CMPSC 260, Spring 2004 – Homework Assignment 1
Due April 12th

1. Consider the following program that computes the x-th power of the number stored in y:

$$[\mathtt{z} := 1]^1; \mathtt{while}[\mathtt{x} > 0]^2 \mathtt{do}([\mathtt{z} := \mathtt{z} * \mathtt{y}]^3; [\mathtt{x} := \mathtt{x} - 1]^4)$$

Formulate a system of equations F defining the Reaching Definitions for this program (as in Section 1.3.1). Compute the least solution to these equations 1) using the sequence $(F^n(\vec{\emptyset}))_n$ discussed in Section 1.3.1 and 2) using the Chaotic Iteration discussed in Section 1.7. Show each step of the iterations and the result.

2. Consider the program given in Problem 1. Formulate a system of equations G defining the Collecting Semantics for this program (as in Section 1.5). Can you compute the collecting semantics for this program using Chaotic Iteration? Explain.

Show the sets $CS_{entry}(2)$ and $CS_{exit}(3)$. Compute $\alpha(CS_{exit}(3))$ and $\gamma(RD_{exit}(3))$ based on the definitions of abstraction function α and concretisation function γ given in Section 1.5.

Show that the equation you gave in Problem 1 for $RD_{exit}(2)$ can be derived from $\vec{\alpha} \circ G \circ \vec{\gamma}$ using induced analysis technique.

3. Given the following program

$$[\mathtt{x} := 1]^1; [\mathtt{y} := 2]^2; \mathtt{if}[\mathtt{x} > 0]^3 \mathtt{then}[\mathtt{y} := 2 + \mathtt{x}]^4 \mathtt{else}[\mathtt{x} := 1]^5; [\mathtt{z} := \mathtt{x} + \mathtt{y}]^6$$

Write the least solution for Reaching Definitions analysis for this program. Using the Constant Folding transformation given in Section 1.8 transform the above program to a more efficient program.

4. Extend the Constant Folding transformations given in Section 1.8 by adding transformation rules for boolean expressions and transformations such as

$$RD \vdash ([\mathtt{skip}]^\ell; S) \rhd S$$

$$RD \vdash (\mathtt{if}\ [\mathtt{true}]^\ell\ \mathtt{then}\ S_1\ \mathtt{else}\ S_2) \rhd S_1$$

Discuss if and how the transformation algorithm discussed in Section 1.8 needs to be modified to handle these transformations. Transform the program given in Problem 4 using these new transformations.

5. Given a complete lattice $(L, \sqsubseteq, \sqcup, \sqcap, \bot, \top)$ that satisfies the ascending chain condition and a monotone function $f : L \to L$, prove that the sequence $(f^n(\bot))_n$ eventually stabilizes, and when it stabilizes the value it converges to is $lfp(f)$.