

CS 267 – Spring 2023 – FINAL

Due June 13th at 10:00AM. Turn in a hard copy to the instructor's mailbox at HFH 2108, or send a pdf file (please include 267 in the subject line) via email.

1. 1. Consider the transition system $T = (S, I, R)$ where $I = \{0\}$, $S = \{0, 1, 2, 3\}$, and $R = \{(0, 1), (1, 2), (2, 3), (2, 0), (3, 1), (1, 3)\}$, the set of atomic propositions $AP = \{p, q, r\}$ and the labeling function $L : S \rightarrow 2^{AP}$ where $L(0) = \{p\}$, $L(1) = \{p, q\}$, $L(2) = \{p\}$ and $L(3) = \{p, q, r\}$.

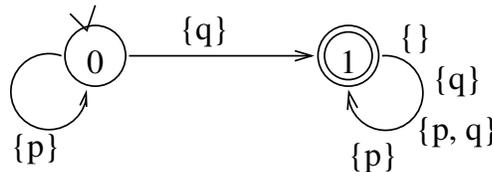
(a) For each of the following CTL formulas show the states which satisfy them:

$$EG(p \wedge q), AF(p \wedge q), AF(r), AG(r), AG(pAUq)$$

(b) Which of the following LTL formulas hold for the above transition system? For the ones that do not hold, give a counter-example path.

$$Gp, Fq, Fr, GFq, FGq.$$

2. You are given the following Büchi automaton A_f :



What is the LTL property f represented by the Büchi automaton above?

Given the transition system $T = (S, I, R)$ where $I = \{0\}$, $S = \{0, 1, 2\}$, and $R = \{(0, 1), (1, 2), (2, 1), (1, 1)\}$, the set of atomic propositions $AP = \{p, q\}$ and the labeling function $L : S \rightarrow 2^{AP}$ where $L(0) = \{p\}$, $L(1) = \{p\}$ and $L(2) = \{q\}$, show the Büchi automaton A_T that corresponds to this transition system, and the product automaton $A_T \times A_f$.

Show an accepting run of the product automaton and show the path in the transition system T which corresponds to this run and satisfies the LTL formula f .

Show a non-accepting run of the product automaton and show the path in the transition system T which corresponds to this run and does not satisfy the LTL formula f .

3. Draw the transition system that corresponds to a binary counter which counts from 0 (initial state) to 3, and after 3 it goes back to 0.

Use two boolean variables x_1 (most significant bit) and x_0 (the least significant bit) to encode the bits of the binary counter. Write the boolean logic formula for the transition relation.

Show the BDD that corresponds to the transition system for the ordering $x_1 < x'_1 < x_0 < x'_0$.

Show the iterations of the fixpoint computation $EF(x_1 \wedge x_0)$ by deriving the boolean logic formulas that corresponds to each iteration.

4. Given a transition system $TS = (I, S, R)$ where S is the set of states, $I \subseteq S$ is the set of initial states, and $R \subseteq S \times S$ is the transition relation, we define the following function $\text{POST} : 2^S \rightarrow 2^S$ where given $p \subseteq S$, $\text{POST}(p) = \{s' : (s, s') \in R \wedge s \in p\}$.

Using the function POST , write a fixpoint characterization of the states that are reachable from the initial states I .

Given the following transition system $TS = (I, S, R)$ with $S = \{0, 1, 2, 3, 4, 5\}$, $I = \{0\}$, and $R = \{(0, 0), (0, 1), (1, 2), (2, 1), (2, 4), (3, 5), (4, 4), (5, 3), (5, 5)\}$, show the steps of the fixpoint computation for the reachable states.