Abstract Interpretation Framework

- Associate each string variable at each program point with an automaton that accepts an over approximation of its possible values.
- Use these automata to perform symbolic executions on string variables.
- Iteratively
    - Compute the next state of current automata against string operations and
    - Update automata by joining the result to the automata at the next statement
- Terminate the execution upon reaching a fixed point.

Challenges

- Precision: Need to deal with sanitization routines having decent PHP functions, e.g., `ereg_replacement`.
- Complexity: Need to face the fact that the problem itself is undecidable. The fixed point may not exist and even if it exists the computation itself may not converge.
- Performance: Need to perform efficient automata manipulations in terms of both time and memory.

Features of Our Approach

We propose:

- A Language-based Replacement: to model decent string operations in PHP programs.
- An Automata Widening Operator: to accelerate fixed point computation.
- A Symbolic Encoding: using Multi-terminal Binary Decision Diagrams (MBDDs) from MONA DFA packages.

A Language-based Replacement

M=REPLACE(M_1, M_2, M_3)

- M_1, M_2, and M_3 are DFAs.
  - M_1 accepts the set of original strings,
  - M_2 accepts the set of match strings, and
  - M_3 accepts the set of replacement strings
- Let $s \in L(M1)$, $x \in L(M2)$, and $c \in L(M3)$:
  - Replaces all parts of any s that match any x with any c.
  - Outputs a DFA that accepts the result to M.

M=REPLACE(M_1, M_2, M_3)

Some examples:

$L(M_1)$	$L(M_2)$	$L(M_3)$	$L(M)$
{ baaabaa}	{aa}	{c}	{bacbc, bcabc}
{baaabaa}	a^+	ϵ	{bb}
{baaabaa}	a^+b	{c}	{bcaa}
{baaabaa}	a^+	{c}	{bcccbcc, bcccbc, bccbcc, bccbc, bcbcc, bcbc}
ba^+b	a^+	{c}	bc^+b

M=REPLACE(M_1, M_2, M_3)

- An over approximation with respect to the leftmost/longest(first) constraints
- Many string functions in PHP can be converted to this form:
  - htmlspecialchars, tolower, toupper, str_replace, trim, and
  - preg_replace and ereg_replace that have regular expressions as their arguments.

A Language-based Replacement

Implementation of REPLACE(M_1, M_2, M_3):

- Mark matching sub-strings
  - Insert marks to M_1
  - Insert marks to M_2
- Replace matching sub-strings
  - Identify marked paths
  - Insert replacement automata

In the following, we use two marks: $<$ and $>$ (not in Σ), and a duplicate set of alphabet: $\Sigma' = \{\alpha' | \alpha \in \Sigma\}$. We use an example to illustrate our approach.

An Example

Construct $M = \text{REPLACE}(M_1, M_2, M_3)$.

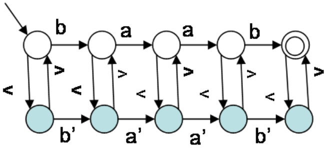
- $L(M_1) = \{baab\}$
- $L(M_2) = a^+ = \{a, aa, aaa, \ldots\}$
- $L(M_3) = \{c\}$

Step 1

Construct M_1' from M_1:

- Duplicate M_1 using Σ'
- Connect the original and duplicated states with $<$ and $>$

For instance, M_1' accepts $b < a'a' > b$, $b < a' > ab$.

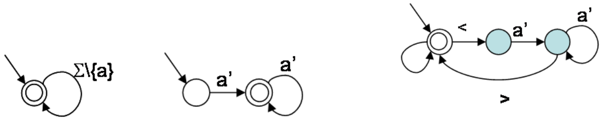


Step 2

Construct M_2' from M_2:

- Construct $M_{\bar{2}}$ that accepts strings do not contain any substring in $L(M_2)$. (a)
- Duplicate M_2 using Σ'. (b)
- Connect (a) and (b) with marks. (c)

For instance, M_2' accepts $b < a'a' > b$, $b < a' > bc < a' >$.



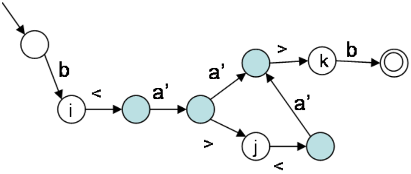
(a) (b) (c)

Step 3

Intersect M_1' and M_2'.

- The matched substrings are marked in Σ'.
- Identify (s, s'), so that $s \rightarrow^{<} \ldots \rightarrow^{>} s'$.

In the example, we idenitfy three pairs:(i,j), (i,k), (j,k).

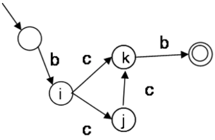


Step 4

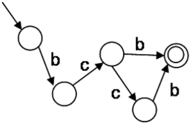
Construct M:

- Insert M_3 for each identified pair. (d)
- Determinize and minimize the result. (e)

$L(M) = \{bcb, bccb\}$.



(d)



(e)

$M \nabla M'$

The operator was originally proposed by Bartzis and Bultan [BB, CAV04]. Intuitively, we

- Identify equivalence classes, and
- Merge states in an equivalence class

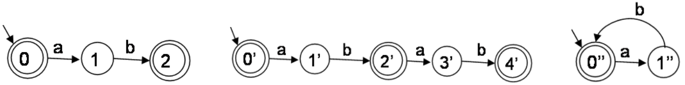
State Equivalence

q, q' are equivalent if one of the following condition holds:

- $\forall w \in \Sigma*$, w is accepted by M from q then w is accepted by M' from q', and vice versa.
- $\exists w \in \Sigma*$, M reaches state q and M' reaches state q' after consuming w from its initial state respectively.
- $\exists q$", q and q" are equivalent, and q' and q"are equivalent.

An Example for $M \nabla M'$

- $L(M) = \{\epsilon, ab\}$ and $L(M') = \{\epsilon, ab, abab\}$.
- The set of equivalence classes: $C = \{q''_0, q''_1\}$, where $q''_0 = \{q_0, q'_0, q_2, q'_2, q'_4\}$ and $q''_1 = \{q_1, q'_1, q'_3\}$.



(a) M (b) M' (c) $M \nabla M'$

Figure: Widening automata

A Fixed Point Computation

Recall that we want to compute the least fixpoint that corresponds to the reachable values of string expressions.

- The fixpoint computation will compute a sequence M_0, M_1, ..., M_i, ..., where $M_0 = I$ and $M_i = M_{i-1} \cup post(M_{i-1})$

A Fixed Point Computation

Consider a simple example:

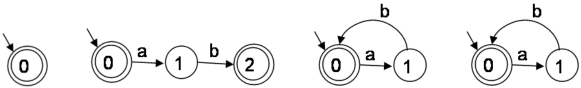
- Start from an empty string and concatenate *ab* at each iteration
- The exact computation sequence M_0, M_1, ..., M_i, ... will never converge, where $L(M_0) = \{\epsilon\}$ and $L(M_i) = \{(ab)^k \mid 1 \leq k \leq i\} \cup \{\epsilon\}$.

Accelerate The Fixed Point Computation

Use the widening operator ∇.

- Compute an over-approximate sequence instead: M'_0, M'_1, ..., M'_i, ...
- $M'_0 = M_0$, and for $i > 0$, $M'_i = M'_{i-1} \nabla (M'_{i-1} \cup post(M'_{i-1}))$.

An over-approximate sequence for the simple example:



(a) M'_0 (b) M'_1 (c) M'_2 (d) M'_3