CS110a 2005 Final

`Name:`

`Perm No:`

Open book and notes, no computers. Raise your hand if you have any doubt. There are 10 problems, each of 10 points.

1. Let x, p, and A be defined by the following Matlab statements:

```
x = [27 18 28 18];
p = [3 4 2 1];
A = [1 2 3 ; 4 5 6; 7 8 9; 10 11 12];
```

What is the value of ...

(a) `B` after: `B = A(p,:)`

(b) `y` after: `y = x(p)`

(c) `y` after: `y(p) = x`

(d) `z` after: `z = A * ones(3,1)`

(e) `A` after: `A(3:4, 2:3) = [x(1:2) ; x(3:4)]`

2. (a) Let `A = [-1 0 3 ; 1 6 8 ; 2 4 4]`
Show the result of LU factorization of A with partial pivoting, so `A(p,:) = L * U`.

`L =`

`U =`

`p =`

(b) Let `A = [2 2 6 ; 3 3 3 ; 2 4 0]`
Show the result of LU factorization of A with complete pivoting, so `A(p,q) = L * U` .

`L =`

`U =`

`p =`

`q =`

3. Norman, Evan, and Hans work in the AA grocery shop. They are all paid the same amount per hour. Sometimes they take home groceries instead of pay.

Last week, Norman worked 18 hours and took home 2 loaves of bread and one box of fruit. Evan worked 12 hours and took home 3 loaves of bread and 4 boxes of fruit. Hans worked 20 hours and took home one box of fruit. At the end of the week, Norman was paid \$344, Evan \$196, and Hans \$392.

We want to find out three things: the pay per hour, the cost of a loaf of bread, and the cost of a box of fruit. Write down the equations you would solve, and the Matlab code to solve them. (This should be at most a few lines of Matlab.) You don't have to actually solve the equations, but explain clearly which outputs correspond to which unknowns.

4. How many times does each of the following loops iterate? (Full credit if you're within 20% of the right answer.)

```
(a) x = 1; while x-1 < x; x = 100*x; end;
```

```
(b) x = 1; while x+x > x; x = 100*x; end;
```

```
(c) x = 1; while x+x > 0; x = 100*x; end;
```

5. Choose the right method to solve the following problems. Your choices are: Interpolation, Least Squares, Zero finding (specify which method: Bisection, Newton's method, Secant method), ODEs. In some cases, more than one technique may be required, so mention both, clearly showing the order in which they are used. eg. Least Squares, and then Interpolation.

(a) The NE Heat Arrester company fights forest fires. A fire has spread in a huge redwood forest and it is burning very rapidly. To model the fire, the engineers notice that the rate of depletion of trees depends on the amount of oxygen in the air, and the number of trees in the vicinity. Also, since the fire consumes oxygen, the rate of depletion of oxygen also depends on the amount of trees in the vicinity. How do they model this sort of scenario?

(b) The Nigerian Email Handlers Association sends out marketing information via email. They want to buy new servers for sending out more mail, and want to be able to handle the increasing traffic. They have records of how many mails were sent in the past months, and would like to use this data to guess how many mails they will be sending in the future. What technique do they use?

(c) The New Epidemic and Health Agency is growing a bacteria in their laboratory. The growth rate of the bacteria depends on the existing number of bacteria cells, and the amount of sugar solution that is in the test-tubes. A scientist has put a few cells and some sugar solution in a test-tube, and is waiting for the test-tube to contain 1 million cells. Assuming that the equations of population growth are known, what method is used to find out when the test-tube will contain 1 million cells?

(d) A single electron is shot in a 2-D electrically charged field. Miss NE Hansa has set up equipment to take fifty photographs of the motion of the electron through this field. The motion of the electron is in two dimensions only, so the photographs capture exactly the position of the electron at the particular time. For her talk, she would like to show a continuous movie of the motion of the electron. What method would she use?

(e) A particular physicist is trying to model the behavior of sunspots. He has data on the number of sunspots over a 1 year period, and he would like to approximate this so that he can predict occurances of sunspots in the future.

6. Suppose we want to compute the cube root of 5 by finding a zero of the function $f(x) = x^3 - 5$. It is easy to see that the zero lies between $x = 1$ and $x = 2$.

(a) Sketch the graph of the function

(b) Take one step of the secant method, starting at x=1 and x=2, to find an approximation to the zero.

(c) Take one step of Newton's method, starting at x=2, to find an approximation to the zero.

7. You are given the following data:

time	*Temp*
0.1	1
2.2	14
4.3	20
6.3	32
8.3	45
10.5	45
12.1	60

This is the temperature T at time t inside a pipe that is being heated from outside, and also being cooled and heated periodically by liquid flowing through it.

We want to fit a curve of the form:

$$T = \alpha t + \beta sin(t)$$

Write down the matrices involved, and write down Matlab code to solve this problem and find α and β.

8. The standard form of an ODE initial value problem is:

$$\dot{y} = f(t, y), y(t_0) = y_0$$

Express this ODE problem in the standard form.

$$\ddot{y} = t^2 - \dot{y} - z^2 + 1$$

$$\ddot{z} = t + \dot{z} + y^3 - 2$$

where all function values and derivatives are zero at $t = 0$.

9. This is a simpler version of exercise 7.15 in the online version of the book (exercise 7.18 in the SIAM paperback).

Consider a 2-dimensional coordinate system that has a horizontal x-*axis* and a vertical y-*axis*. A cannon standing on a 100-meter cliff, at position $x = 0$ and $y = 100$, fires a cannonball horizontally (in the $+x$ direction, parallel to the x axis) at an initial velociy of $v_0 = 100$ meters per second.

Let θ be the angle between the ball's direction and the x axis (initially zero), let (x, y) be its coordinates, and let v be its velocity. If we pick the units to make all the various constants cancel out, the equations describing the motion of the ball are

$$\dot{x} = v \cos\theta$$

$$\dot{y} = v \sin\theta$$

$$\dot{\theta} = -\frac{\cos\theta}{v}$$

$$\dot{v} = -D - \sin\theta$$

where $D(t) = (\dot{x} + 10)^2 + \dot{y}^2$ is the aerodynamic drag on the ball in a 10 meter per second headwind.

Write Matlab code to plot the path of the ball from time $t = 0$ to time $t = 10$. Your code should include a (short) function to compute the derivatives, and a (short) main program that calls ode45 to solve the equations and then calls "plot" appropriately on the output of ode45.

We won't take off points for minor Matlab errors, but your code should be substantially correct for full credit.

10. Using pencil and paper, take one step of the forward Euler algorithm for the ODE problem

$$\dot{y} = 2y + 1$$

$$y(0) = 1$$

Express your results in terms of an arbitrary step size h.