Chapter 6: Transition Graphs *

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- The corresponding textbook chapter should be read before attending this lecture.
- These notes are not intended to be complete. They are supplemented with figures, and other material that arises during the lecture period in response to questions.

*Based on **Theory of Computing**, 2nd Ed., D. Cohen, John Wiley & Sons, Inc.

Definition of a Transition Graph

A **transition graph** is defined by a 5-tuple:

- A finite set of *states*, Q.
- A finite set of *input symbols*, Σ.
- A non-empty set set of *start states*, $S \subseteq Q$.
- A set of *final* or *accepting states* $F \subseteq Q$.
- A finite set, δ of *transitions*, (directed edge labels) (u, s, v), where $u, v \in Q$ and $s \in \Sigma^*$.

Illustrate.

The Language Accepted by a Transition Graph

- Let $A = (Q, \Sigma, S, F, \delta)$ be a transition graph.
- A **successful path** in A is one that starts in some start state and ends in some accepting state of A.
- Let P be the set of all successful paths in A.
- Let L be the set of words that are the concatenation of the sequence of *edge labels* of A corresponding to some successful path in A.
- The **language accepted by** A, denoted $L(A) = L$.

Illustrate.

Transition Graphs: Some Observations

- If *there is no factoring of a word* w *that is the concatenation of edge labels of a successful path in* A, then $w \notin L(A)$.
- Every finite automaton can be viewed as a transition graph.
- Since the reverse is not true, transition graphs *generalize* finite automata.

Transition Graphs: Basic Building Blocks

Illustrate transition graphs for the following building blocks:

- $L(A) = \emptyset$
- $L(A) = \{\Lambda\}$
- $L(A) = \Sigma$
- $L(A) = L(B)L(C)$, for transition graphs B and C.
- $L(A) = L(B) \cup L(C)$, for transition graphs B and C.
- $L(A) = \overline{L(B)}$, for transition graph B.

Transition Graphs: Some Observations

- Every finite language is accepted by some transition graph. Illustrate.
- Given a transition graph A, it is unclear how to determine $L(A)$.
- We see soon why transitions graphs are introduced.

Generalized Transition Graphs

A **generalized transition graph** is defined by a 5-tuple:

- A finite set of *states*, Q.
- A finite set of *input symbols*, Σ.
- A non-empty set set of *start states*, $S \subseteq Q$.
- A set of *final* or *accepting states* $F \subseteq Q$.
- A finite set, δ of *transitions*, (directed edge labels) (u, s, v), where $u, v \in Q$ and s is a *regular expression* over Σ.

Illustrate.

Generalized transition graphs are **nondeterministic**: Given a state and a [partially consumed] input, there may be more than 1 possible successor state.

$\forall$ TG, $\exists$ an equivalent GTG with 1 final state

Proof: (Illustrate)

1. Construct a new GTG, A'. Initially, $A' \leftarrow A$.
2. A has 0, 1, or more than 1 final state.

   Case 0 final states Add a final state with no transition to it.

   Case 1 final state Do nothing; the given transition graph has the desired property.

   Case more than 1 final state

   (a) Add state f to Q'.

(b) $F' = \{f\}$.

(c) For each $s \in F$, add a Λ-transition from s to f'.

3. Every string that could reach a final state in A can reach a final state in A', using a Λ-transition: $L(A) \subseteq L(A')$.
4. Every string that can reach a final state in A' must first reach a final state in A: $L(A') \subseteq L(A)$.
5. Thus, $L(A') = L(A)$.

$\forall$ TG A, $\exists$ a GTG A', $L(A') = L^+(A)$.

Proof: (Illustrate)

1. Construct a new GTG, A'. Initially, $A' \leftarrow A$.
2. For each final state in A, add a Λ-transition in A' from it to the start state.
3. If a substring reaches a final state in A, it can reach the start state in A': $L^+(A) \subseteq L(A')$.
4. Since the *only* transitions that are added are Λ-transitions from a final state to the start state, $w \in L(A') \Rightarrow w = w_1 \cdots w_i \cdots w_k$, where $w_i \in L(A)$, for $1 \leq i \leq k$: $L(A') \subseteq L^+(A)$.
5. Thus, $L(A') = L^+(A)$.