PROVING IMPLICATIONS

In the assertions below, $\mathcal{N}$ denotes the natural numbers, $\{1, 2, 3, \ldots\}$, and $\mathcal{Z}$ denotes the integers. Prove the following assertions:

1. $((p \vee r) \Rightarrow q) \equiv ((p \Rightarrow q) \wedge (r \Rightarrow q))$.
2. The cube of any integer is either of the form $9n, 9n + 1$, or $9n + 8$.
3. The equation $x^3 - 117y^3 = 5$ has no integer solutions.
4. $\forall n \in \mathcal{N}, n(n + 1)(2n + 1)/6$ is an integer.
5. If n is a 2-digit decimal number and $5n$ is a 2-digit decimal number, then n's first digit is a 1.
6. $\forall m, n \in \mathcal{Z}, ((m > 2 \wedge n > 2) \Rightarrow mn > m + n)$.
7. $f(x) = x^2 - x - a$ has no integral roots for odd a.