Game Theory in Ad Hoc Networks

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- Our goal : algorithms to take into account selfish behavior on parts of agents.

Plan

- Game Theory and Equilibrium
  - Games and their properties
  - Nash equilibrium
- Repeated Games
  - Iterated Prisoner's Dilemma
  - Randomly matched opponents
- Algorithmic Mechanism Design
  - The problem of mechanism design
  - The VCG Mechanism

Game Theory

Game Theory describes interaction of *selfish* and *rational* individuals.

Two Person Prisoner's Dilemma

A, B	Cooperate	Defect
Cooperate	1, 1	10, 0
Defect	0, 10	8, 8

- Strategy : Cooperate vs Defect
- Payoffs/Utilities : Rewards and punishment
- Dominance

Question: given these strategies and payoffs or utilities, what would a rational selfish person do?

Nash Equilibrium

Nash Equilibrium : strategies for the players such that neither player can improve his payoff by switching strategy unilaterally.

- Not always optimal. (Defect, Defect) is a Nash equilibrium for Prisoner's dilemma.
- Not always unique. Other arguments may be needed to choose between multiple alternatives.
- Declarative, but not a constructive concept
- Pure vs mixed strategy equilibriums
- Problems of Nash Equilibrium
  - Appropriateness of equilibrium analysis
  - Strength of equlibrium vs dominance
  - Multiplicity of equilibrium

D Fudenberg, J. Tirole, *Game Theory*, MIT Press 1991

Questions for Computer Scientists

- Nash Equilibrium
  - What are possible Nash equilibria in distributed systems?
  - What is the computational complexity of Nash equilibria?
- Cost of Anarchy
  - Compared to optimal solution, what is the cost of Nash equilibrium solution?
- Designing Systems for Selfish Users
  - How can we design systems so that we have desirable outcomes?

Repeated Games

- Finite horizon games
  - Backward induction
  - Prisoner's dilemma : non-cooperative equilibrium
- Infinite horizon games
  - Discount factor
  - Prisoner's dilemma strategies
    - Always defect
    - A : alternate cooperate/defect, B: cooperate
    - tit for tat
  - Unlimited number of Nash equilibria

Repeated Games With Many Opponents

- The randomly matched opponents
- Infinite memory
  - Tit For Tat and the Axelrod tournament
  - The white wash problem
- Finite Memory
  - Keep memory of last game's strategy
    - Tit for Tat : hard to distinguish between defection and punishment
  - Keep memory of last game's outcome
    - If last game's outcome was coop-coop, then cooperate, else defect
    - Good equilibrium, but "society" crumbles if anybody defects

Mechanism Design

- A collective decision x that needs to be made
- Set of agents and a principal
- Private information for each agent $\theta_i, u_i = u_i(x, \theta_i)$
- Social choice function $f : \Theta \rightarrow X$, needs to be efficient
- Mechanism consists of strategies s_i and outcome function $g : S \rightarrow X$.
- Focus on mechanisms which rely on
  - revelation
  - incentive compatible $\rightarrow$ truth telling is the equilibrium
  - dominant strategy

Vickrey-Clarke-Groves Mechanism

- Outcome : decision and transfers
- $x = (k, t_1, t_2, \ldots, t_I)$
- Groves : Utilities are quasilinear $u_i(x, \theta_i) = v_i(k, \theta_i) + (m_i + t_i)$
- Clarke : Transfer from i = $t_i(\theta)$ = (Total utility of system except i) - (Total utility of system without i)
  - The Pivotal Mechanism
- Bad news
  - Budget balance : Total transfer = 0 ($\sum_i t_i(\theta) = 0$)
  - No dominant strategy mechanism which is efficient, truthful and budget balanced
- Algorithmic problem : how fast can we compute the social choice function?