1. Give a B"uchi automaton $A_f$ that corresponds to the LTL property $GFp$.

Given the transition system $T = (S, I, R)$ where $I = \{0\}$, $S = \{0, 1, 2\}$ and $R = \{(0, 1), (1, 2), (2, 1), (1, 1)\}$, assume that the only state which satisfies the atomic proposition $p$ is 1. Show the B"uchi automaton $A_T$ that corresponds to this transition system (based on the construction given in the lecture notes), and the product automaton $A_T \times A_f$.

If there is one, show an accepting run of the product automaton and show the path in the transition system $T$ which corresponds to this run and satisfies the LTL formula $GFp$.

Does the transition system $T$ satisfy the property $FG\neg p$? Why?

2. Construct a B"uchi automaton that corresponds to the LTL property $p U (q \land X (\neg q))$ using the LTL-B"uchi automata translation algorithm. Show the intermediate steps (like the example in the lecture notes).

3. Given the following piece of code:

```plaintext
x = y;
while (x < z) {
    x++;
}
assert(x == z);
```

demonstrate the verification approach used by the CBMC model checker by 1) converting it to a loop free code by unwinding the loop 2 times, 2) converting the resulting code to the static single assignment form, 3) generating the constraint for the verification of the assertion. Determine if the generated constraint is satisfiable and give a satisfying assignment if it is.

4. Consider the following two transition systems:

$M_1 = (AP, S, R, S_0, L)$ with the set of states $S = \{0, 1, 2, 3\}$, the initial set of states $S_0 = \{0\}$, the transition relation $R = \{(0, 1), (1, 2), (2, 3), (1, 0), (3, 2)\}$, the set of atomic propositions $AP = \{p, q\}$ and the labeling function $L : S \rightarrow 2^{AP}$ where $L(0) = \{p\}$, $L(1) = \{p\}$, $L(2) = \{q\}$, and $L(3) = \{q\}$.

$M_2 = (AP, S, R, S_0, L)$ with the set of states $S = \{0, 1\}$, the initial set of states $S_0 = \{0\}$, the transition relation $R = \{(0, 0), (0, 1), (1, 1)\}$, the set of atomic propositions $AP = \{p, q\}$ and the labeling function $L : S \rightarrow 2^{AP}$ where $L(0) = \{p\}$, and $L(1) = \{q\}$.

Is there a simulation relation between these two transition systems? If there is, show the simulation relation.
Determine if $M_2$ satisfies $AGp$, $AGq$, $AFp$, $AFq$ by identifying the states of $M_2$ that satisfy these properties. Given these results, can you determine if $M_1$ satisfies these properties?

5. Assume that you are given the statement “$y := x + 1$” and two predicates $y > x$ and $y > 0$. Show how predicate abstraction technique would abstract this statement by 1) computing the preconditions, 2) checking the implications, and 3) generating the abstract code. (Assume that $x$ and $y$ are unbounded integer variables). In the second step (checking the implications) use the web interface for the Z3 theorem prover which is available at: https://microsoft.github.io/z3guide/docs/logic/intro/ (use the Playground tab to enter a formula and run Z3)

In addition to the results of the steps 1, 2 and 3, turn in the formulas that you checked with Z3.