CMPSC 260, Spring 2004 – Homework Assignment 4
Due June 2nd, BEFORE THE CLASS

1. Given two integers $c_1, c_2 \in Z$, $c_1 \leq c_2$, the $\mathbf{FiniteInterval}_{c_1,c_2}$ lattice is defined such that $\mathbf{FiniteInterval}_{c_1,c_2} \subseteq \mathbf{Interval}$ and for all $int \in \mathbf{FiniteInterval}_{c_1,c_2}$, $\inf(int) = -\infty$ or $\inf(int) \geq c_1$, and, $\sup(int) = +\infty$ or $\sup(int) \leq c_2$.

(a) Define a monotone framework for the Interval Analysis (by defining ι, E, F, $\mathcal{F}$ and the transfer functions f_ℓ for the basic statements) based on the lattice $\mathbf{Var}_\star \to \mathbf{FiniteInterval}_{c_1,c_2}$.

(b) Assume that $c_1 = -5$ and $c_2 = +5$. Show the result of the Interval Analysis for the program:

$$[\texttt{x} := 1]^1;\ (\texttt{while}\ [\texttt{x} < 10]^2\ \texttt{do}\ [\texttt{x} := \texttt{x} + 1]^3);\ [\texttt{skip}]^4$$

(c) Give the complexity of computing the MFP solution to the Interval Analysis you defined above.

2. Consider the complete lattice **Interval** of Figure 4.2 (page 219 in the textbook) and the complete lattice **Sign**' of Figure 4.10 (page 274 in the textbook). Define a Galois connection between these two lattices by defining the abstraction function $\alpha_{IS'}$ and the concretisation function $\gamma_{IS'}$. Prove that (**Interval**, $\alpha_{IS'}$, $\gamma_{IS'}$, **Sign**') is a Galois connection.

3. Let ∇_K be the widening operator defined in Example 4.15 (page 226 of the textbook) on the lattice **Interval** and define $\nabla_{int'}$ as

$$int_1 \nabla_{int'} int_2 = \begin{cases} int_1 \sqcup int_2 & \text{if } int_1 \sqsubseteq int' \vee int_2 \sqsubseteq int_1 \\ int_1 \nabla_K int_2 & \text{otherwise} \end{cases}$$

where int' is an interval satisfying $\mathbf{inf}(int') > -\infty$ and $\mathbf{sup}(int') < \infty$. Show that

$$\forall int_1, int_2 \ :\ int_1 \nabla_{int'} int_2 \ \sqsubseteq\ int_1 \nabla_K int_2$$

and that the inequality may be strict. Show that $\nabla_{int'}$ is an upper bound operator. Determine whether or not $\nabla_{int'}$ is a widening operator and prove it.