To prove: *If R is a symmetric relation, then $R \cap R^{-1} = R$.*

1. Let $R \subseteq S \times S$ be symmetric, for arbitrary set S.
2. Assume that $R \cap R^{-1} \neq R$. (Proof by contradiction)
3. $\exists (x, y) \in R \wedge (x, y) \notin R^{-1}$. (Step 2 and defn. of $\cap$)
4. $(y, x) \in R$. (Steps 1 and 3)
5. $(x, y) \in R^{-1}$, contradicting step 3. (Step 4 and defn. of R^{-1})
6. The assumption of step 2 is false: $R \cap R^{-1} = R$.