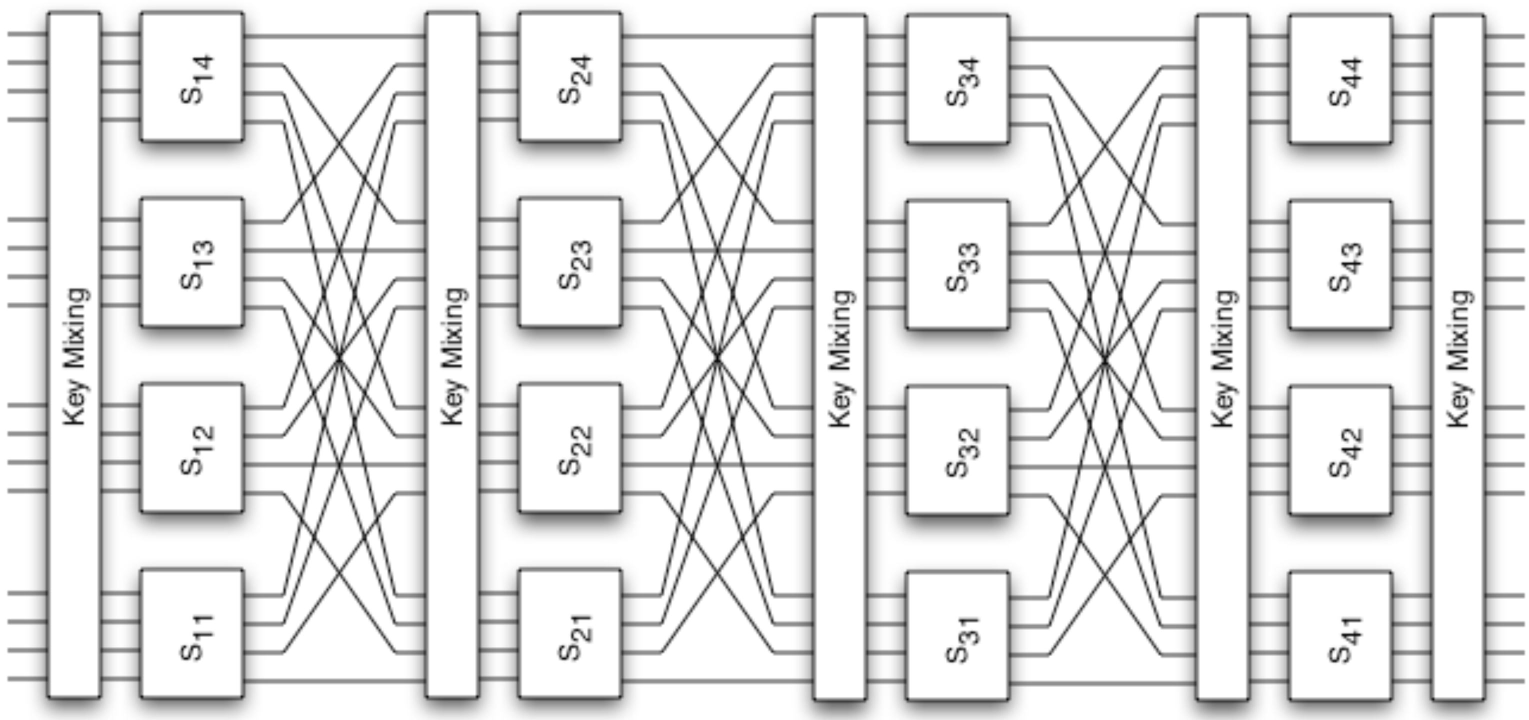
CS 177 - Computer Security

Linear Cryptanalysis

Linear Cryptanalysis



Linear Cryptanalysis

- Linear cryptanalysis
  - known plaintext attack
  - exploits high probability occurrences of linear relationships between plaintext, ciphertext, and key bits
  - linear with regards to bitwise operation modulo 2 (i.e., XOR)
  - expressions of form $X_{i1} \oplus X_{i2} \oplus X_{i3} \oplus \ldots \oplus X_{iu} \oplus Y_{j1} \oplus Y_{j2} \oplus \ldots \oplus Y_{jv} = 0$

    X_i = i-th bit of input plaintext [X_1, X_2 , ...]

    Y_j = j-th bit of output ciphertext [Y_1, Y_2, …]

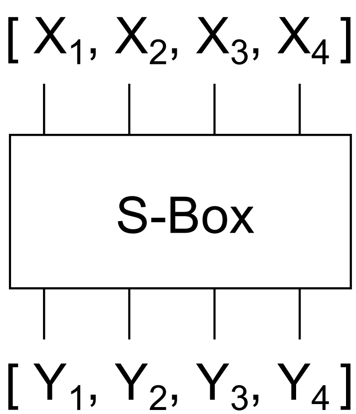
  - for a perfect cipher, such relationships hold with probability 1/2
  - for vulnerable cipher, the probability p might be different from 1/2

  → a bias |p - 1/2| is introduced

Linear Cryptanalysis

- 2 steps
  - analyze the linear vulnerability of a single S-Box
  - connect the output of an S-Box to the input of the S-Box in the next round and “pile up” probability bias

- To analyze a single S-Box, check all possible linear approximations



0	1	2	3	4	5	6	7	8	9	A	B	C	D	E	F
E	4	D	1	2	F	B	8	3	A	6	C	5	9	0	7

Linear Cryptanalysis

X1	X2	X3	X4	Y1	Y2	Y3	Y4	$X1 \oplus X3 \oplus X4 = Y2$	$X2 = Y2 \oplus Y4$
0	0	0	0	1	1	1	0	F	F
0	0	0	1	0	1	0	0	T	F
0	0	1	0	1	1	0	1	T	T
0	0	1	1	0	0	0	1	T	F
0	1	0	0	0	0	1	0	T	F
0	1	0	1	1	1	1	1	T	F
0	1	1	0	1	0	1	1	F	T
0	1	1	1	1	0	0	0	T	F
1	0	0	0	0	0	1	1	F	F
1	0	0	1	1	0	1	0	T	T
1	0	1	0	0	1	1	0	F	F
1	0	1	1	1	1	0	0	T	F
1	1	0	0	0	1	0	1	T	F
1	1	0	1	1	0	0	1	T	T
1	1	1	0	0	0	0	0	T	F
1	1	1	1	0	1	1	1	T	F

Linear Cryptanalysis

- Linear approximations with many true or many false entries are interesting

$p(X1 \oplus X3 \oplus X4 = Y2) = 12/16 = 0.75$ [bias = 0.25]

$p(X2 = Y2 \oplus Y4) = 4/16 = 0.25$ [bias = -0.25]

- How to connect probabilities between different rounds?

consider the following equations, when bias of X1 is b1, and bias of X2 is b2

$$
\begin{aligned}
p(X1 \oplus X2 = 0) &= p(X1)*p(X2) + (1-p(X1))*(1-p(X2)) \\
&= (1/2+b1)*(1/2+b2) + (1/2-b1)*(1/2-b2) \\
&= 1/2 + 2*b1*b2
\end{aligned}
$$

Linear Cryptanalysis

- Now, we show how we can eliminate intermediate variables

$p(X1 \oplus X2 = 0) = 1/2 + b1,2$

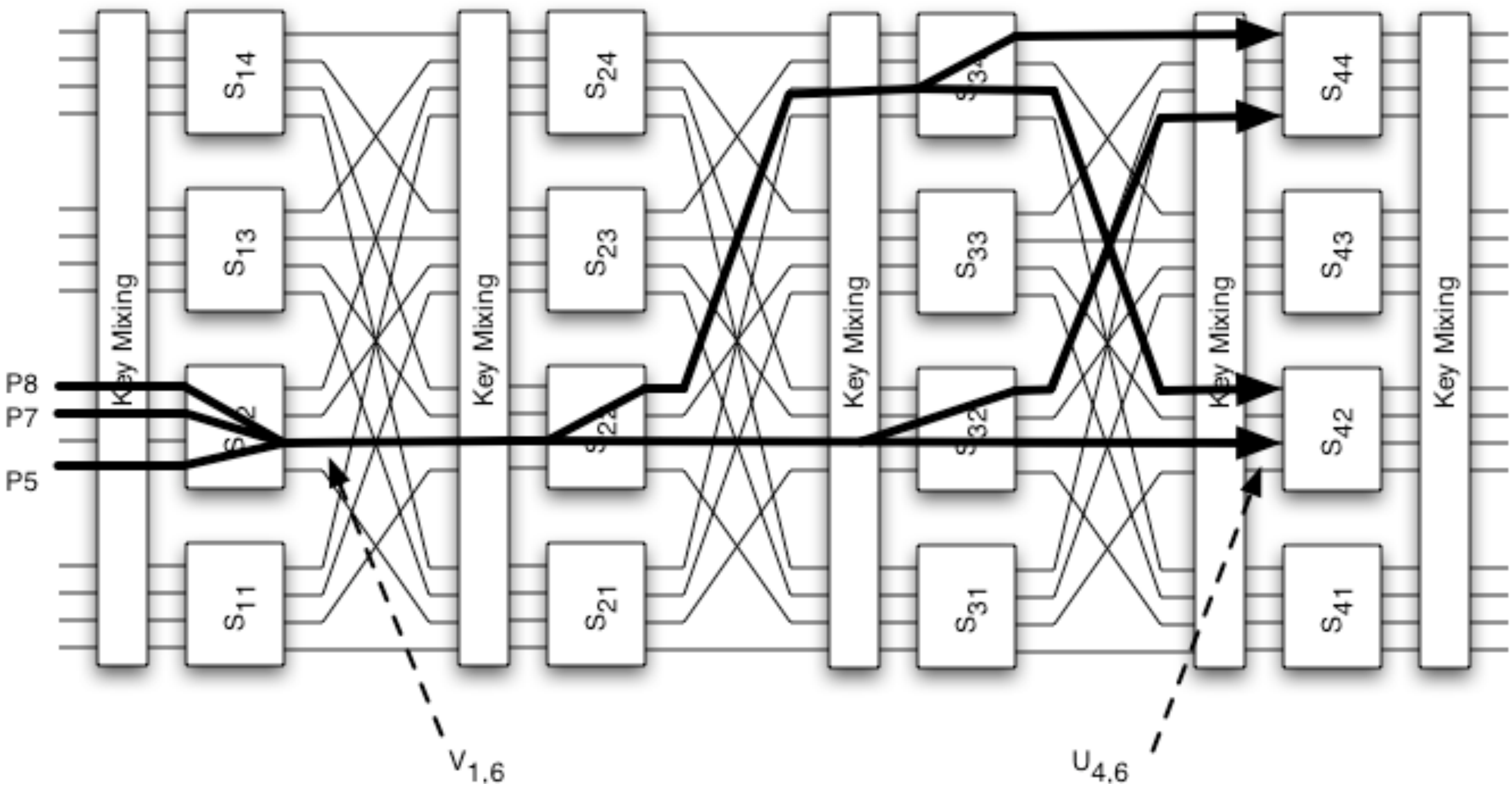
$p(X2 \oplus X3 = 0) = 1/2 + b2,3$

$p(X1 \oplus X3 = 0) = p([X1 \oplus X2] \oplus [X2 \oplus X3] = 0)$

$= 1/2 + 2*b1,2 *b2,3$

- Let $U_i(V_i)$ represent the 16-bit block of bits at the input (output) of the S-Box of round i. Then, let $U_{i,k}$ denote the k-th bit of the i-th round of the cipher. Similarly, let K_i represent the key of round i.

Linear Cryptanalysis



Linear Cryptanalysis

- With probability 0.75 (and bias = 0.25), we have

  $V_{1,6} = U_{1,5} \oplus U_{1,7} \oplus U_{1,8}$

  $= (P_5 \oplus K_{1,5}) \oplus (P_7 \oplus K_{1,7}) \oplus (P_8 \oplus K_{1,8})$

- For the second round, we obtain with probability 0.25 (bias = -0.25)

  $V_{2,6} \oplus V_{2,8} = U_{2,6}$

- Because $U_{2,6} = V_{1,6} \oplus K_{2,6}$ we can connect these two equations and get

  $V_{2,6} \oplus V_{2,8} = (P_5 \oplus K_{1,5}) \oplus (P_7 \oplus K_{1,7}) \oplus (P_8 \oplus K_{1,8}) \oplus K_{2,6}$

  which can be rewritten as

  $V_{2,6} \oplus V_{2,8} \oplus P_5 \oplus P_7 \oplus P_8 \oplus K_{1,5} \oplus K_{1,7} \oplus K_{1,8} \oplus K_{2,6} = 0$

  This holds with a probability (see before) of 1/2 + 2*0.25*(-0.25) = 0.375

Linear Cryptanalysis

- We continue to eliminate intermediate variables in intermediate rounds to obtain

  $U_{4,6} \oplus U_{4,8} \oplus U_{4,14} \oplus U_{4,16} \oplus P_5 \oplus P_7 \oplus P_8 \oplus \sum = 0$

  where $\sum$ is a constant factor (either 0 or 1 that depends on a number of key bits)

  This equation holds with a probability of 15/32 (with a bias of -1/32).

  Because $\sum$ is fixed, we know the following linear approximation of the cipher that holds with probability 15/32 or 17/32 (depending on whether $\sum$ is 0 or 1):

  $U_{4,6} \oplus U_{4,8} \oplus U_{4,14} \oplus U_{4,16} \oplus P_5 \oplus P_7 \oplus P_8 = 0$

Linear Cryptanalysis

- Given an equation that relates the input to the last round of S-Boxes to the plaintext, how can we get the key?
- We attack parts of the key (called target subkey) of the last round, in particular those bits of the key that connect the output of our S-Boxes of interest with the ciphertext

Given the equation $U_{4,6} \oplus U_{4,8} \oplus U_{4,14} \oplus U_{4,16} \oplus P_5 \oplus P_7 \oplus P_8 = 0$, we look at the 8 bits $K_{5,5}$ - $K_{5,8}$ and $K_{5,13}$-$K_{5,16}$

Linear Cryptanalysis

- Idea
  - for a large number of ciphertext and plaintext pairs, we first feed the ciphertext back into the active S-Boxes S_{42} and S_{44}
  - because we do not know the target subkey, we have to repeat this feedback procedure for all possible 256 keys
  - for each subkey, we keep a count on how often the linear equation holds
  - when the wrong subkey is used
    - the equation will hold with probability 1/2 (similar to using random values)
  - when the correct subkey is used
    - the equation will hold with more or less often than 1/2 (depending on the bias)

→ after all pairs of plaintext and ciphertext are checked, we take the subkey with the count that differs most from 1/2