8. The standard form of an ODE initial value problem is:

$$\dot{y} = f(t, y), y(t_0) = y_0$$

Express this ODE problem in the standard form.

$$\ddot{y} = t^2 - \dot{y} - z^2 + 1$$

$$\ddot{z} = t + \dot{z} + y^3 - 2$$

where all function values and derivatives are zero at $t = 0$.

9. This is a simpler version of exercise 7.15 in the online version of the book (exercise 7.18 in the SIAM paperback).

Consider a 2-dimensional coordinate system that has a horizontal x-*axis* and a vertical y-*axis*. A cannon standing on a 100-meter cliff, at position $x = 0$ and $y = 100$, fires a cannonball horizontally (in the $+x$ direction, parallel to the x axis) at an initial velociy of $v_0 = 100$ meters per second.

Let θ be the angle between the ball's direction and the x axis (initially zero), let (x, y) be its coordinates, and let v be its velocity. If we pick the units to make all the various constants cancel out, the equations describing the motion of the ball are

$$\dot{x} = v \cos\theta$$

$$\dot{y} = v \sin\theta$$

$$\dot{\theta} = -\frac{\cos\theta}{v}$$

$$\dot{v} = -D - \sin\theta$$

where $D(t) = (\dot{x} + 10)^2 + \dot{y}^2$ is the aerodynamic drag on the ball in a 10 meter per second headwind.

Write Matlab code to plot the path of the ball from time $t = 0$ to time $t = 10$. Your code should include a (short) function to compute the derivatives, and a (short) main program that calls ode45 to solve the equations and then calls "plot" appropriately on the output of ode45.

We won't take off points for minor Matlab errors, but your code should be substantially correct for full credit.

10. Using pencil and paper, take one step of the forward Euler algorithm for the ODE problem

$$\dot{y} = 2y + 1$$

$$y(0) = 1$$

Express your results in terms of an arbitrary step size h.