Computer Science 160
Translation of Programming Languages

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Building a Handle Recognizing Machine:
[now, with a look-ahead token, which is LR(1)]

LR(*k*) items

An LR(*k*) item is a pair [*A , B*], where

- *A* is a production $\alpha \rightarrow \beta\gamma\delta$ with a • at some position in the *rhs*
- *B* is a look-ahead string of length ≤ *k* (terminal symbols or $)

Examples: [$\alpha \rightarrow \bullet\beta\gamma\delta$, a], [$\alpha \rightarrow \beta\bullet\gamma\delta$, a], [$\alpha \rightarrow \beta\gamma\bullet\delta$, a], & [$\alpha \rightarrow \beta\gamma\delta\bullet$, a]

The • in an item indicates the position of the top of the stack

LR(0) items [$\alpha \rightarrow \beta \bullet \gamma\delta$] (no look-ahead symbol)

LR(1) items [$\alpha \rightarrow \beta \bullet \gamma\delta$, a] (one token look-ahead)

LR(2) items [$\alpha \rightarrow \beta \bullet \gamma\delta$, a b] (two token look-ahead) ...

LR(k) items

The • in an item indicates the position of the top of the stack

[$\alpha \rightarrow \bullet\beta\gamma\delta$, **a**] means that the input seen so far is consistent with the use of $\alpha \rightarrow \beta\gamma\delta$ immediately after the symbol on top of the stack

[$\alpha \rightarrow \beta\gamma\bullet\delta$, **a**] means that the input seen so far is consistent with the use of $\alpha \rightarrow \beta\gamma\delta$ at this point in the parse, *and* that the parser has already recognized $\beta\gamma$.

[$\alpha \rightarrow \beta\gamma\delta\bullet$, **a**] means that the parser has seen $\beta\gamma\delta$, *and* the lookahead **a** is consistent with reducing to α (for LR(k) parsers, **a** is a string of terminal symbols of length k)

The table construction algorithm uses items to represent valid configurations of an LR(1) parser

LR(1) Items

The production $\alpha \rightarrow \bullet\beta\gamma\delta$, with lookahead **a**, generates 4 items

$[\alpha \rightarrow \bullet\beta\gamma\delta$, **a**], $[\alpha \rightarrow \beta\bullet\gamma\delta$, **a**], $[\alpha \rightarrow \beta\gamma\bullet\delta$, **a**], & $[\alpha \rightarrow \beta\gamma\delta\bullet$, **a**]

The set of LR(1) items for a grammar is finite

What's the point of all these look-ahead symbols?

- Carry them along to choose correct reduction
- Look-ahead symbols are bookkeeping, *unless* item has • at right end
  - Has no direct use in $[\alpha \rightarrow \beta\gamma\bullet\delta$, **a**]
  - In $[\alpha \rightarrow \beta\gamma\delta\bullet$, **a**], a look-ahead of **a** implies a reduction by $\alpha \rightarrow \beta\gamma\delta$
  - For { $[\alpha \rightarrow \gamma\bullet$, **a**],$[\beta \rightarrow \gamma\bullet\delta$, **b**] }

    lookahead = **a** $\Rightarrow$ ***reduce*** to α

    lookahead $\in$ FIRST(δ) $\Rightarrow$ ***shift***

$\Rightarrow$ Limited right context is enough to pick the actions

Back to Finding Handles

Parser in a state where the stack (the fringe) was

Expr – *Term*

With look-ahead of *

How did it choose to expand *Term* rather than reduce to *Expr?*

- *Look-ahead* symbol is the key
- With look-ahead of + or –, parser should reduce to *Expr*
- With look-ahead of * or /, parser should shift
- Parser uses look-ahead to decide
- All this context from the grammar is encoded in the handle recognizing mechanism

Back to x - 2 * y

Stack	Input	Handle	Action
$	id – num * id $	*none*	shift
$ id	– num * id $	9,1	red. 9
$ *Factor*	– num * id $	7,1	red. 7
$ *Term*	– num * id $	4,1	red. 4
$ *Expr*	– num * id $	*none*	shift
$ *Expr* –	num * id $	*none*	shift
$ *Expr* – num	* id $	8,3	red. 8
$ *Expr* – *Factor*	* id $	7,3	red. 7
$ *Expr* – *Term*	* id $	*none*	shift
$ *Expr* – *Term* *	Id $	*none*	shift
$ *Expr* – *Term* * id	$	9,5	red. 9
$ *Expr* – *Term* * *Factor*	$	5,5	red. 5
$ *Expr* – *Term*	$	3,3	red. 3
$ *Expr*	$	1,1	red. 1
$ *S*	$	*none*	accept

shift here

reduce here

1. **Shift until TOS is the right end of a handle**
2. **Find the left end of the handle & reduce**

LR(1) Table Construction

High-level overview

1 Build the handle recognizing DFA (aka *Canonical Collection* of sets of LR(1) items), $C = \{ I_0 , I_1 , ... , I_n \}$

a) Introduce a new start symbol S' which has only one production

$S' \rightarrow S$

b) Initial state, I_0 should include

- [$S' \rightarrow \bullet S$, \$], along with any equivalent items
- Derive equivalent items as *closure*(I_0)

c) Repeatedly compute, for each I_k, and each grammar symbol α, *goto*(I_k, α)

- If the set is not already in the collection, add it
- Record all the transitions created by *goto( )*

This eventually reaches a fixed point

2 Fill in the ACTION and GOTO tables using the DFA

Computing Closures

closure(I) adds all the items implied by items already in *I*

- Any item $[\alpha \rightarrow \beta \bullet A\delta\ , a]$ implies $[A \rightarrow \bullet\ \tau\ , x]$ for each production with A on the *lhs*, and $x \in \text{FIRST}(\delta a)$
- Since A is valid, any way to derive A is valid, too
- FIRST(δa) tells us the set of things that could possibly come *after* this particular use of A (and would tell us the production to use)

The algorithm

```
Closure( I )
  while ( I is still changing )
    for each item [α → β • γδ , a] ∈ I
      for each production γ → τ ∈ P
        for each terminal b ∈ FIRST(δa)
          if [γ → • τ , b] ∉ I
            then add [γ → • τ , b] to I
```

Fixpoint computation

Example Grammar

Initial step builds the item [S $\rightarrow \bullet$ *Z* ,$]
and takes its *closure( )*

1	*S*	$\rightarrow$	*Z*
2	*Z*	$\rightarrow$	*Z* **z**
3		|	**z**

Closure([*S* $\rightarrow \bullet$ *Z* , $] *)*

Item	***From***
[*S* $\rightarrow \bullet$ *Z* , $]	**Original item**
[*Z* $\rightarrow \bullet$ *Z* z , $]	**1, δ a is $**
[*Z* $\rightarrow \bullet$ z , $]	**1, δ a is $**
[*Z* $\rightarrow \bullet$ *Z* z , z]	**2, δ a is z $**
[*Z* $\rightarrow \bullet$ z , z]	**2, δ a is z $**

So, initial state s_0 is

{ [*S* $\rightarrow \bullet$ *Z* ,$], [*Z* $\rightarrow \bullet$ *Z* z, $],[*Z* $\rightarrow \bullet$ z , $], [*Z* $\rightarrow \bullet$ *Z* z , z], [*Z* $\rightarrow \bullet$ z , z] }

Computing Gotos

goto(I , x) computes the state that the parser would reach if it recognized an *x* while in state *I*

- *goto(* { [$\alpha \rightarrow \beta \bullet \gamma\delta$, a] }, γ *)* produces [$\alpha \rightarrow \beta\gamma \bullet \delta$, a]
- It also includes *closure(* [$\alpha \rightarrow \beta\gamma \bullet \delta$, a] *)* to fill out the state

The algorithm

```
Goto( I, x )
   new = Ø
   for each [α → β • x δ , a] ∈ I
         new = new ∪ [α → βx • δ , a]
   return closure(new)
```

- Not a fixpoint method
- Uses closure

Example Grammar

s_0 is { [$S \rightarrow \bullet Z$,\$], [$Z \rightarrow \bullet Z$ z, \$],[$Z \rightarrow \bullet$ z , \$], [$Z \rightarrow \bullet Z$ z , z], [$Z \rightarrow \bullet$ z , z] }

goto(S_0 , z)

- Loop produces

Item	***From***
[$Z \rightarrow$ z • , \$]	**Item 3 in s_0**
[$Z \rightarrow$ z • , z]	**Item 5 in s_0**

- Closure adds nothing since • is at end of *rhs* in each item

In the construction, this produces s_2

{ [$Z \rightarrow$ z • , {\$, z}]}

New, but obvious, notation for two distinct items

[$Z \rightarrow$z • , \$] and [$Z \rightarrow$z • , z]

Canonical Collection of LR(1) Items

This is where we build the handle recognizing DFA!

Start from I_0 = *closure(* [*S'* → • *S* , $] *)*

Repeatedly construct new states, until no new states are generated

The algorithm

```
I0 = closure( [S' → • S , $] )
C  = { I0 }
while ( C is still changing )
   for each Ii ∈ C and for each  x ∈ ( T∪NT )
      Inew  =  goto(Ii , x)
      if  Inew ∉ C then
         C = C ∪ Inew
         record transition Ii → Inew on x
```

- **Fixed-point computation**
- **Loop adds to *C***
- **$C \subseteq 2^{ITEMS}$, so *C* is finite**

Algorithms - Overview

Computing closure of set of LR(1) items:

Closure(I **)**
  while (I **is still changing)**
    for each item $[\alpha \rightarrow \beta \bullet \gamma\delta , a] \in I$
      for each production $\gamma \rightarrow \tau \in P$
        for each terminal $b \in FIRST(\delta a)$
          if $[\gamma \rightarrow \bullet \tau , b] \notin I$
            then add $[\gamma \rightarrow \bullet \tau , b]$ **to** I

Computing goto for set of LR(1) items:

Goto(I, x **)**
  new $= \emptyset$
  for each $[\alpha \rightarrow \beta \bullet x\,\delta , a] \in I$
    new = new $\cup$ $[\alpha \rightarrow \beta x \bullet \delta , a]$
  return closure(new)

Constructing canonical collection of LR(1) items:

I_0 **= closure(** $[S' \rightarrow \bullet S , \$]$ **)**
$C = \{ I_0 \}$
while (C **is still changing)**
  for each $I_i \in C$ **and for each** $x \in (T \cup NT)$
    I_{new} = **goto(**I_i , x**)**
    if $I_{new} \notin C$ **then**
      $C = C \cup I_{new}$
      record transition $I_i \rightarrow I_{new}$ **on** x

- Canonical collection construction algorithm is the algorithm for constructing handle recognizing DFA
- Uses Closure to compute the states of the DFA
- Uses Goto to compute the transitions of the DFA

Practical Approach to LR(1) Parsing

Start with Grammar

→

Construct a DFA representing all possible legal transition on terminal and non-terminals. Technically, this is a bunch of NFAs grouped together via e-closure

→

Canonical Collection

This is the DFA which represents all valid transitions through the grammar. We need this for efficient **handle** finding

↓

Use the CC to fill in the LR tables (ACTION and GOTO tables), which is the way to program an automated LR(1) parser

Parser

Input Sentence

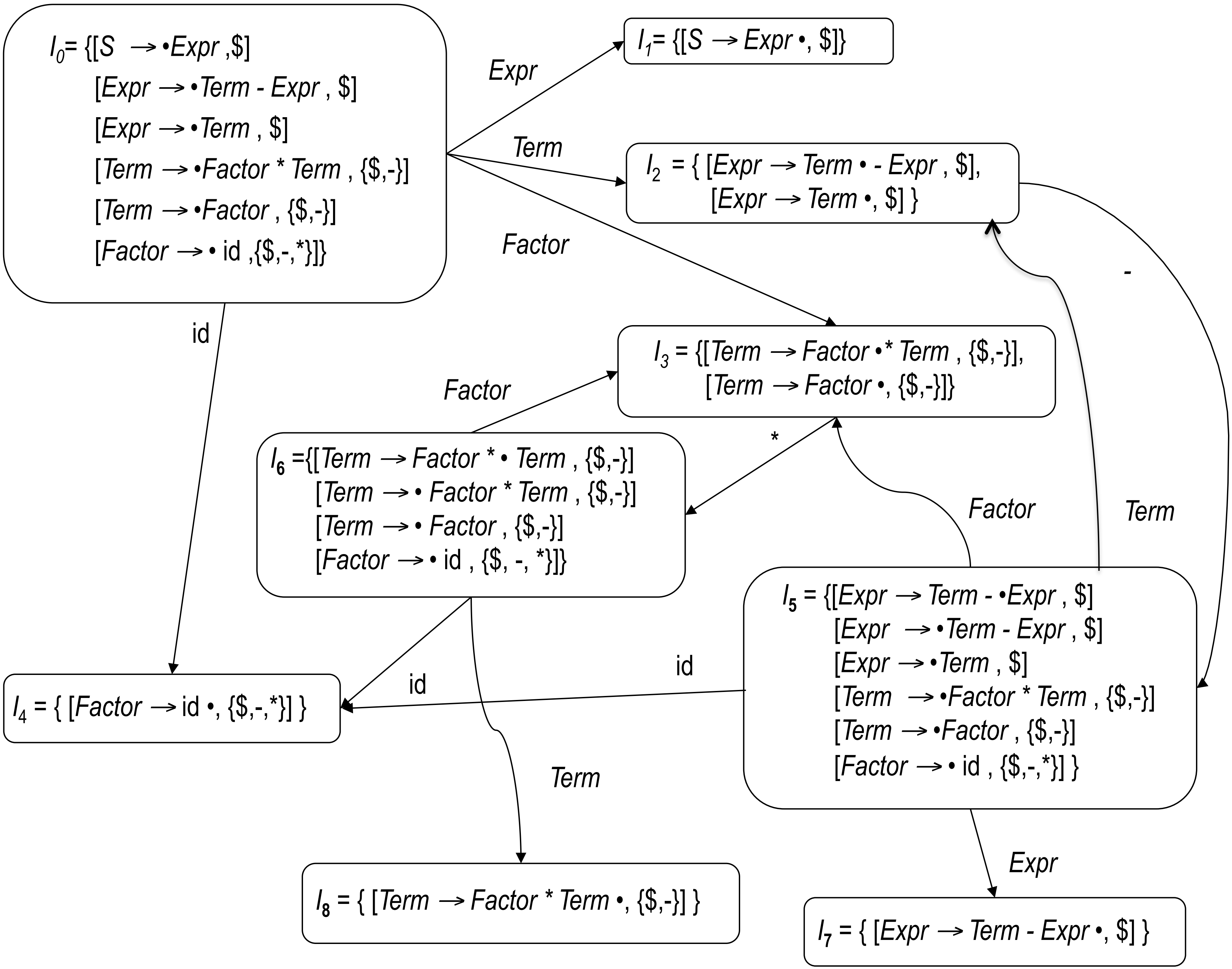
Reverse rightmost derivation

Example

Simplified, right recursive expression grammar

S → *Expr*
Expr → *Term* - *Expr*
Expr → *Term*
Term → *Factor* * *Term*
Term → *Factor*
Factor → id

Symbol	FIRST
S	{ id }
Expr	{ id }
Term	{ id }
Factor	{ id }
-	{ - }
*	{ * }
id	{ id }



I0= {[S → •Expr ,$]
[Expr → •Term - Expr , $]
[Expr → •Term , $]
[Term → •Factor * Term , {$,-}]
[Term → •Factor , {$,-}]
[Factor → • id ,{$,-,*}]}
I1= {[S → Expr •, $]}
I2 = { [Expr → Term • - Expr , $],
[Expr → Term •, $] }
I3 = {[Term → Factor •* Term , {$,-}],
[Term → Factor •, {$,-}]}
I4 = { [Factor → id •, {$,-,*}] }
I5 = {[Expr → Term - •Expr , $]
[Expr → •Term - Expr , $]
[Expr → •Term , $]
[Term → •Factor * Term , {$,-}]
[Term → •Factor , {$,-}]
[Factor → • id , {$,-,*}] }
I6 ={[Term → Factor * • Term , {$,-}]
[Term → • Factor * Term , {$,-}]
[Term → • Factor , {$,-}]
[Factor → • id , {$, -, *}]}
I7 = { [Expr → Term - Expr •, $] }
I8 = { [Term → Factor * Term •, {$,-}] }
Expr
Term
Factor
id
Factor
*
-
Factor
Term
id
id
Term
Expr

Constructing the ACTION and GOTO Tables

The algorithm

for each set of items $I_x \in C$
for each *item* $\in I_x$
if *item* is $[\alpha \rightarrow \beta \bullet a\gamma, b]$ and $a \in T$ and goto(I_x,a) = I_k ,
then ACTION[x,a] $\leftarrow$ "shift k"
else if *item* is $[S' \rightarrow S \bullet, \$]$
then ACTION[x ,\$] $\leftarrow$ "accept"
else if *item* is $[\alpha \rightarrow \beta \bullet, a]$
then ACTION[x,a] $\leftarrow$ "reduce $\alpha \rightarrow \beta$"

for each $n \in NT$
if goto(I_x ,n) = I_k
then GOTO[x,n] $\leftarrow$ k

***x* is the state number**
Each state corresponds to a set of LR(1) items

Example (Constructing the LR(1) tables)

The algorithm produces the following table

	ACTION				GOTO		
	id	-	*	$	*Expr*	*Term*	*Factor*
0	s 4				1	2	3
1				acc			
2		s 5		r 3			
3		r 5	s 6	r 5			
4		r 6	r 6	r 6			
5	s 4				7	2	3
6	s 4					8	3
7				r 2			
8		r 4		r 4			

Parsing Example (x-z*y)

Stack	Input	Action
S_0	$id * id - **id**	S4
S_0 id S_4	$id * id –	R6, G3
S_0 F S_3	$id * id –	R5,G2
S_0 T S_2	$id * id –	S5
S_0 T S_2 – S_5	$id * **id**	S4
S_0 T S_2 – S_5 id S_4	$id *	R6, G3
S_0 T S_3 – S_5 F S_3	$id *	S6
S_0 T S_3 – S_5 F S_3 * S_6	**$id**	S4
S_0 T S_3 – S_5 F S_3 * S_6 id S_4	**$**	R6,G3
S_0 T S_3 – S_5 F S_3 * S_6 F S_3	**$**	R5,G8
S_0 T S_3 – S_5 F S_3 * S_6 T S_8	**$**	R4,G2
S_0 T S_3 – S_5 T S_2	**$**	R3,G7
S_0 T S_3 – S_5 E $S7$	**$**	R2,G1
S_0 E S_1	**$**	ACC

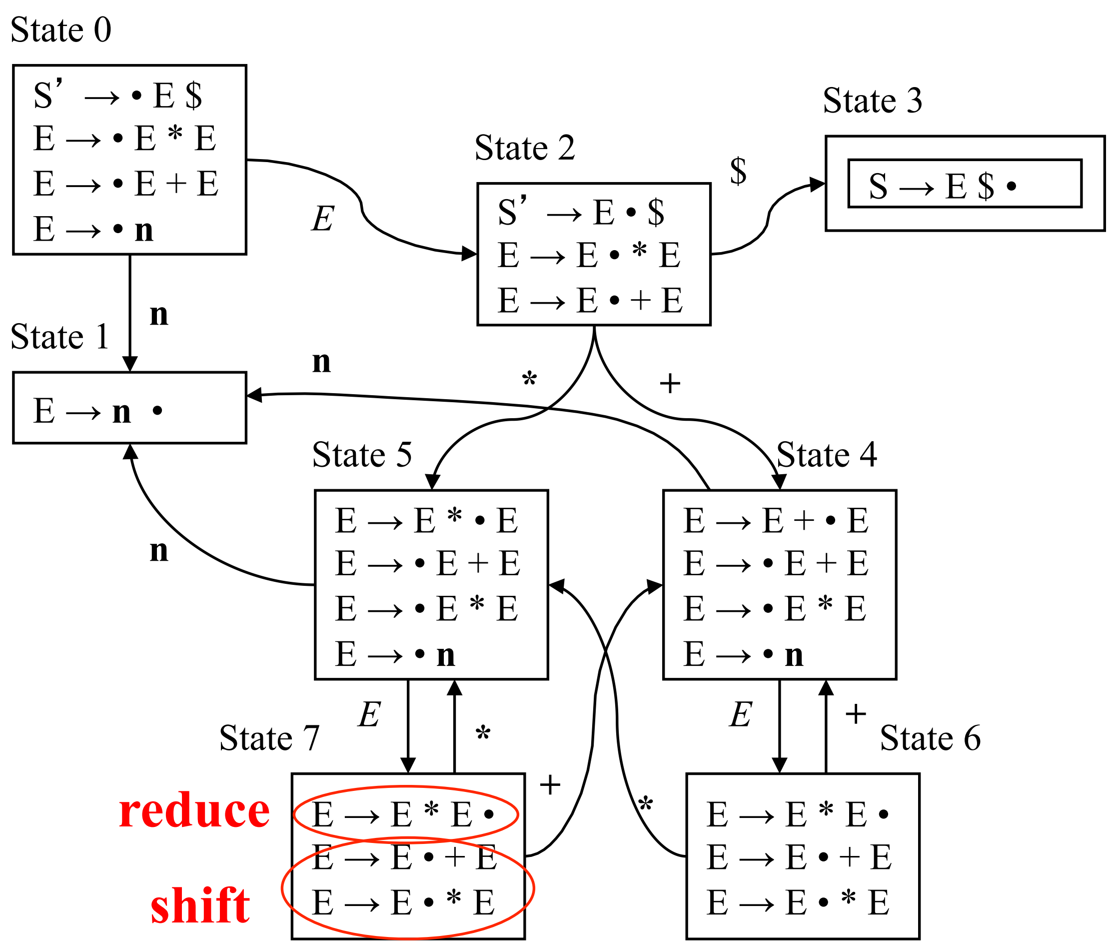
	ACTION				GOTO		
	id	-	*	$	*Expr*	*Term*	*Factor*
0	s 4				1	2	3
1				acc			
2		s 5		r 3			
3		r 5	s 6	r 5			
4		r 6	r 6	r 6			
5	s 4				7	2	3
6	s 4					8	3
7				r 2			
8		r 4		r 4			

1. ***S* → *Expr***
2. ***Expr* → *Term* - *Expr***
3. ***Expr* → *Term***
4. ***Term* → *Factor* * *Term***
5. ***Term* → *Factor***
6. ***Factor* → id**

Conflicts and Associativity/Precedence

example
0 S' → E $
1 E → E * E
2 E → E + E
3 E → **n**

	*	+	$	n	E
0				S1	2
1	R3	R3	R3		
2	S5	S4	S3		
3			A		
4				S1	6
5				S1	7
6	S5/R1	S4/R2	R2	R2	
7	S5/R1	S4/R1	R1	R1	



Conflicts and Associativity/Precedence

example

0 S’ → E $

1 E → E + T

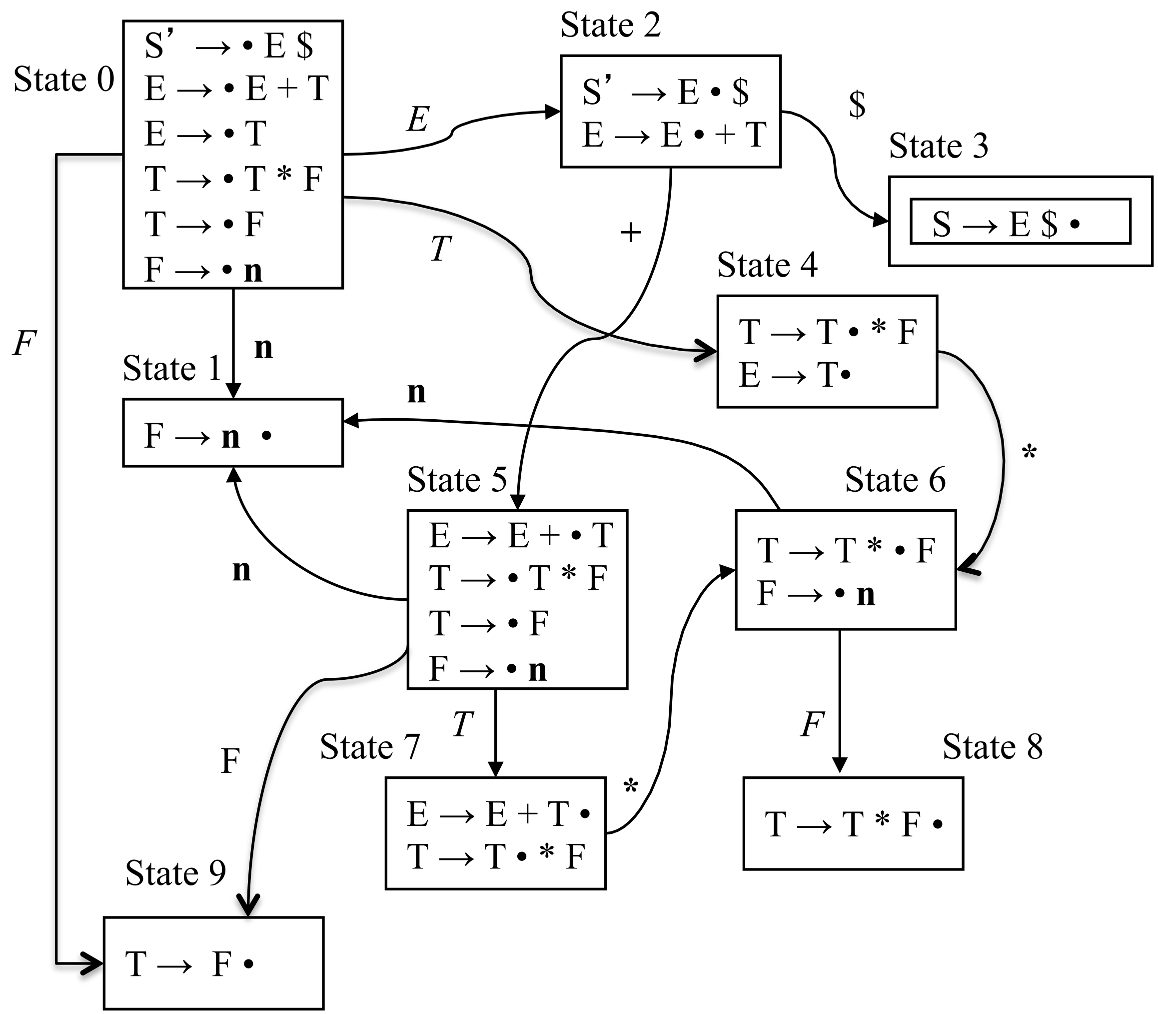
2 E → T

3 T → T * F

4 T → F

5 F → **n**

	+	*	$	n	E	T	F
0				S1	2	4	9
1	R5	R5	R5				
2	S5		S3				
3			A				
4	R2	S6	R2				
5				S1		7	9
6				S1			8
7	R1	S6	R1				
8	R3	R3	R3				
9	R4	R4	R4				



What can go wrong in LR(1) parsing?

What if state *s* contains [$\alpha \rightarrow \beta \bullet a\gamma$, b] and [$\alpha \rightarrow \beta \bullet$, a] ?

- First item generates "shift", second generates "reduce"
- Both define ACTION[*s*,a] — cannot do both actions
- This is called a *shift/reduce conflict*
- Modify the grammar to eliminate it
- Shifting will often resolve it correctly (dangling else problem?)

What if set *s* contains [$\alpha \rightarrow \beta \bullet$, a] and [$\gamma \rightarrow \beta \bullet$, a] ?

- Each generates "reduce", but with a different production
- Both define ACTION[*s*,a] — cannot do both reductions
- This is called a *reduce/reduce conflict*
- Modify the grammar to eliminate it

In either case, the grammar is not LR(1)

Error recovery

- **Panic-mode recovery:** On discovering an error, discard input symbols one at a time until one synchronizing token is found
  - For example delimiters such as “;” or “}” can be used as synchronizing tokens
- **Phrase-level recovery:** On discovering an error make local corrections to the input
  - For example replace “,” with “;”
- **Error-productions:** If we have a good idea about what type of errors occur, we can augment the grammar with error productions and generate appropriate error messages when an error production is used
- **Global correction:** Given an incorrect input string try to find a correct string which will require minimum changes to the input string
  - In general too costly

Direct Encoding of Parse Tables

Rather than using a table-driven interpreter …

- Generate spaghetti code that implements the logic
- Each state becomes a small case statement or if-then-else
- Analogous to direct coding a scanner

Advantages

- No table lookups and address calculations
- No representation for don't care states
- No outer loop — it is implicit in the code for the states

This produces a faster parser with more code but no table

LR Parsers

- LR(k) parsers are table-driven, bottom-up, shift-reduce parsers that use a limited right context (k-token look-ahead) for handle recognition

- LR(k): Left-to-right scan of the input, rightmost derivation in reverse with k token look-ahead

A grammar is LR(k) if, given a rightmost derivation

$$S \Rightarrow \gamma_0 \Rightarrow \gamma_1 \Rightarrow \gamma_2 \Rightarrow \dots \Rightarrow \gamma_{n-1} \Rightarrow \gamma_n \Rightarrow \textit{sentence}$$

We can

1. *isolate the handle of each right-sentential form* γ_i, and
2. *determine the production by which to reduce,*

by scanning γ_i from left-to-right, going at most k symbols beyond the right end of the handle of γ_i

Summary

	Advantages	*Disadvantages*
Top-down recursive descent	**Fast** **Simplicity** **Good error detection**	**Hand-coded** **High maintenance** **Right associativity**
LR(1)	**Fast** **Deterministic langs.** **Automatable** **Left associativity**	**Large working sets** **Poor error messages** **Large table sizes**