CS 267: Automated Verification

Lecture 12: Bounded Model Checking

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Remember Symbolic Model Checking

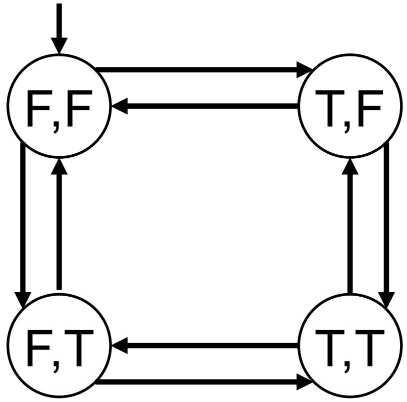
- Represent sets of states and the transition relation as Boolean logic formulas

- Fixpoint computation becomes formula manipulation
  - pre-condition (EX) computation: Existential variable elimination
  - conjunction (intersection), disjunction (union) and negation (set difference), and equivalence check

- Use an efficient data structure for boolean logic formulas
  - Binary Decision Diagrams (BDDs)

An Extremely Simple Example

Variables: x, y: boolean



Set of states:
$S = \{(F,F), (F,T), (T,F), (T,T)\}$
$S \equiv \text{True}$

Initial condition:
$I \equiv \neg x \wedge \neg y$

Transition relation (negates one variable at a time):
$R \equiv x' = \neg x \wedge y' = y \vee x' = x \wedge y' = \neg y$ (= means $\leftrightarrow$)

An Extremely Simple Example

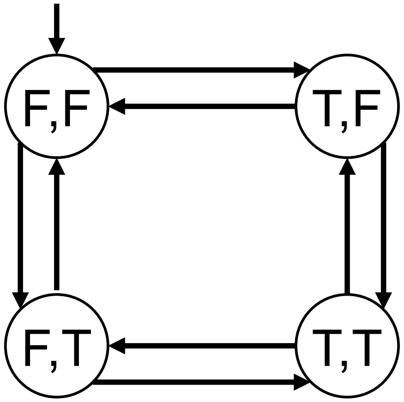
- Assume that we want to check if this transition system satisfies the property $AG(\neg x \vee \neg y)$
- Instead of checking $AG(\neg x \vee \neg y)$ we can check $EF(x \wedge y)$
  - Since $AG(\neg x \vee \neg y) \equiv \neg\ EF(x \wedge y)$

    $I \subseteq AG(\neg x \vee \neg y)$ if and only if $I \cap EF(x \wedge y) = \emptyset$

- If we find an initial state which satisfies $EF(x \wedge y)$ (i.e., there exists a path from an initial state where eventually x and y both become true at the same time)
  - Then we conclude that the property $AG(\neg x \vee \neg y)$ does not hold for this transition system

- If there is no such initial state, then property $AG(\neg x \vee \neg y)$ holds for this transition system

An Extremely Simple Example



Given $p \equiv x \wedge y$, compute EX(p)

$$EX(p) \equiv \exists V' \; R \wedge p[V' / V]$$

$$\equiv \exists V' \; R \wedge x' \wedge y'$$

$$\equiv \exists V' \; (x' = \neg x \wedge y' = y \vee x' = x \wedge y' = \neg y) \wedge x' \wedge y'$$

$$\equiv \exists V' \; (x' = \neg x \wedge y' = y) \wedge x' \wedge y' \vee (x' = x \wedge y' = \neg y) \wedge x' \wedge y'$$

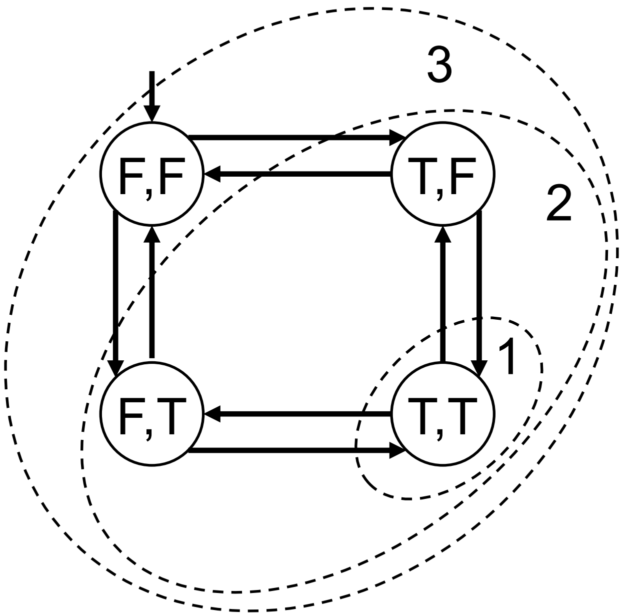
$$\equiv \exists V' \; \neg x \wedge y \wedge x' \wedge y' \vee x \wedge \neg y \wedge x' \wedge y'$$

$$\equiv \neg x \wedge y \vee x \wedge \neg y$$

$EX(x \wedge y) \equiv \neg x \wedge y \vee x \wedge \neg y$

In other words $EX(\{(T,T)\}) \equiv \{(F,T), (T,F)\}$

An Extremely Simple Example



Let’s compute compute EF($x \wedge y$)

The fixpoint sequence is

False, $x \wedge y$, $x \wedge y \vee \mathrm{EX}(x \wedge y)$, $x \wedge y \vee \mathrm{EX}\,(x \wedge y \vee \mathrm{EX}(x \wedge y))$, ...

If we do the EX computations, we get:

$$\underbrace{\text{False},}_{0}\qquad \underbrace{x \wedge y}_{1}\,,\qquad \underbrace{x \wedge y \vee \neg x \wedge y \vee x \wedge \neg y,}_{2}\qquad \underbrace{\text{True}}_{3}$$

EF($x \wedge y$) $\equiv$ True $\equiv$ {(F,F),(F,T), (T,F),(T,T)}

This transition system violates the property AG($\neg x \vee \neg y$)

since it has an initial state that satisfies the property EF($x \wedge y$)

Bounded Model Checking

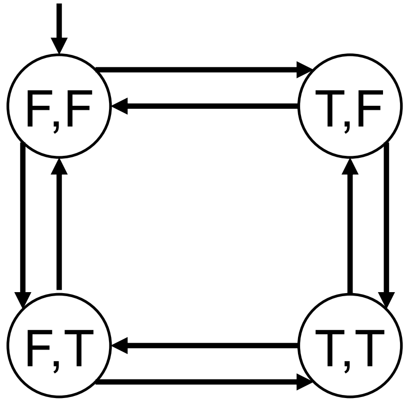
- Represent sets of states and the transition relation as Boolean logic formulas

- Instead of computing the fixpoints, unroll the transition relation up to certain fixed bound and search for violations of the property within that bound

- Transform this search to a Boolean satisfiability problem and solve it using a SAT solver

Same Extremely Simple Example

Variables: x, y: boolean



Set of states:

$S = \{(F,F), (F,T), (T,F), (T,T)\}$

$S \equiv \text{True}$

Initial condition:

$I(x,y) \equiv \neg x \wedge \neg y$

Transition relation (negates one variable at a time):

$R(x,y,x',y') \equiv x' = \neg x \wedge y' = y \vee x' = x \wedge y' = \neg y$ (= means $\leftrightarrow$)

Bounded Model Checking

- Assume that we like to check that if the initial states satisfy the formula EF($x \wedge y$)
- Instead of computing a backward fixpoint, we will unroll the transition relation a fixed number of times starting from the initial states
- ***For each unrolling we will create a new set of variables:***
  - The initial states of the system will be characterized with the variables x_0 and y_0
  - The states of the system after executing one transition will be characterized with the variables x_1 and y_1
  - The states of the system after executing two transitions will be characterized with the variables x_2 and y_2

Unrolling the Transition Relation

- Initial states: $I(x_0,y_0) \equiv \neg x_0 \wedge \neg y_0$
- Unrolling the transition relation once (bound k=1):

  $I(x_0,y_0) \wedge R(x_0,y_0,x_1,y_1)$

  $\equiv \neg x_0 \wedge \neg y_0 \wedge (x_1=\neg x_0 \wedge y_1=y_0 \vee x_1=x_0 \wedge y_1=\neg y_0)$
- Unrolling the transition relation twice (bound k=2):

  $I(x_0,y_0) \wedge R(x_0,y_0,x_1,y_1) \wedge R(x_1,y_1,x_2,y_2)$

  $\equiv \neg x_0 \wedge \neg y_0 \wedge (x_1=\neg x_0 \wedge y_1=y_0 \vee x_1=x_0 \wedge y_1=\neg y_0)$

  $\wedge (x_2=\neg x_1 \wedge y_2=y_1 \vee x_2=x_1 \wedge y_2=\neg y_1)$
- Unrolling the transition relation thrice (bound k=3):

  $I(x_0,y_0) \wedge R(x_0,y_0,x_1,y_1) \wedge R(x_1,y_1,x_2,y_2) \wedge R(x_2,y_2,x_3,y_3)$

  $\equiv \neg x_0 \wedge \neg y_0 \wedge (x_1=\neg x_0 \wedge y_1=y_0 \vee x_1=x_0 \wedge y_1=\neg y_0)$

  $\wedge (x_2=\neg x_1 \wedge y_2=y_1 \vee x_2=x_1 \wedge y_2=\neg y_1)$

  $\wedge (x_3 =\neg x_2 \wedge y_3=y_2 \vee x_3=x_2 \wedge y_3 =\neg y_2)$

Expressing the Property

- How do we represent the property we wish to verify?

- Remember the property: We were interested in finding out if some initial state satisfies EF($x \wedge y$)

  - This is equivalent to checking if $x \wedge y$ holds in some reachable state

  - If we are doing bounded model checking with bound k=3, we can express this property as:

    $x_0 \wedge y_0 \vee x_1 \wedge y_1 \vee x_2 \wedge y_2 \vee x_3 \wedge y_3$

Converting to Satisfiability

- We end up with the following formula for bound k=3:

$$F \equiv I(x_0,y_0) \wedge R(x_0,y_0,x_1,y_1) \wedge R(x_1,y_1,x_2,y_2) \wedge R(x_2,y_2,x_3,y_3)$$

$$\wedge(x_0 \wedge y_0 \vee x_1 \wedge y_1 \vee x_2 \wedge y_2 \vee x_3 \wedge y_3)$$

$$\equiv \neg x_0 \wedge \neg y_0 \wedge (x_1{=}\neg x_0 \wedge y_1{=}y_0 \vee x_1{=}x_0 \wedge y_1{=}\neg y_0)$$

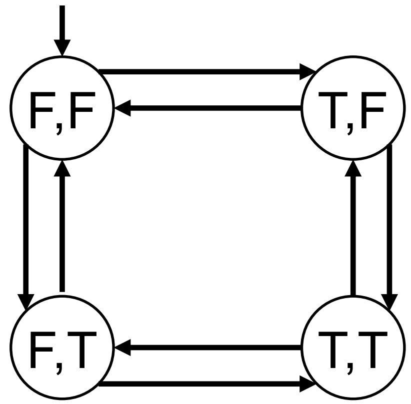
$$\wedge (x_2{=}\neg x_1 \wedge y_2{=}y_1 \vee x_2{=}x_1 \wedge y_2{=}\neg y_1)$$

$$\wedge (x_3 {=}\neg x_2 \wedge y_3{=}y_2 \vee x_3{=}x_2 \wedge y_3 {=}\neg y_2)$$

$$\wedge (x_0 \wedge y_0 \vee x_1 \wedge y_1 \vee x_2 \wedge y_2 \vee x_3 \wedge y_3)$$

- Here is the main observation: if F is a satisfiable formula then there exists an initial state which satisfies EF($x \wedge y$)
  - A satisfying assignment to the boolean variables in F corresponds to a counter-example for AG($\neg x \vee \neg y$) (i.e., a witness for EF($x \wedge y$))

The Result



$F \equiv$

$\neg x_0 \wedge \neg y_0 \wedge (x_1{=}\neg x_0 \wedge y_1{=}y_0 \vee x_1{=}x_0 \wedge y_1{=}\neg y_0)$

$\wedge\ (x_2{=}\neg x_1 \wedge y_2{=}y_1 \vee x_2{=}x_1 \wedge y_2{=}\neg y_1)$

$\wedge\ (x_3 {=}\neg x_2 \wedge y_3{=}y_2 \vee x_3{=}x_2 \wedge y_3 {=}\neg y_2)$

$\wedge\ (x_0 \wedge y_0 \vee x_1 \wedge y_1 \vee x_2 \wedge y_2 \vee x_3 \wedge y_3)$

Here is a satisfying assignment:

x_0=F, y_0=F, x_1=F, y_1=T, x_2=T, y_2=T, x_3=F, y_3=T

which corresponds to the (bounded) path:

(F,F), (F,T), (T,T), (F,T)

What Can We Guarantee?

- We converted checking property AG(p) to Boolean SAT solving by looking for bounded paths that satisfy EF(¬p)
- Note that we are checking only for bounded paths (paths which have at most k+1 distinct states)
  - So if the property is violated by only paths with more than k+1 distinct states, we would not find a counter-example using bounded model checking
  - Hence if we do not find a counter-example using bounded model checking we are not sure that the property holds
- However, if we find a counter-example, then we are sure that the property is violated since the generated counter-example is never spurious (i.e., it is always a concrete counter-example)

Bounded Model Checking for LTL

- It is possible to extend the basic ideas we discussed for verifying properties of the form AG(p) to all LTL (and even ACTL*) properties.

- The basic observation is that we can define a bounded semantics for LTL properties so that if a path satisfies an LTL property based on the bounded semantics, then it satisfies the property based on the unbounded semantics
  - This is why a counter-example found on a bounded path is guaranteed to be a real counter-example
  - However, this does not guarantee correctness

Bounded Model Checking: Proving Correctness

- One can also show that given an LTL property f, if E f holds for a finite state transition system, then E f also holds for that transition system using bounded semantics for some bound k

- So if we keep increasing the bound, then we are guaranteed to find a path that satisfies the formula
  - And, if we do not find a path that satisfies the formula, then we decide that the formula is not satisfied by the transition system
  - Is there a problem here?

Proving Correctness

- We can modify the bounded model checking algorithm as follows:
  - Start from an initial bound.
  - If no counter-examples are found using the current bound, increment the bound and try again.

- The problem is: We do not know when to stop

Proving Correctness

- If we can find a way to figure out when we should stop then we would be able to provide guarantee of correctness.

- There is a way to define a ***diameter*** of a transition system so that a property holds for the transition system if and only if it is not violated on a path bounded by the diameter.

- So if we do bounded model checking using the diameter of the system as our bound, then we can guarantee correctness if no counter-example is found.

Bounded Model Checking

- What are the differences between bounded model checking and BDD-based symbolic model checking?
  - In bounded model checking we are using a SAT solver instead of a BDD library
  - In symbolic model checking we do not unroll the transition relation as in bounded model checking
  - In bounded model checking we do not compute the fixpoint as in symbolic model checking
  - In symbolic model checking for finite state systems both verification and falsification results are guaranteed
    - In bounded model checking we can only guarantee the falsification results, in order to guarantee the verification results we need to know the diameter of the system

Bounded Model Checking

- Boolean satisfiability problem (SAT) is an NP-complete problem

- A bounded model checker needs an efficient SAT solver
  - zChaff SAT solver is one of the most commonly used ones
  - However, in the worst case any SAT solver we know will take exponential time

- Most SAT solvers require their input to be in Conjunctive Normal Form (CNF)
  - So the final formula has to be converted to CNF

Bounded Model Checking

- Similar to BDD-based symbolic model checking, bounded model checking was also first used for hardware verification

- Later on, it was applied to software verification

Bounded Model Checking for Software

CBMC is a bounded model checker for ANSI-C programs

- Handles function calls using inlining
- Unwinds the loops a fixed number of times
- Allows user input to be modeled using non-determinism
  - So that a program can be checked for a set of inputs rather than a single input
- Allows specification of assertions which are checked using the bounded model checking

Loops

- Unwind the loop n times by duplicating the loop body n times
  - Each copy is guarded using an if statement that checks the loop condition
- At the end of the n repetitions an unwinding assertion is added which is the negation of the loop condition
  - Hence if the loop iterates more than n times in some execution, the unwinding assertion will be violated and we know that we need to increase the bound in order to guarantee correctness
- A similar strategy is used for recursive function calls
  - The recursion is unwound up to a certain bound and then an assertion is generated stating that the recursion does not go any deeper

A Simple Loop Example

Original code

```
x=0;
while (x < 2) {
   y=y+x;
   x++;
}
```

Unwinding the loop 3 times

```
x=0;
if (x < 2) {
   y=y+x;
   x++;
}
if (x < 2) {
   y=y+x;
   x++;
}
if (x < 2) {
   y=y+x;
   x++;
}
```

Unwinding assertion: →

```
assert (! (x < 2))
```

From Code to SAT

- After eliminating loops and recursion, CBMC converts the input program to the static single assignment (SSA) form
  - In SSA each variable appears at the left hand side of an assignment only once
  - This is a standard program transformation that is performed by creating new variables
- In the resulting program each variable is assigned a value only once and all the branches are forward branches (there is no backward edge in the control flow graph)
- CBMC generates a Boolean logic formula from the program using bit vectors to represent variables

Another Simple Example

Original code

```
x=x+y;
if (x!=1)
  x=2;
else
  x++;
assert(x<=3);
```

Convert to static single assignment

```
$x_1=x_0+y_0$;
if ($x_1$!=1)
  $x_2$=2;
else
  $x_3=x_1+1$;
$x_4=(x_1!=1)?x_2:x_3$;
assert($x_4$<=3);
```

Generate constraints

$$C \equiv x_1=x_0+y_0 \wedge x_2=2 \wedge x_3=x_1+1 \wedge (x_1!=1 \wedge x_4=x_2 \vee x_1=1 \wedge x_4=x_3)$$

$$P \equiv x_4 <= 3$$

Check if $C \wedge \neg P$ is satisfiable, if it is then the assertion is violated

$C \wedge \neg P$ is converted to boolean logic using a bit vector representation for the integer variables $y_0, x_0, x_1, x_2, x_3, x_4$

Bounded Verification Approaches

- What we have discussed above is bounded verification by bounding the number of steps of the execution.
- For this approach to work, the variable domains also need to be bounded, otherwise we cannot convert the problems to boolean SAT
- Bounding the execution steps and bounding the data domain are two orthogonal approaches.
  - When people say bounded verification it may refer to either of these
  - When people say bounded model checking, it typically refers to bounding the execution steps