1) each summary graph $G_{s_i} \in G_S$ is a 1-summary graph of a group of answer graphs $G_{p_i} \subseteq \mathcal{G}$, (2) the answer graph sets G_{p_i} form a K-partition of $\mathcal{G}$, *i.e.,* $\mathcal{G} = \bigcup G_{p_i}$, and $G_{p_i} \cap G_{p_j} = \emptyset$ ($i, j \in [1, K]$, $i \neq j$); and (3) the total size of G_S, *i.e.,* $\sum_{G_{s_i} \in G_S} |G_{s_i}|$ is minimized. The KSUM problem can also be extended to support α-summarization.

Instead of finding only a single summary graph, KSUM finds K summary graphs such that each "groups" a set of similar answer graphs together and covers all the keyword pairs appeared in the cluster. This may also provide a reasonable clustering for answer graphs [11].

The following result tells us that the problem is hard to approximate. We will prove the result in Section 5, and provide a heuristic algorithm for KSUM.

Theorem 3: KSUM *is* NP-*complete and* APX-*hard.* □

4. Computing α-Summarization

In this section we investigate the α-summarization problem. We first investigate PSUM in Section 4.1, as a special case of MSUM. We then discuss MSUM in Section 4.2.

4.1 Computing 1-Summary Graphs

To show Theorem 2, we characterize the 1-summary graph with a sufficient and necessary condition. We then provide an algorithm to check the condition in polynomial time.

We first introduce the notion of dominance relation.

Dominance relation. The *dominance relation* $R_{\preceq}(k, k')$ for keyword pair (k, k') over an answer graph $G = (V, E, L)$ is a binary relation over the intermediate nodes of G, such that for each node pair $(v_1, v_2) \in R_{\preceq}(k, k')$, (1) $L(v_1) = L(v_2)$, and (2) for any path ρ_1 between keyword node pair v_{k_1} of k and v_{k_2} of k' passing v_1, there is a path ρ_2 with the same label between two keyword nodes v'_{k_1} of k and $v_{k'_2}$ of k' passing v_2. We say v_2 *dominates* v_1 *w.r.t.* (k, k'); moreover, v_1 is *equivalent* to v_2 if they dominate each other. In addition, two keyword nodes are equivalent if they have the same label, and correspond to the same keyword.

The dominance relation is as illustrated in Fig. 4. Intuitively, (1) $R_{\preceq}(k, k')$ captures the nodes that are "redundant" in describing the relationship between a keyword pair (k, k') in G; (2) moreover, if two nodes are equivalent, they play the same "role" in connecting keywords k and k', *i.e.,* they cannot be distinguished in terms of path labels. For example, when the keyword pair (a, c) is considered in G'_1, the node b_1 is dominated by b_2, as illustrated in Fig. 4.

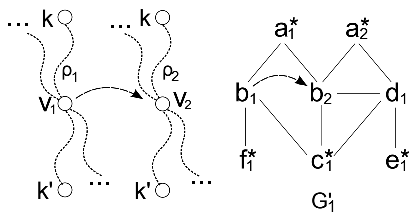


Figure 4: Dominance relation: $(v_1, v_2) \in R_{\preceq}$

Remarks. The relation $R_{\preceq}$ is similar to the *simulation* relation [2,14], which computes node similarity over the entire graph by neighborhood similarity. In contrast to simulation, $R_{\preceq}$ captures dominance relation induced by the paths connecting keyword nodes only, and only consider intermediate nodes. For example, the node b_1 and b_2 is not in a simulation relation in G'_1, unless the keyword pair (a, c) is considered (Fig. 4). We shall see that this leads to effective summarizations for specified keyword pairs.

Sufficient and necessary condition. We now present the sufficient and necessary condition, which shows the connection between $R_{\preceq}$ and a 1-summary graph.

Proposition 4: *Given* Q *and* $\mathcal{G}$*, a summary graph* G_s *is a minimum* 1-*summary graph for* $\mathcal{G}$ *and* Q*, if and only if for each keyword pair* (k, k') *from* Q*, (a) for each intermediate node* v_s *in* G_s*, there is a node* v_i *in* $[v_s]$*, such that for any other node* v_j *in* $[v_s]$*,* $(v_j, v_i) \in R_{\preceq}(k, k')$*; and (b) for any intermediate nodes* v_{s_1} *and* v_{s_2} *in* G_s *with same label and any nodes* $v_1 \in [v_{s_1}]$*,* $v_2 \in [v_{s_2}]$*,* $(v_2, v_1) \notin R_{\preceq}(k, k')$*.* □

Proof sketch: We prove Proposition 4 as follows.

(1) We first proof by contradiction that G_s is a 1-summary graph if and only if Condition (a) holds. Assume G_s is a 1-summary graph while Condition (a) does not hold. Then there exists an intermediate node v_s, and two nodes v_i and v_j that cannot dominate each other. Thus, there must exist two paths in the union of answer graphs as $\rho = \{v_1, \ldots, v_i, v_{i+1}, \ldots, v_m\}$ and $\rho' = \{v'_1, \ldots, v_j, v_{j+1}, \ldots, v_n\}$ with different labels, for a keyword pair (k, k'). Since v_i, v_j is merged as v_s in G_s, there exists, *w.l.o.g.*, a false path in G_s as ρ'' with label $L(v_1) \ldots L(v_i) L(v_{j+1}) \ldots L(v_m)$, which contradicts the assumption that G_s is a 1-summary graph. Now assume Condition (a) holds while G_s is not a 1-summary graph. Then there at least exists a path from keywords k to k' that is not in G_s. Thus, there exists at least an intermediate node v_s on the path with $[v_s]$ in G_s which contains two nodes that cannot dominate each other. This contradicts the assumption that Condition (a) holds.

(2) For the summary minimization, we show that Conditions (a) and (b) together guarantee if there exists a 1-summary G'_s where $|G'_s| \leq |G_s|$, there exists a one to one function mapping each node (resp. edge) in G'_s to a node (resp. edge) in G_s, *i.e.,* $|G_s| = |G'_s|$. Hence, G_s is a minimum 1-summary graph by definition. □

We next present an algorithm for PSUM following the sufficient and necessary condition, in polynomial time.

Algorithm. Fig. 5 shows the algorithm, denoted as pSum. It has the following two steps.

Initialization (lines 1-4). pSum first initializes an empty summary graph G_s (line 1). For each keyword pair (k, k') from Q, pSum computes a "connection" graph of (k, k') in-

Input: A keyword query Q, an answer graph set $\mathcal{G}$.
Output: A minimum 1-summary graph G_s.

1. Initialize $G_s = \emptyset$;
2. **for each** keyword pair (k, k') $(k, k' \in Q, k \neq k')$ **do**
3. build $G_{(k,k')}$ as an induced connection graph of (k, k');
4. merge G_s with $G_{(k,k')}$;
5. $R_{\preceq}$:= $\mathsf{DomR}(G_s)$; remove dominated nodes from G_s;
6. merge each v_{s_1}, v_{s_2} in G_s where there is a node $v_1 \in [v_{s_1}]$ such that for $\forall v_2 \in [v_{s_2}]$, $(v_2, v_1) \in R_{\preceq}(k, k')$;
7. **return** G_s;

Procedure DomR

Input: a graph G_s, $\mathcal{G}$;
Output: the dominance relation $R_{\preceq}$ over G_s.

1. **for** each node v in G_s **do**
2. dominant set $[v] = \{v' | L(v') = L(v)\}$;
3. **while** $[v]$ is changed for some v **do**
4. **for** each edge (u, v) in G **do** $[u] = [u] \cap N([v])$; $[v] = [v] \cap N([u])$;
5. **for each** v and $v' \in [v]$ **do**
6. $R_{\preceq} = R_{\preceq} \cup \{(v, v')\}$;
7. **return** $R_{\preceq}$;

Figure 5: Algorithm pSum

duced from $\mathcal{G}$ (line 2-3). Let G be the union of the answer graphs in $\mathcal{G}$. A connection graph of (k, k') is a subgraph of G induced by (1) the keyword nodes of k and k', and (2) the intermediate nodes on the paths between the keyword nodes of k and those of k'. Once $G_{(k,k')}$ is computed, pSum sets G_s as the union graph of G_s and $G_{(k,k')}$ (line 4).

Reducing (lines 5-7). pSum then constructs a summary graph by removing nodes and edges from G_s. It computes the dominance relation $R_{\preceq}$ by invoking a procedure DomR, which removes the nodes v as well as the edges connected to them, if they are dominated by some other nodes (line 5). It next merges the nodes in G_s that have dominate relation, *i.e.,* line 6 (as defined in 4(a)), into a set $[v_s]$, until no more nodes in G_s can be merged. For each set $[v_s]$, a new node v_s as well as its edges connected to other nodes are created. G_s is then updated with the new nodes and edges, and is returned as a minimum 1-summary graph (line 7).

Procedure DomR. The idea of DomR is similar as the process to compute a simulation relation [14], while it extends the process to undirected connection graphs. For each node v in G_s, DomR first initializes a dominant set, denoted as $[v]$, as $\{v' | L(v') = L(v)\}$ (lines 1-2). For each edge $(u, v) \in G_s$, it identifies the neighborhood set of u (resp. v) as $N(u)$ (resp. $N(v)$), and removes the nodes that are not in $N(v)$ (resp. $N(u)$) from $[u]$ (resp. $[v]$) (lines 4). Note that a node $u' \in [u]$ cannot dominant u if $u' \notin N(v)$, since there exists a path connecting two keyword nodes passing edge (u, v) and contains "$L(u)L(v)$" in its label, while for u', such path does not exist. The process repeats until no changes can be made to any dominant set (lines 3-4). $R_{\preceq}$ is then collected from the dominant sets and returned (line 5-7).

Analysis. pSum correctly returns a summary graph G_s. Indeed, G_s is initialized as the union of the connection graphs, which is a summary graph (lines 2-4). Each time G_s is updated, pSum keeps the invariants that G_s remains to be a summary graph. When pSum terminates, one may verify that the sufficient and necessary condition as in Proposition 4 is satisfied. Thus, the correctness of pSum follows.

It takes $O(|Q|^2|\mathcal{G}|)$ to construct G_s as the union of the connection graphs for each keyword pairs (lines 2-4). It takes DomR in total $O(|\mathcal{G}|^2)$ time to compute $R_{\preceq}$. To see this, observe that (a) it takes $O(|\mathcal{G}|^2)$ time to initialize the dominant sets (line 1), (b) during each iteration, once a node is removed from $[u]$, it will no longer be put back, *i.e.,* there are in total $|G_s|^2$ iterations, and (c) the checking at line 4 can be done in constant time, by looking up a dynamically maintained map recording $|[u] \setminus N(v)|$ for each edge (u, v), leveraging the techniques in [14]. Thus, the total time complexity of pSum is in $O(|Q|^2|\mathcal{G}| + |\mathcal{G}|^2)$.

Theorem 2 follows from the above analysis.

Example 6: Recall the query Q and the answer graph set $\mathcal{G}$ in Fig. 2. The algorithm pSum constructs a minimum 1-summary graph G_s for $\mathcal{G}$ as follows. It initializes G_s as the union of the connection graphs for the keyword pairs in Q, which is the union graph of G_1, G_2 and G_3. It then invokes procedure DomR, which computes dominance sets for each intermediate node in G_s, partly shown as follows.

Nodes in G_s	dominance sets
offer	$\{\mathsf{offer}_i\}(i \in [1, m])$
city	$\{\mathsf{city}_i\}(i \in [1, k]), \{\mathsf{city}_j\}(j \in [k+1, p])$
company	$\{\mathsf{company}_i\}(i \in [1, l-1]), \{\mathsf{company}_l\}$

pSum then reduces G_s by removing dominated nodes and merging equivalent nodes until no change can be made. For example, (1) $\mathsf{company}_x$ $(x \in [1, l-1])$ are removed, as all are dominated by $\mathsf{company}_l$; (2) all the offer nodes are merged as a single node, as they dominate each other. G_s is then updated as the union of G_{s_1} and G_{s_2} (Fig. 2). ☐

From Theorem 2, the result below immediately follows.

Corollary 5: *It is in $O(|S||\mathcal{G}| + |\mathcal{G}|^2)$ to find a minimum 1-summary graph of $\mathcal{G}$ for a given keyword pair set S.* ☐

Indeed, pSum can be readily adapted for specified keyword pair set S, by specifying G_s as the union of the connection graphs induced by S (line 4). The need to find 1-summary graphs for specified keyword pairs is evident in the context of *e.g.,* relation discovery [7], where users may propose specified keyword pairs to find their relationships in graph data.

4.2 Minimum α-summarization

We next investigate the MSUM problem: finding the minimum α-summarization. We first prove Theorem 1, *i.e.,* the decision problem for MSUM is NP-complete. Given Q, a set of answer graphs $\mathcal{G}$ induced by Q, a coverage ratio α, and a size bound B, the decision problem of MSUM is to determine if there exists a α-summary graph G_s with size no more than B. Observe that MSUM is equivalent to the following problem (denoted as MSUM*): find an m-element set $S_m \subseteq S$ from a set of keyword pairs S, such that $|G_s| \leq B$, where (a) $m = \alpha \cdot \frac{|Q||Q-1|}{2}$, (b) $S = \{(k, k') | k, k' \in Q, k \neq k'\}$, and (c) G_s is the minimum 1-summary graph for $\mathcal{G}$ and S_m. It then suffices to show MSUM* is NP-complete.

Complexity. We show that MSUM* is NP-complete as follows. (1) MSUM* is in NP, since there exists a polynomial time algorithm to compute G_s for a keyword pair set S, and determine if $|G_s| \leq B$ (Corollary 5). (2) To show the lower bound, we construct a reduction from the maximum coverage problem, a known NP-complete problem [9]. Given a set X and a set T of its subsets $\{T_1, \ldots, T_n\}$, as well as integers

K and N, the problem is to find a set $T' \subseteq T$ with no more than K subsets, where $|\bigcup T' \cap X| \geq N$. Given an instance of maximum coverage, we construct an instance of MSUM^* as follows. (a) For each element $x_i \in X$, we construct an intermediate node v_i. (b) For each set $T_j \in T$, we introduce a keyword pair (k_{T_j}, k'_{T_j}), and construct an answer graph G_{T_j} which consists of edges (k_{T_j}, v_i) and (v_i, k'_{T_j}), for each v_i corresponding to $x_i \in T_j$. We set S as all such (k_{T_j}, k'_{T_j}) pairs. (c) We set $m = |T|$-K, and $B = |X|$-N. One may verify that there exists at most K subsets that covers at least N elements in X, if and only if there exists a 1-summary graph that covers at least $|S|$-K keyword pairs, with size at most $2 * (|X|$-$N+ m)$. Thus, MSUM^* is NP-hard. Putting (1) and (2) together, MSUM^* is NP-complete.

The APX-hardness can be proved by constructing an approximation ratio-preserving reduction [35] from the weighted maximum coverage problem, a known APX-hard problem, via a similar transformation as discussed above.

The above analysis completes the proof of Theorem 1.

A greedy heuristic algorithm. As shown in Theorem 1, it is unlikely to find a polynomial time algorithm with good approximation ratio for MSUM. Instead, we resort to an efficient heuristic algorithm, mSum.

Given Q and $\mathcal{G}$, mSum (1) dynamically maintains a set of connection graphs $\mathcal{G}_C$, and (2) greedily selects a keyword pair (k, k') and its connection graph G_c, such that the following "merge cost" is minimized:

$$\delta_{r(\mathcal{G}_C, G_c)} = |G_{s(\mathcal{G}_C \cup \{G_c\})}| - |G_{s(\mathcal{G}_C)}|$$

where $G_{s(\mathcal{G}_C \cup \{G_c\})}$ (resp. $G_{s(\mathcal{G}_C)}$) is the 1-summary graph of the answer graph set $\mathcal{G}_C \cup \{G_c\}$ (resp. $(\mathcal{G}_C)$). Intuitively, the strategy always chooses a keyword pair with a connection graph that "minimally" introduces new nodes and edges to the dynamically maintained 1-summary graph.

The algorithm mSum is shown in Fig. 6. It first initializes a summary graph G_s (as empty), as well as an empty answer graph set $\mathcal{G}_C$ to maintain the answer graphs to be selected for summarizing (line 1). For each keyword pair (k, k'), it computes the connection graph $G_{c(k,k')}$ from the union of the answer graphs in $\mathcal{G}$, and puts $G_{c(k,k')}$ to $\mathcal{G}_C$ (line 2-3). This yields a set $\mathcal{G}_C$ which contains in total $O(\frac{|Q|(|Q|-1)}{2})$ connection graphs. It then identifies a subset of connection graphs in $\mathcal{G}$ by greedily choosing a connection graph G_c that minimizes a dynamically updated merge cost $\delta_{r(\mathcal{G}_C, G_c)}$, as remarked earlier (line 5). In particular, we use an efficiently estimated merge cost, instead of the accurate cost via summarizing computation (as will be discussed). Next, it either computes G_s as a 1-summary graph for $G_{c(k,k')}$ if G_s is $\emptyset$, by invoking pSum (line 6), or updates G_s with the newly selected G_c, by invoking a procedure merge (line 7). G_c is then removed from G_S (line 8), and the merge cost of all the rest connection graphs in $\mathcal{G}_C$ are updated according to the selected connection graphs (line 10-11). The process repeats until $m = \lceil \frac{\alpha|Q|(|Q|-1)}{2} \rceil$ pairs of keywords are covered by G_s, *i.e., m* connection graphs are processed (line 9). The updated G_s is returned (line 12).

Procedure. The procedure merge (not shown in Fig. 6) is invoked to update G_s upon new connection graphs. It takes as input a summary graph G_s and a connection graph G_c. It also keeps track of the union of the connection graphs G_s corresponds to. It then updates G_s via the following actions:

Input: A keyword query Q, a set of answer graphs $\mathcal{G}$,
  a coverage ratio α
Output: An α-summary graph G_s.
1. Initialize G_s; Set $\mathcal{G}_C := \emptyset$;
2. **for each** pair (k, k') where k, $k' \in Q$ **do**
3. compute connection graph $G_{c(k,k')}$; $\mathcal{G}_C := \mathcal{G}_C \cup \{G_{c(k,k')}\}$;
4. **while** $G_S \neq \emptyset$ **do**
5. **for each** $G_{c(k,k')} \in \mathcal{G}_C$ with minimum merge cost **do**
6. **if** $G_s = \emptyset$ **then** $G_s := \mathsf{pSum}((k, k'), \mathcal{G})$;
7. **else** $\mathsf{merge}(G_s, G_{c(k,k')})$;
8. $\mathcal{G}_C := \mathcal{G}_C \setminus \{G_{c(k,k')}\}$;
9. **if** m connection graphs are merged **then break** ;
10. **for each** $G_c \in \mathcal{G}_C$ **do**
11. update merge cost of G_c;
12. **return** G_s;

Figure 6: Algorithm mSum

(1) it removes all the nodes in G_c that are dominated by the nodes in itself or the union graph; (2) it identifies equivalent nodes from the union graph and G_c (or have the same identification); (3) it then splits node v_s in G_s if $[v_s]$ contains two nodes that cannot dominate each other, or merge all the nodes in G_s that have dominance relation. G_s is then returned if no more nodes in G_s can be further updated.

Optimization techniques. The computation of the merge cost (line 5) of mSum takes in total $O(|\mathcal{G}|^2)$ time, which requires a merge process between a summary and each connection graph. Instead, we use an estimation of the merge cost that can be efficiently computed as follows.

Given a set of answer graphs $\mathcal{G}$, a neighborhood containment relation R_r captures the containment of the label sets from the neighborhood of two nodes in the union of the graphs in $\mathcal{G}$. Formally, R_r is a binary relation over the nodes in $\mathcal{G}$, such that a pair of nodes $(u, v) \in R_r$ if and only if for u (resp. v) from $G_1 = (V_1, E_1, L_1)$ (resp. $G_2 = (V_2, E_2, L_2)$) in $\mathcal{G}$, (1) $L_1(u) = L_2(v)$, and (2) for each neighbor u' of u, there is a neighbor v' of v, such that $L(u') = L(v')$. Moreover, we denote as $D(R_r)$ the union of the edges attached to the node u, for all $(u, v) \in R_r$. We have the following result.

Lemma 6: *For a set of answer graphs $\mathcal{G}$ and its 1-summary G_s, $|\mathcal{G}| \geq |G_s| \geq |\mathcal{G}|$ - $|R_r(\mathcal{G})|$ - $|D(R_r)|$.* □

To see this, observe the following. (1) $|\mathcal{G}|$ is clearly no less than $|G_s|$. (2) Denote G as the union of the answer graphs in $\mathcal{G}$, we have $|G_s| \geq |\mathcal{G}|$ - $|R_\prec(G)|$ - $|D(R_\prec|)$, where $R_\prec(G)$ is the dominance relation over G, and $D(R_\prec)$ is similarly defined as $D(R_r)$. (3) For any $(u, v) \in R_\prec(G)$, (u, v) is in $R_r(\mathcal{G})$. In other words, $|R_\prec(G)| \leq |R_r(\mathcal{G})|$, and $|D(R_\prec)| \leq |D(R_r)|$. Putting these together, the result follows.

The above result tells us that $|\mathcal{G}|$ - $|R_r(\mathcal{G})|$ - $|D(R_r)|$ is a lower bound for G_s of $\mathcal{G}$. We define the merge cost $\delta_{r(\mathcal{G}_C, G_c)}$ as $|\mathcal{G}|$ - $|R_r(\mathcal{G})|$ - $|D(R_r)|$ - $|G_{s(\mathcal{G}_C)}|$. Using an index structure that keeps track of the neighborhood labels of a node in $\mathcal{G}$, $\delta_{r(\mathcal{G}_C, G_c)}$ can be evaluated in $O(|\mathcal{G}|)$ time.

Analysis. The algorithm mSum correctly outputs an α-summary graph, by preserving the following invariants. (1) During each operation in merge, G_s is correctly maintained as a minimum summary graph for a selected keyword pair set. (2) Each time a new connection graph is selected, G_s is updated to a summary graph that covers one more pair of keywords, until m pairs of keywords are covered by G_s.

For complexity, (1) it takes in total $O(m \cdot |\mathcal{G}|)$ time to

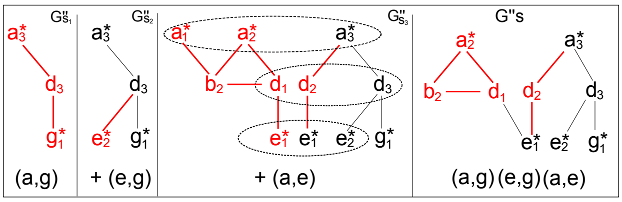


Figure 7: Computing minimum α-summary graph

induce the connection graphs (line 1-3); (2) the **while** loop is conducted m times (line 4); In each loop, it takes $O((|\mathcal{G}|^2)$ time to select a G_c with minimum merge cost, and to update G_s (line 7). Thus, the total time complexity is $O(m|\mathcal{G}|^2)$. Note that in practice m is typically small.

Example 7: Recall the query $Q' = \{a, c, e, f, g\}$ and the answer graph set $\mathcal{G} = \{G'_1, G'_2\}$ in Fig. 3. There are in total 10 keyword pairs. Suppose $\alpha = 0.3$. mSum finds a minimum 0.3-summary graph for $\mathcal{G}$ and Q' as follows. It first constructs the connection graphs for each keyword pair. It starts with a smallest connection graph induced by *e.g.,* (a, g), and computes a 1-summary graph as G''_{s_1} shown in Fig. 7. It then identifies that the connection graph G_c induced by (e, g) introduces least merge cost. Thus, G_{s_1} is updated to G_{s_2} by merging G_c, with one more node e_2 and edge (d_3, e_2) inserted. It then updates the merge cost, and merges the connection graph of (a, e) to G''_{s_2} to form G''_{s_3}, by invoking merge. merge identifies that in G''_{s_3} (1) a_1 is dominated by a_2, (2) the two e^*_1 nodes refer to the same node. Thus, it removes a_1 and merges e^*_1, updating G''_{s_3} to G''_s, and returns G''_s as a minimum 0.3-summary graph. □

Remarks. The algorithm mSum can be adapted to (approximately) find a solution to the following problem: find a summary graph within a size bound B which maximizes the coverage ratio. To this end, mSum is invoked in $O(\log |Q|)$ times to find the summary graph, by checking the maximum coverage ratio via a binary search. At each iteration, it computes a minimum α-summary graph G_s for a fixed α. If $|G_s|$ is larger than B, it changes α to $\frac{\alpha}{2}$; otherwise, it changes α to $2 \cdot \alpha$. The process repeats until a proper α is identified.

5. Computing K Summarizations

In this section we study how to construct K summary graphs for answer graphs, *i.e.,* the KSUM problem.

Complexity. We start by proving Theorem 3 (Section 2). Given Q, $\mathcal{G}$, an integer K and a size bound B, the decision problem of KSUM asks if there exists a K-partition of $\mathcal{G}$, such that the sum of the 1-summary graph for each partition is no more than B. (1) The problem is in NP, as there exists a polynomial time algorithm to check if a given partition satisfies the constraints. (2) To show the lower bound, we construct a reduction from the graph decomposition problem shown to be NP-hard [28]. Given a complete graph G where each edge is assigned with an integer weight, the problem is to identify K' partitions of edges, such that the sum of the maximum edge weight in each partition is no greater than a bound W. We construct a transformation from an instance of the graph decomposition problem to KSUM, in polynomial time. (a) We identify the maximum edge weight w_m in G, and construct w_m intermediate nodes $V_I = \{v_1, \ldots, v_{w_m}\}$, where each intermediate node has a distinct label. (b) For each edge in G with weight w_i, we construct an answer graph with two fixed keyword nodes k_1, k_2 and edges (k_1, v_j) and (v_j, k_2), where $v_j \in V_I$, and $j \in [1, w_i]$. (c) We set $K = K'$, and $B = W$. One may verify that if a K'-partition of edges in G has a total weight within W, then there exists a K-partition of $\mathcal{G}$ with total summary size within $3W+2K$, and vice versa. Thus, KSUM is NP-hard. This verifies that KSUM is NP-complete.

The APX-hardness of the K summarization problem can be shown similarly, by conducting an approximation preserving reduction from the graph decomposition problem, which is shown to be APX-hard [28]. The above analysis completes the proof of Theorem 3.

We next present a heuristic algorithm for the KSUM problem. To find K summary graphs, a reasonable partition G_P of $\mathcal{G}$ is required. To this end, we introduce a similarity measure between two answer graphs.

Graph distance metric. Given two answer graphs G_1 and G_2, we introduce a similarity function $F(G_1, G_2)$ as follows.

$$F(G_1, G_2) = \frac{|R_r(G_{1,2})| + |D(R_r)|}{|G_1| + |G_2|}$$

where $G_{1,2}$ is the union of G_1 and G_2, and $R_r(G_{1,2})$ and $D(R_r)$ are as defined in Section 4. Intuitively, the similarity function F captures the similarity of two answer graphs, by measuring "how well" a summary graph may compress the union of the two graphs [11]. Thus a distance function $\delta(G_1, G_2)$ of G_1 and G_2 can be defined as

$$\delta(G_1, G_2) = 1 - F(G_1, G_2)$$

Based on the distance measure, we propose an algorithm, kSum, which partitions $\mathcal{G}$ into K clusters G_P, such that the total set distance $F(G_{p_i})$ in each cluster G_{p_i} is minimized. This intuitively leads to K small summary graphs.

Algorithm. The algorithm kSum works similarly as a K-center clustering process [4]. It has the following three steps.

(1) *Initialization.* kSum first initializes (a) a set G_P to maintain the partition of $\mathcal{G}$, (b) an answer graph set $\mathcal{G}_K$ to maintain the K "centers", *i.e.,* the selected graphs to form the cluster, from $\mathcal{G}$, and (c) a summarization set $\mathcal{G}_S$ to keep record of K 1-summary graphs, each corresponds to a cluster G_{p_i} in G_P; in addition, the total difference θ is initialized as a large number, *e.g.,* $K\ |\mathcal{G}|^2$. It initializes $\mathcal{G}_K$ with randomly selected K answer graphs from $\mathcal{G}$.

(2) *Clustering.* It then iteratively refines the partition G_P as follows. (1) For each answer graph $G \in \mathcal{G}$, it selects the "center" graph G_{c_j} in $\mathcal{G}_K$, which minimizes $\delta(G, G_{c_j})$, *i.e.,* is the closest one to G_{c_j}, and extends the cluster G_{p_j} with G. (2) The updated clusters G_P forms a partition of $\mathcal{G}$. For each cluster $G_{p_i} \in G_P$, a new "center" graph G'_{c_i} is selected, which minimizes the sum of the distance from G'_{c_i} to all the rest graphs in G_{p_i}. The newly identified K graphs replace the original graphs in $\mathcal{G}_K$. (3) The overall distance $\theta = \sum_i \sum_{G \in G_{p_i}} \delta(G, G_{c_i})$ is recomputed for G_P. kSum repeats the above process until θ converges.

(3) *Summarizing.* If G_P can no longer be improved in terms of θ, kSum computes the 1-summary graph by invoking the algorithm pSum for each cluster $G_{p_i} \in G_P$, and returns K 1-summary graphs maintained in $\mathcal{G}_S$.

Example 8: Recall the answer graphs G'_1, G'_2 and G'_3 in Fig. 3. Let K= 2, The algorithm pSum identifies a 2-

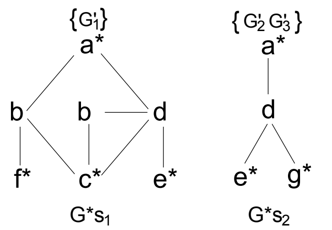


Figure 8: summary graphs for a 2-partition

partition for $\mathcal{G}$ ={G'_1, G'_2, G'_3} as follows. It first selects two graphs as "center" graphs, *e.g.,* G'_1 and G'_3. It then computes the distance between the graphs. One may verify that $\delta(G'_1, G'_2) > \delta(G'_2, G'_3)$. Thus, G'_2 and G'_3 are much "closer," and are grouped together to form a cluster. This produces a 2-partition of $\mathcal{G}$ as $\{\{G'_1\}, \{G'_2, G'_3\}\}$. The 1-summary graphs are then computed for each cluster. pSum finally returns G'_{s_1} and G'_{s_2} as the minimized 2 1-summary graphs, with total size 22 (shown in Fig. 8). □

Analysis. The algorithm kSum correctly computes K 1-summary graphs for a K-partition of $\mathcal{G}$. It heuristically identifies K clusters with minimized total distance of each answer graph in the cluster to its "center" graph. Intuitively, the closer the graphs are to a center answer graph, the more nodes are likely to be merged in a summarization. kSum can also be used to compute K α-summary graphs.

For complexity, (1) it takes kSum $O(\mathcal{G})$ time for initialization; (2) the clustering phase takes in total $O(I \cdot K \cdot |G_m|^2)$ time, where I is the number of iterations, and G_m is the largest answer graph in $\mathcal{G}$; and (3) the total time of summarization is in $O(|Q|^2||\mathcal{G}| + |\mathcal{G}|^2)$. In our experiments, we found that I is typically small, *e.g.,* it is no more than 3 over both real-life and synthetic datasets.

Remark. While determining the optimal value of the cluster number K is an open issue, in practice, it may be determined by empirical rules [24] or information theory.

6. Experimental Evaluation

In this section, we experimentally verify the effectiveness and efficiency of the proposed algorithms.

6.1 Experimental Settings

Datasets. We use the following three real-life datasets in our tests. (1) *DBLP* (`http://dblp.uni-trier.de/xml/`), a bibliographic dataset with in total 2.47 million nodes and edges, where (a) each node has a type from in total 24 types (*e.g.,*'paper', 'book', 'author'), and a set of attribute values (*e.g.,*'network', 'database', etc), and (b) each edge denotes *e.g.,* authorship or citation. (2) *DBpedia* (`http://dbpedia.org`), a knowledge graph which includes 1.2 million nodes and 16 million edges. Each node represents an entity with a type (*e.g.,*'animal', 'architectures', 'famous places') from in total 122 types, with a set of attributes (*e.g.,*'jaguar', 'Ford'). (3) *YAGO* (`http://www.mpi-inf.mpg.de/yago`) is also a knowledge graph. Compared with *DBLP* and *DBpedia*, it is "sparser" (1.6 million nodes, 4.48 million edges) and much richer with diverse schemas (2595 types).

Keyword queries. We design keyword queries as follows. (1) For *DBLP*, we select 5 common queries as shown in Table 1. The keyword queries are for searching information related with various topics or authors. For example, Q_1 is to search the mining techniques for temporal graphs.

Query	Keywords	card($\mathcal{G}$)	$\overline{\lvert V\rvert}, \overline{\lvert E\rvert}$
Q_1	mining temporal graphs	355	(5,6)
Q_2	david parallel computing ACM	1222	(5,4)
Q_3	distributed graphs meta-data integration	563	(5,5)
Q_4	improving query uncertain database conference	1617	(9,14)
Q_5	keyword search algorithm evaluation XML conference	7635	(7,8)

Table 1: Queries for DBLP

Query	Keywords template	$\lvert Q_T\rvert$	$\overline{\text{card}(\mathcal{G})}$	$\overline{\lvert V\rvert}, \overline{\lvert E\rvert}$
Q_{T_1}	*Jaguar* place	136	75	(5,7)
Q_{T_2}	*united_states* politician award	235	177	(6,7)
Q_{T_3}	album music genre *american_music_awards*	168	550	(11,25)
Q_{T_4}	fish bird mammal protected_area *north_american*	217	1351	(12,24)
Q_{T_5}	player club manager league city country	52	1231	(17,28)
Q_{T_6}	actor film award company *hollywood*	214	1777	(12,27)

Table 2: Queries and the answer graphs for *DBpedia*. The templates are also applied for *YAGO*.

(2) For *DBpedia* and *YAGO*, we design 6 query templates Q_{T_1} to Q_{T_6}, each consists of *type* keywords and *value* keywords. The *type* keywords are taken from the type information in *DBpedia* (*resp.* *YAGO*), *e.g.,* country in Q_{T_5}, and the value keywords are from the attribute values of a node, *e.g.,United States* in Q_{T_2}. Each query template Q_{T_i} is then extended to a set of keyword queries (simply denoted as Q_{T_i}), by keeping all the value keywords, and by replacing some type keywords (*e.g.,*place) with a corresponding value (*e.g.,America*). Table 2 shows the query templates Q_T and the total number of its corresponding queries $|Q_T|$. For example, for Q_{T_1}, 136 keyword queries are generated for *DBpedia*. One such query is {'Jaguar', 'America'}.

Answer graph generator. We generate a set of answer graphs $\mathcal{G}$ for each keyword query, leveraging [17,20]. Specifically, (1) the keyword search algorithm in [17] is used to produce a set of trees connecting all the keywords, and (2) the trees are expanded to a graph containing all the keywords, with a bounded diameter 5, using the techniques in [20]. Table 1 and Table 2 report the average number of the generated answer graphs card($\mathcal{G}$) and their average size, for *DBLP* and *DBpedia*, respectively. For example, for Q_{T_3}, an answer graph has 11 nodes and 25 edges (denoted as (11, 25)) on average. For *YAGO*, card($\mathcal{G}$) ranges from 200 to 2000, with answer graph size from (5, 7) to (10, 20). On the other hand, various methods exist *e.g.,* top-k graph selection [34], to reduce possibly large answer graphs.

Implementation. We implemented the following algorithms in Java: (1) pSum, mSum and kSum for answer graph summarization; (2) SNAP [33] to compare with pSum, which generates a summarized graph for a single graph, by grouping nodes such that the pairwise group connectivity strength is maximized; (3) kSum $_{td}$, a revised kSum using a top-down strategy: (a) it randomly selects two answer graphs G_1 and G_2, and constructs 2 clusters by grouping the graphs that are close to G_1 (resp. G_2) together; (b) it then iteratively splits the cluster with larger total inter-cluster distance to two clusters by performing (a), until K clusters

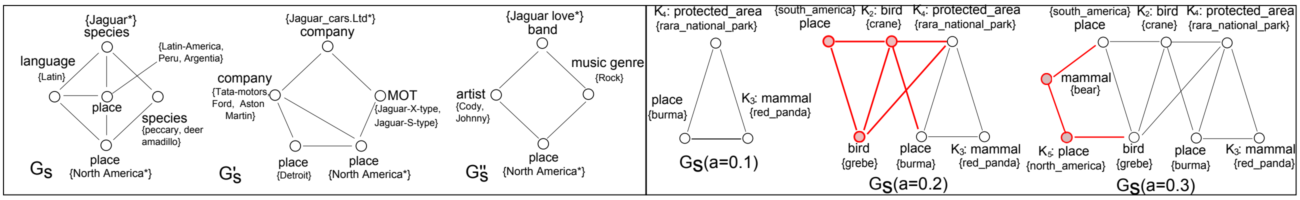


Figure 9: Case study: summarizing real-life answer graphs

are constructed, and the K summary graphs are computed.

All experiments were run on a machine with an Intel Core2 Duo 3.0GHz CPU and 4GB RAM, using Linux. Each experiment was run 5 times and the average is reported here.

6.2 Case Study: K and α-summarization

We first provide a case study using *DBpedia*. (1) Fixing $K = 10$ and $Q = \{$Jaguar,America$\}$, we select 3 summary graphs generated by kSum, as shown in Fig. 9 (left). The summary graph suggests three types of connection patterns between Jaguar and America, where Jaguar is a type of animal, car, and a band, respectively. Each intermediate node (*e.g.,*company) contains the entities connecting the keyword nodes, (*e.g.,*Ford). Observe that each summary graph can also be treated as a suggested graph query for Q. (2) Fig. 9 (right) depicts three α-summary graphs for a keyword query Q from the query template Q_{T_4}. $G_{s(\alpha=0.1)}$ covers a single pair of keyword "protected area" and "mammal". With the increase of α, new keywords are added to form new α-summary graphs. When $\alpha = 0.3$, we found that $G_{s(\alpha=0.3)}$ already covers 67% of the path labels for all keyword pairs.

6.3 Performance on Real-life Datasets

Exp-1: Effectiveness of pSum. We first evaluate the effectiveness of pSum. To compare the effectiveness, we define the *compression ratio* cr of a summarization algorithm as $\frac{|G_s|}{|\mathcal{G}|}$, where $|G_s|$ and $|\mathcal{G}|$ are the size of the summary graph and answer graphs. For pSum, G_s refers to the 1-summary graph for $\mathcal{G}$ and Q. Since SNAP is not designed to summarize a set of graphs, we first union all the answer graphs in $\mathcal{G}$ to produce a single graph, and then use SNAP to produce a summarized graph G_s. To guarantee that SNAP generates a summarized graph that preserves path information between keywords, we carefully selected parameters such as participation ratio [33]. We verify the effectiveness of pSum, by comparing cr of pSum with that of SNAP.

Fixing the query set as in Table 1, we compared the compression ratio of pSum and SNAP over *DBLP*. Fig. 10(a) shows the results, which tell us the following. (a) pSum generates summary graphs much smaller than the original answer graph set. For example, , cr of pSum is only 7% for Q_2. On average, cr of pSum is 23%. (b) pSum generates much smaller summary graphs than SNAP. For example, for Q_2 over *DBLP*, the G_s generated by pSum reduces the size of its counterparts from SNAP by 67%. On average, pSum outperforms SNAP by 50% over all the datasets. It is observed that while SNAP may guarantee path preserving via carefully set parameters, it cannot identify dominated nodes, thus produces larger G_s.

Using Q_{T_i} ($i \in [1, 6]$), we compared cr of pSum and SNAP over *DBpedia* and *YAGO*. Fig. 10(b) and Fig. 10(c) illustrate their performance, respectively. The results show that (1) pSum produces summary graphs on average 50% (resp. 80%) smaller of the answer graphs, and are on average 62% (resp. 72%) smaller than their counterparts generated by SNAP over *DBpedia* (resp. *YAGO*). (2) For both algorithms, cr is highest over *DBpedia*. The reason is that *DBpedia* has more node labels than *DBLP*, and the answer graphs constructed from *DBpedia* are much denser than *YAGO* (Table 2). Hence, fewer nodes can be removed or grouped in the answer graphs for *DBpedia*, leading to larger summary graphs. To further increase the compression ratio, one can resort to α-summarization with information loss.

Exp-2: Effectiveness of mSum. In this set of experiments, we verify the effectiveness of mSum. We compare the average size of α-summary graphs by mSum (denoted as $|G_s^\alpha|$) with that of 1-summary graphs by pSum (denoted as $|G_s|$). Using real-life datasets, we evaluated $\frac{|G_s^\alpha|}{|G_s|}$ by varying α.

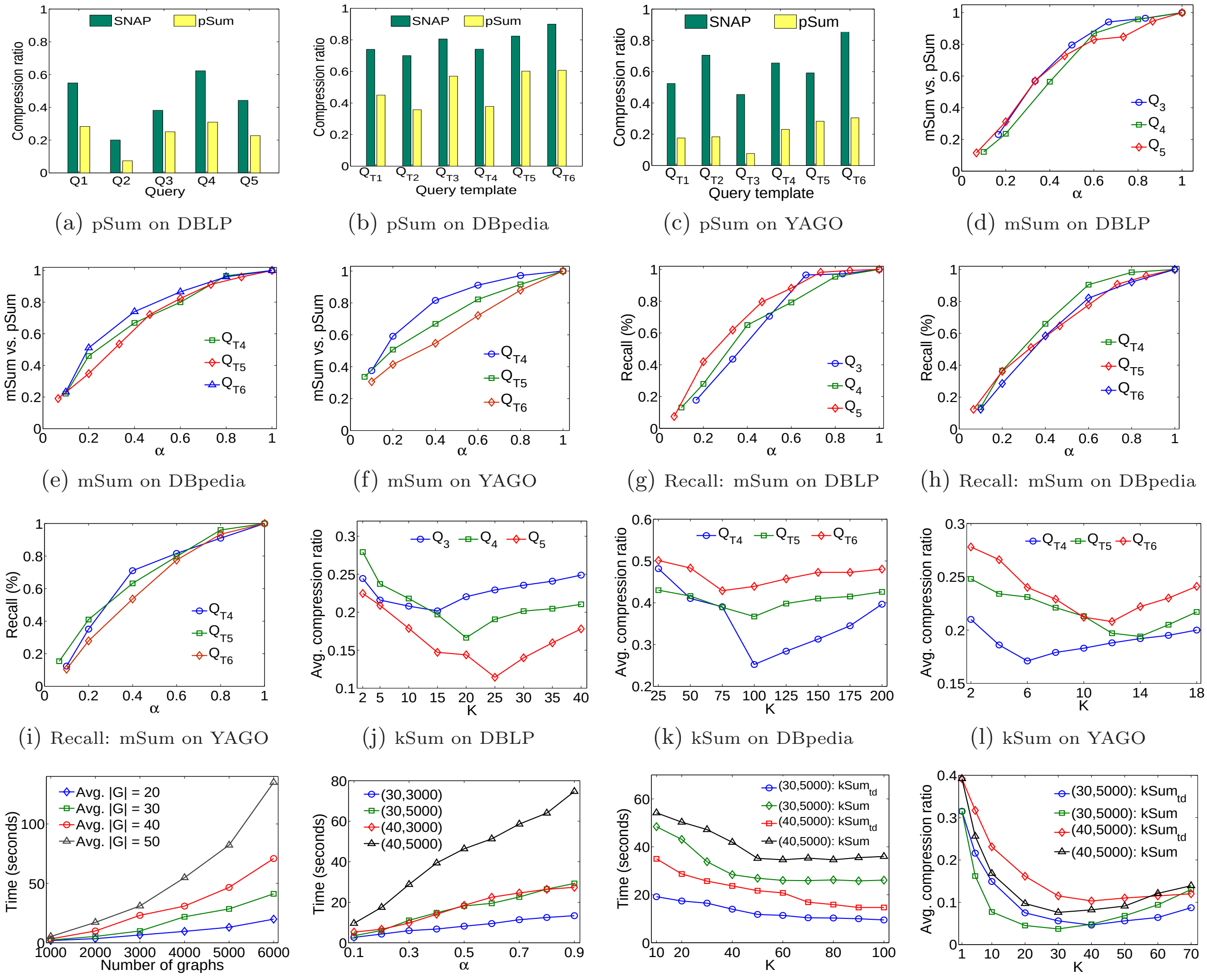
Fixing the keyword query set as $\{Q_3, Q_4, Q_5\}$, we show the results over *DBLP* in Fig. 10(d). (1) $|G_s^\alpha|$ increases for larger α. Indeed, the smaller coverage ratio a summary graph has, the fewer keyword pair nodes and the paths are summarized, which usually reduce $|G_s^\alpha|$ and make it more compact. (2) The growth of $|G_s^\alpha|$ is slower for larger α. This is because new keyword pairs are more likely to have already been covered with the increment of α. Fig. 10(e) and Fig. 10(f) illustrate the results over *DBpedia* and *YAGO* using the query templates $\{Q_{T_4}, Q_{T_5}, Q_{T_6}\}$ (Table 2). The results are consistent with Fig. 10(d).

We also evaluated the *recall* merit of mSum as follows. Given a keyword query Q, we denote the recall of mSum as $\frac{|P'|}{|P|}$, where P (resp. P') is the set of path labels between the keyword nodes of k and k' in $\mathcal{G}$ (resp. α-summary graph by mSum), for all $(k, k') \in Q$. Figures 10(g), 10(h) and 10(i) illustrate the results over the three real-life datasets. The recall increases with larger α, since more path labels are preserved in summary graphs, as expected. Moreover, we found that mSum covers on average more than 85% path labels for all keyword pairs over *DBLP*, even when $\alpha = 0.6$.

In addition, we compared the performance of mSum with an algorithm that identifies the minimum summary graph by exhaust searching. Using *DBpedia* and its query templates, and varying α from 0.1 to 1 (we used pSum when $\alpha = 1.0$), we found that mSum always identifies summary graphs with size no larger than 1.07 times of the minimum size.

Exp-3: Effectiveness of kSum. We next evaluate the effectiveness of kSum, by evaluating the *average compression ratio*, $\mathsf{cr}_K = \frac{1}{K}\sum_{i=1}^{K}\frac{|G_{s_i}|}{|G_{p_i}|}$ for each cluster G_{p_i} and its corresponding 1-summary graph G_{s_i}.

Fixing the query set $\{Q_3, Q_4, Q_5\}$ and varying K, we tested cr_K over *DBLP*. Fig. 10(j) tells us the following. (1) For all queries, cr_K first decreases and then increases with the increase of K. This is because a too small K induces large clusters that contain many intermediate nodes that are not dominated by any node, while a too large K leads to many small clusters that "split" similar intermediate nodes.



(a) pSum on DBLP (b) pSum on DBpedia (c) pSum on YAGO (d) mSum on DBLP

(e) mSum on DBpedia (f) mSum on YAGO (g) Recall: mSum on DBLP (h) Recall: mSum on DBpedia

(i) Recall: mSum on YAGO (j) kSum on DBLP (k) kSum on DBpedia (l) kSum on YAGO

(m) Runtime: pSum (n) Runtime: mSum (o) Runtime: kSum$_{td}$ vs. kSum (p) kSum$_{td}$ vs. kSum

Figure 10: Performance evaluation

Both cases increase cr_K. (2) cr_K is always no more than 0.3, and is also smaller than its counterpart of pSum in Fig. 10(a). By using kSum, each cluster G_{p_i} contains a set of similar answer graphs that can be better summarized.

The results in Fig. 10(k) and 10(l) are consistent with their counterparts in Fig. 10(a). In addition, cr_K is in general higher in *DBpedia* than its counterparts over *DBLP* and *YAGO*. This is also consistent with the observation in Exp-1.

Summary: effectiveness. We found the following. (1) The summarization effectively constructs summary graphs: the compression ratio of pSum is on average 24%, and the average compression ratio is 20% for kSum. Moreover, mSum can provide more compact summary results with some information loss. (2) Graphs with simpler schema (less types) and topology can be better summarized. In addition, our algorithms take up to several seconds over all real-life datasets.

6.4 Performance on Synthetic Dataset

We next evaluate the efficiency of pSum, mSum and kSum using synthetic graphs. (1) We randomly generate synthetic keyword queries with on average 5 keywords, where each keyword is taken from a set Σ of 40 random labels. (2) We generate a set of answer graphs $\mathcal{G}$ with size $\mathsf{card}(\mathcal{G})$ and average graph size Avg. $|G|$ as follows. We first select 5 labels as keywords from Σ, and randomly generate 50 path templates, where a path template connects two keywords with the selected labels. We then construct an answer graph by (a) constructing a path from a path template by replacing the labels with nodes, and (b) merge a set of paths, until the answer graph has size Avg. $|G|$.

Exp-4: Summarization efficiency. Varying $\mathsf{card}(\mathcal{G})$ from 1000 to 6000 and Avg. $|G|$ from 20 to 50, we test the efficiency of pSum. Fig. 10(m) shows that (1) it takes more time for pSum to find summary graphs over larger answer graphs, and over larger $\mathsf{card}(\mathcal{G})$, and (2) pSum scales well with the number and the size of answer graphs. Note that pSum seldom perform its worst case complexity.

Varying α from 0.1 to 0.9, we tested the efficiency of mSum where $\mathsf{card}(\mathcal{G})$ (resp. Avg. $|G|$) varies from 3000 to 5000 (resp. 30 to 40). Fig. 10(n) shows that mSum scales well with α, and takes more time when $\mathsf{card}(\mathcal{G})$ and Avg. $|G|$ increase.

Fixing $\mathsf{card}(\mathcal{G}) = 5000$, we evaluated the efficiency of kSum and its baseline version kSum $_{td}$, by varying K (resp. Avg. $|G|$) from 10 to 100 (resp. 30 to 40). Figure 10(o) tells us that both algorithms take less time with the increase of K, since they take less total time over smaller clusters induced by larger K. Both algorithms take more time for larger answer graphs. In general, kSum_{td} takes less time than kSum, due to a faster top-down partitioning strategy.

Fixing $\mathsf{card}(\mathcal{G}) = 5000$, we compared cr_K, *i.e.,* average compression ratio of kSum_{td} and kSum, by varying K (resp. Avg. $|G|$) from 1 to 70 (resp. 30 to 40). As shown in Fig. 10(p), cr_K first decreases, and then increases with the increasing of K, the same as Fig. 10(j) and Fig. 10(k). Although kSum_{td} is faster, kSum outperforms kSum_{td} with lower cr_K, due to better iterative clustering strategy.

Summary: efficiency. We found that the summarization algorithms scale well with the size of answer graphs, and efficiently compute summary graphs under coverage and conciseness constraints. Also, our algorithms take more time over random graphs than over real datasets, due to (1) larger answer graph number and size, and (2) more diversity in connection patterns. Techniques such as incremental computation for simulation [6] may apply for dynamic and interactive scenarios, over large number of answer graphs.

7. Conclusion

In this paper we have developed summarization techniques for keyword search in graph data. By providing a succinct summary of answer graphs induced by keyword queries, these techniques can improve query interpretation and result understanding. We have proposed a new concept of summary graphs and their quality metrics. Three summarization problems were introduced to find the best summarizations with minimum size. We established the complexity of these problems, which range from PTIME to NP-complete. We proposed exact and heuristic algorithms to find the best summarizations. As experimentally verified, the proposed summarization methods effectively compute small summary graphs for capturing keyword relationships in answer graphs.

For future work, we will compare the summarization results for different keyword search strategies. Our work can also be extended to enhance keyword search with summary structures so that the access to graph data becomes easier.

8. References

[1] C. C. Aggarwal and H. Wang. A survey of clustering algorithms for graph data. In *Managing and Mining Graph Data*, pages 275–301. 2010.

[2] D. Bustan and O. Grumberg. Simulation-based minimization. *TOCL*, 4(2):181–206, 2003.

[3] S. Chakrabarti, S. Sarawagi, and S. Sudarshan. Enhancing search with structure. *IEEE Data Eng. Bull.*, 33(1):3–24, 2010.

[4] M. Charikar, S. Guha, É. Tardos, and D. Shmoys. A constant-factor approximation algorithm for the k-median problem. In *STOC*, pages 1–10, 1999.

[5] Y. Chen, W. Wang, Z. Liu, and X. Lin. Keyword search on structured and semi-structured data. In *SIGMOD*, 2009.

[6] W. Fan, J. Li, J. Luo, Z. Tan, X. Wang, and Y. Wu. Incremental graph pattern matching. In *SIGMOD*, 2011.

[7] L. Fang, A. D. Sarma, C. Yu, and P. Bohannon. Rex: Explaining relationships between entity pairs. *PVLDB*, 5(3):241–252, 2011.

[8] H. Fu and K. Anyanwu. Effectively interpreting keyword queries on rdf databases with a rear view. In *ISWC*, 2011.

[9] M. Garey and D. Johnson. *Computers and Intractability: A Guide to the Theory of NP-Completeness.* 1979.

[10] R. Gentilini, C. Piazza, and A. Policriti. From bisimulation to simulation: Coarsest partition problems. *J. Automated Reasoning*, 2003.

[11] J. Goldstein, V. Mittal, J. Carbonell, and M. Kantrowitz. Multi-document summarization by sentence extraction. In *NAACL-ANLPWorkshop on Automatic summarization*, pages 40–48, 2000.

[12] L. Guo, F. Shao, C. Botev, and J. Shanmugasundaram. Xrank: ranked keyword search over xml documents. In *SIGMOD*, 2003.

[13] H. He, H. Wang, J. Yang, and P. S. Yu. Blinks: ranked keyword searches on graphs. In *SIGMOD*, pages 305–316, 2007.

[14] M. R. Henzinger, T. A. Henzinger, and P. W. Kopke. Computing simulations on finite and infinite graphs. In *FOCS*, 1995.

[15] Y. Huang, Z. Liu, and Y. Chen. Query biased snippet generation in xml search. In *SIGMOD*, pages 315–326, 2008.

[16] P. K., S. P. Kumar, and D. Damien. Ranked answer graph construction for keyword queries on rdf graphs without distance neighbourhood restriction. In *WWW*, 2011.

[17] V. Kacholia, S. Pandit, S. Chakrabarti, S. Sudarshan, R. Desai, and H. Karambelkar. Bidirectional expansion for keyword search on graph databases. In *VLDB*, 2005.

[18] R. Kaushik, P. Shenoy, P. Bohannon, and E. Gudes. Exploiting local similarity for indexing paths in graph-structured data. In *ICDE*, pages 129–140, 2002.

[19] G. Koutrika, Z. M. Zadeh, and H. Garcia-Molina. Data clouds: summarizing keyword search results over structured data. In *EDBT*, pages 391–402, 2009.

[20] G. Li, B. Ooi, J. Feng, J. Wang, and L. Zhou. Ease: an effective 3-in-1 keyword search method for unstructured, semi-structured and structured data. In *SIGMOD*, 2008.

[21] Z. Liu and Y. Chen. Query results ready, now what? *IEEE Data Eng. Bull.*, 33(1):46–53, 2010.

[22] Z. Liu and Y. Chen. Return specification inference and result clustering for keyword search on xml. *TODS*, 35(2):10, 2010.

[23] Z. Liu, S. Natarajan, and Y. Chen. Query expansion based on clustered results. *PVLDB*, 4(6):350–361, 2011.

[24] K. V. Mardia, J. T. Kent, and J. M. Bibby. Multivariate analysis. 1980.

[25] T. Milo and D. Suciu. Index structures for path expressions. In *ICDT*, 1999.

[26] S. Navlakha, R. Rastogi, and N. Shrivastava. Graph summarization with bounded error. In *SIGMOD*, 2008.

[27] D. Petkova, W. B. Croft, and Y. Diao. Refining keyword queries for xml retrieval by combining content and structure. In *ECIR*, pages 662–669, 2009.

[28] J. Plesník. Complexity of decomposing graphs into factors with given diameters or radii. *Mathematica Slovaca*, 32(4):379–388, 1982.

[29] N. Sarkas, N. Bansal, G. Das, and N. Koudas. Measure-driven keyword-query expansion. *PVLDB*, 2(1):121–132, 2009.

[30] S. Shekarpour, S. Auer, A.-C. N. Ngomo, D. Gerber, S. Hellmann, and C. Stadler. Keyword-driven sparql query generation leveraging background knowledge. In *Web Intelligence*, pages 203–210, 2011.

[31] M. Sydow, M. Pikula, R. Schenkel, and A. Siemion. Entity summarisation with limited edge budget on knowledge graphs. In *IMCSIT*, pages 513–516, 2010.

[32] S. Tata and G. M. Lohman. Sqak: doing more with keywords. In *SIGMOD*, 2008.

[33] Y. Tian, R. Hankins, and J. Patel. Efficient aggregation for graph summarization. In *SIGMOD*, 2008.

[34] T. Tran, H. Wang, S. Rudolph, and P. Cimiano. Top-k exploration of query candidates for efficient keyword search on graph-shaped (rdf) data. In *ICDE*, 2009.

[35] V. V. Vazirani. *Approximation Algorithms.* Springer, 2003.

[36] H. Wang and C. Aggarwal. A survey of algorithms for keyword search on graph data. *Managing and Mining Graph Data*, pages 249–273, 2010.

[37] N. Zhang, Y. Tian, and J. M. Patel. Discovery-driven graph summarization. In *ICDE*, 2010.