

## CS110a SAMPLE 2005 Final

Name:

Perm No:

Open book and notes, no computers. Raise your hand if you have any doubt.

1. What is the output of the following Matlab code?

```
format long e
x = (1/10) .^ (1:20)
y = 100 + x - 100
```

2. Find the LU Factorization of:

$$A = \begin{pmatrix} 0 & 1 & 2 \\ 1 & 2 & 1 \\ 2 & 1 & 0 \end{pmatrix}$$

3. Consider this program:

```
n = fix(10 * rand)
A = golub(n);
y = rand(n,1);
b = A * y;
x = A \ b;
r = b - A * x;
e = x - y;
```

Match these quantities to the numerical values in the list that follows:

n =

norm(A) =

cond(A) =

norm(y) =

norm(b) =

norm(x) =

norm(r) =

norm(e) =

5.9365e+009  
468.0244  
241.8655  
6  
1.3183  
1.3183  
1.4007e-008  
5.1369e-014

4. Let  $f(x) = 2x^2 - 9x + 8$

(a) Sketch the graph of  $f(x)$  for  $1 \leq x \leq 3$

(b) Take the first two steps of the secant step, starting at point  $x=2$ , to find the zero of the function.

**5.** Problem 5 in the final from year 2004. Match each plot statement to the graph that it produces. Look up the Final in 2004 (on webpage) for the question.

**6.** Take one step of the BS23 algorithm for the initial value problem:

$$\begin{aligned} \dot{y} &= -y, \\ y(0) &= 1 \end{aligned}$$

7.

(a) Suppose  $A$  is an  $n$  by  $n$  symmetric, positive definite matrix and  $b$  is a column vector of dimension  $n$ . Recall that the Cholesky factor  $R$  of  $A$  is the upper triangular matrix such that  $R' * R = A$ . Fill in the blanks below to compute the solution  $x$  to  $A * x = b$ . You may use  $R$  but not  $A$ .

$$R = \text{chol}(A);$$

$$y = \text{-----} \setminus \text{-----} ;$$

$$x = \text{-----} \setminus \text{-----} ;$$

(b) Now suppose in addition that  $p$  is a permutation vector, that is, a vector whose elements are the integers 1 through  $n$  in some order. Fill in the blanks to compute the solution  $x$  to  $A * x = b$ . Again you may not use  $A$ .

$$R = \text{chol}(A(p,p));$$

$$\text{-----} = \text{-----} \setminus \text{-----} ;$$

$$\text{-----} = \text{-----} \setminus \text{-----} ;$$

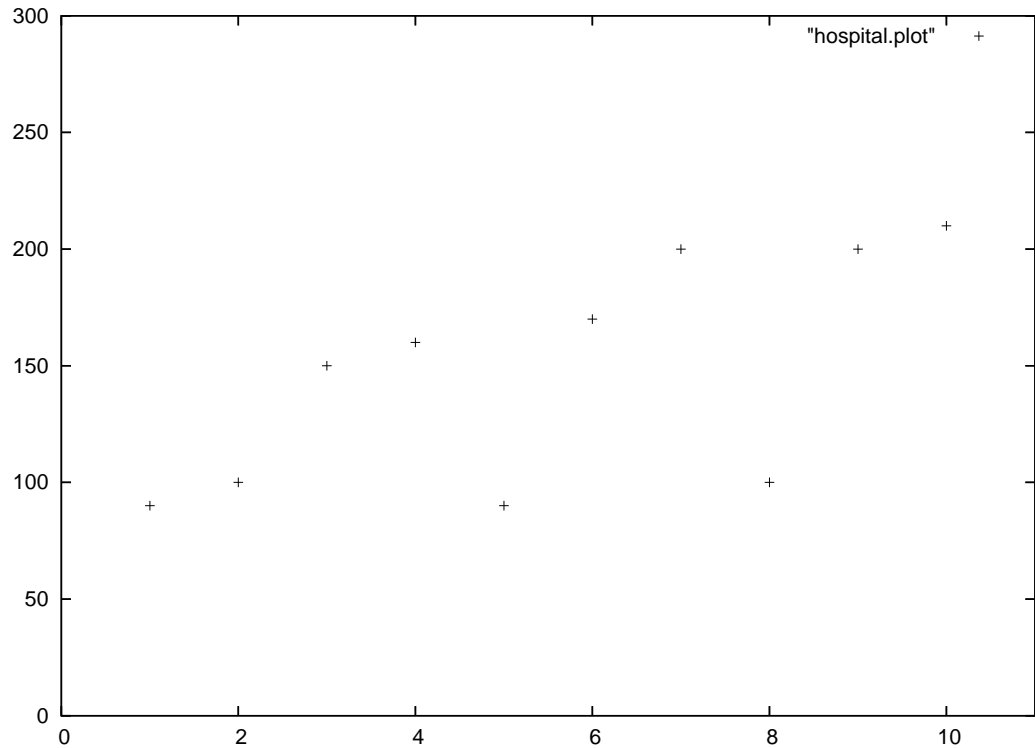


Figure 1: Number of patients in different years.

**8.** Multiple choice: Identify the correct method to solve the problem. Your choices are: Interpolation, ODEs, Least Squares, Zero finding: Newton's method, Zero finding: Secant method.

**8.1** There is a hospital to be built in Goleta to replace the current hospital. The existing hospital has monthly records on how many patients were admitted in the last 15 years, and the new hospital should be able to handle enough people for the next 10 years atleast. The data is given in the figure above. What method would you use?

**8.2** North Eastern Hardware Agents sells large quantities of wooden planks, which they shape from raw wood. They have a record of how many planks they have sold over the last year, but someone lost the account books for June, so they don't have data for June. How can they estimate the amount of wood sold in June?

**8.3** Miss N.E. Harvey has a lot of money invested in various avenues. To calculate how much money she has at the moment, she has written a Matlab program which takes as input the value of interest rate, and outputs the total value of her portfolio.

```
program value = networth(interest)...
```

She doesn't allow us to see the program, but she would like to know at what value of interest would the value of her portfolio be exactly one million, so she can call herself a millionaire. What should we do?

**8.4** N.E.H. Avionics has an experimental aircraft, which they plan to test for very long distance travel. If the flight is travelling without refuelling, it must carry on itself a lot of fuel, which is heavy. The rate of consumption of fuel depends on the velocity, but also on the weight of the plane, which is largely composed of the amount of fuel it is carrying. They found out that the rate of consumption of fuel is equal to  $3v^2 + 21f$  where:

$v$ : velocity of travel (a constant)

$f$ : amount of fuel

We are given the capacity of the fuel tank, which is filled completely at the start of the experiment, and we want to find out how much fuel is left after time  $t$ . How do we solve it?

**8.5** A balloon filled with gas shoots forward when air is allowed to escape from it. The velocity of the balloon is dependant on the air resistance, which is dependent on the density of the air, and on the weight of the balloon. In a large balloon, the weight of the gas inside the balloon is a significant portion of the total weight of the balloon. If you were to find the amount of gas left in the balloon after time  $t$ , what method would you use?



9. The standard form of an ODE initial value problem is:

$$\dot{y} = f(t, y), y(t_0) = y_0$$

Express this ODE problem in the standard form.

$$\frac{d^3 u}{dt^3} = \frac{v}{1+t^2} - \sin r$$

$$\frac{dv}{dt} = \frac{-u}{1+t^2} + \cos r$$

where  $r = \sqrt{\dot{u}^2 + \dot{v}^2}$ . The initial conditions are:

$$u(0) = 1, v(0) = 3, \frac{du}{dt}(0) = 5, \frac{d^2 u}{dt^2}(0) = 12$$

**10.** Given 5 points  $(t_1, y_1), \dots, (t_5, y_5)$ , write Matlab code to find the quadratic  $at^2 + bt + c$  that is the best fit in the least-squares sense. If you would like to use numerical values instead of  $t_i$  and  $y_i$ , then choose  $(12, 2), (13, 5), (15, 10), (20, 15), (25, 30)$