Solving a system?

$$Ax = b$$

Is dividing by the left the same as dividing by the right?

$Ax = b$ is not the same as $xA = b$.

Careful with the divide, Eugene!

$Ax = b$ is not the same as $xA = b$.

So be careful when you do a division!

$$Ax = b$$
$$A^{-1}Ax = A^{-1}b$$
$$x = A^{-1}b$$

Solving the system

In general, you don't want to calculate an inverse. An inverse might lose information.

$$x = 147/49 = 3$$

$$x = (1/49) * 147 = 3.00000$$

What is wrong here?

The LU Decomposition

$$[L, U, P] = lu(A)$$

L is a lower triangular matrix

U is an upper triangular matrix

P is the permulation matrix, and

$$LU = PA$$

Diagonal systems are easier to solve

$Uy = b$ can be solved by back substitution.

$$\begin{pmatrix} 1 & 2 & 3 \\ 0 & 5 & 2 \\ 0 & 0 & 4 \end{pmatrix} x = \begin{pmatrix} 3 \\ 4 \\ 6 \end{pmatrix}$$

Similarly, $Lx = b$ can be solved by forward substitution.

Gaussian Elimination

- Problem

  $2x_1 + 1x_2 + 1x_3 = 4$

  $1x_1 + 4x_2 + 1x_3 = 6$

  $2x_1 + 2x_2 + 6x_3 = 10$

- Step one:

  $A(2,:) = A(2,:) - A(1,:)/2$

  $2x_1 + 1x_2 + 1x_3 = 4$

  $0x_1 + 7/2x_2 + 1/2x_3 = 4$

  $2x_1 + 2x_2 + 6x_3 = 10$

- Step three:
  $A(3,:) = A(3,:)/2 - A(1,:)/2$

  $$2x_1 + 1x_2 + 1x_3 = 4$$
  $$0x_1 + 7/2x_2 + 1/2x_3 = 4$$
  $$0x_1 + 1/2x_2 + 5/2x_3 = 3$$

- Step four:
  $A(3,:) = A(3,:) * 2 - A(2,:) * 2/7$

  $$2x_1 + 1x_2 + 1x_3 = 4$$
  $$0x_1 + 7/2x_2 + 1/2x_3 = 4$$
  $$0x_1 + 0x_2 + 34/7x_3 = 34/7$$

Phew!

But wait!

The A looks suspiciously familiar! Indeed, $U = A$ after doing these steps. So in effect, Gaussian Elimination is doing an LU decomposition. We can get the L as well, if we keep the constants used to obtain U in this fashion. The permutation matrix P, helps us pivot on the correct value, to ensure numerical stability.

Now what?

To solve $Ax = b$, do the following

- $[L, U, P] = lu(A)$
- $Ly = Pb$
- $Ux = y$

The two triangular solves are very easy, and most of the work is in the LU decomposition.