

CS 267 – Spring 2023 – Homework Assignment 1

Due Monday, April 17th at the beginning of the class. Turn in a hard copy (hand written solutions are OK). Do not discuss the problems with anyone other than the instructor.

In the problems below, the labeling functions for transition systems are defined as: $L : S \rightarrow 2^{AP}$ instead of $L' : S \times AP \rightarrow \{\mathbf{true}, \mathbf{false}\}$ where $p \in L(s) \Leftrightarrow L'(s, p) = \mathbf{true}$.

1. Consider the following transition system:

$T = (S, R, I)$ with the set of states $S = \{0, 1, 2, 3\}$, the initial set of states $I = \{0\}$, the transition relation $R = \{(0, 1), (1, 2), (2, 3), (1, 1), (3, 2)\}$, the set of atomic propositions $AP = \{p, q\}$ and the labeling function $L : S \rightarrow 2^{AP}$ which shows which states satisfy a given property, where $L(0) = \{q\}$, $L(1) = \{p, q\}$, $L(2) = \{p\}$, and $L(3) = \{p, q\}$.

(a) For each of the CTL formulas show the states which satisfy them:

EGp , AGp , EGq , AGq , EFp , AFp , $qEUp$, $qAU p$, $EG(EFp)$, $AG(EFp)$, $AG(AFp)$, $EG(AFp)$.

For the ACTL properties that fail for the given transition system, give a counter-example. For the ECTL properties that hold for the given transition systems, give a witness.

(b) Which of the following LTL formulas hold for the above transition system?

Gp , Fp , Gq , Fq , GFp , GFq , FGp , FGq , pUq , qUp .

For the properties that fail, give a counter-example.

2. Assume that p , q and r are atomic properties.

(a) Write LTL formulas that correspond to the following properties: 1) p holds only a finite number of times, 2) if p holds sometime, then q must hold at least once before it, 3) p and q become true in strictly alternating fashion, 4) Whenever p holds q must hold in the next state, 5) q holds infinitely many times.

(b) Write CTL formulas that correspond to the following properties: 1) From each reachable state it is possible to reach a state where p holds, 2) Whenever p holds, eventually q will hold, 3) Eventually, beginning of a path on which p always holds will be reached, 4) It is possible for p and q to hold at the same time, 5) Whenever p holds, eventually r will hold and q will hold until r holds.

3. (a) Give a transition system that satisfies only one of the following two properties: FGp , $AFAGp$.

(b) Give a transition system that satisfies only one of the following two properties: FGp , $AFEGp$.

(c) Give a transition system that satisfies only one of the following two properties: FGp , $EFEGp$.

(d) Give a transition system that satisfies only one of the following two properties: $\text{GF}p$, $\text{AGEF}p$.

4. (a) Show that all CTL formulas can be written using only the temporal operators EX, EU and AU. I.e., for each of the CTL formulas $\text{AX}p$, $\text{AG}p$, $\text{EG}p$, $\text{AF}p$, $\text{EF}p$, write an equivalent formula that only contains the temporal operators EX, EU and AU and Boolean operators and atomic properties.

(b) Assume that the next-until operator XU is defined as follows:

$$p\text{XU}q = \text{X}(p\text{U}q)$$

Show that all LTL formulas can be written using only the temporal operator XU, Boolean operators and atomic properties. I.e., for each of the LTL formulas $\text{X}p$, $p\text{U}q$ and $\text{G}p$ write an equivalent formula that only contains the temporal operator XU, Boolean operators and atomic properties.

5. Consider the following transition system: $T = (S, R, I)$ with the set of states $S = \{0, 1, 2, 3, 4\}$, the initial set of states $I = \{0\}$, the transition relation $R = \{(0, 1), (1, 2), (1, 4), (2, 3), (3, 2), (4, 4)\}$, the set of atomic propositions $AP = \{p, q, r\}$ and the labeling function $L : S \rightarrow 2^{AP}$ where $L(0) = \{p\}$, $L(1) = \{p, q\}$, $L(2) = \{r, q\}$, $L(3) = \{r\}$ and $L(4) = \{q\}$.

Show the steps of the CTL model checking algorithm (by labeling the states with the subformulas that hold in that state) while evaluating the following CTL formulas on this transition system: $\text{EFEG}r$, $\text{AGEF}p$, $\text{AG}(r \vee q)$, $\text{AF}(q)$, and $p\text{EU}(\text{AG}p)$. Which of these formulas hold for this transition system?