Graphs and Matrices

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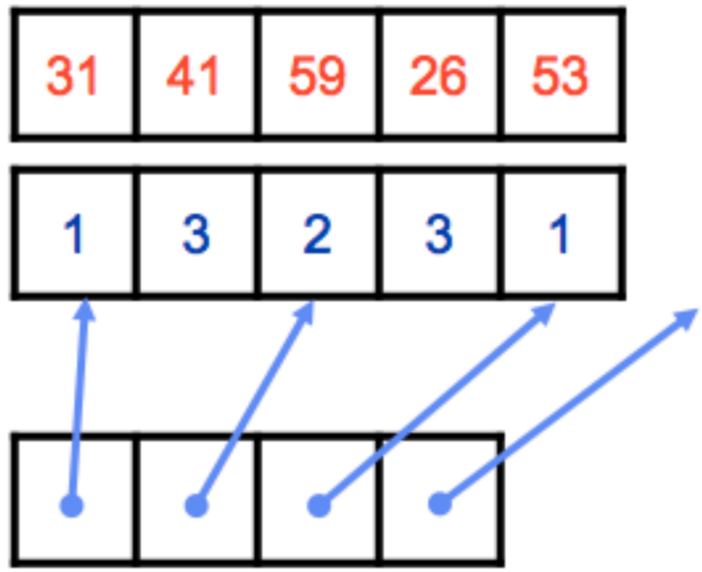
Sparse matrix data structure (one example)

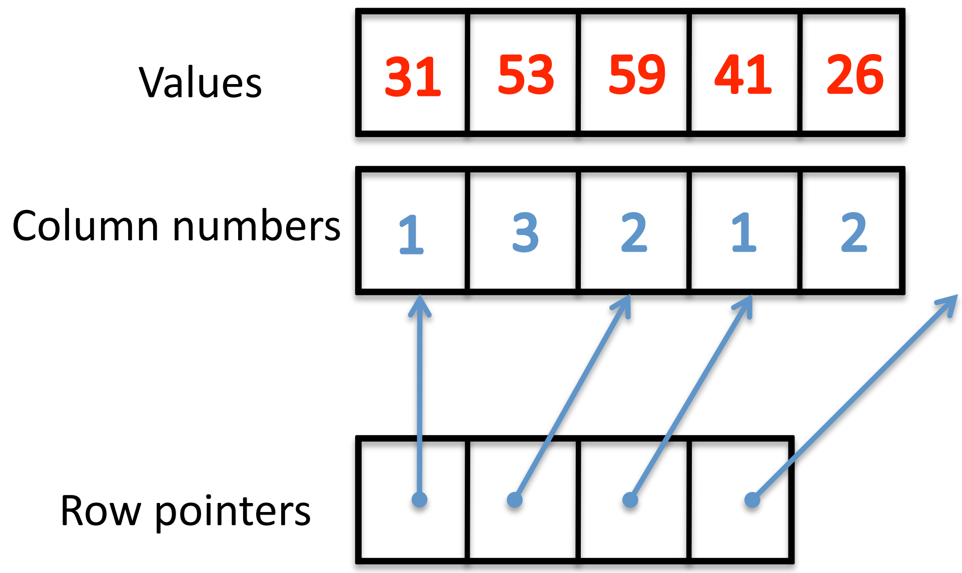
31	0	53
0	59	0
41	26	0



- Full:
  - 2-dimensional array of real or complex numbers
  - (nrows*ncols) memory
- Sparse:
  - compressed column storage (CSC)
  - about (2*nzs + ncols) memory

Compressed Sparse Row (CSR)

31	0	53
0	59	0
41	26	0



Triplet Form

31	0	53
0	59	0
41	26	0

I	J	V
1	1	31
1	3	53
2	2	59
3	1	41
3	2	26

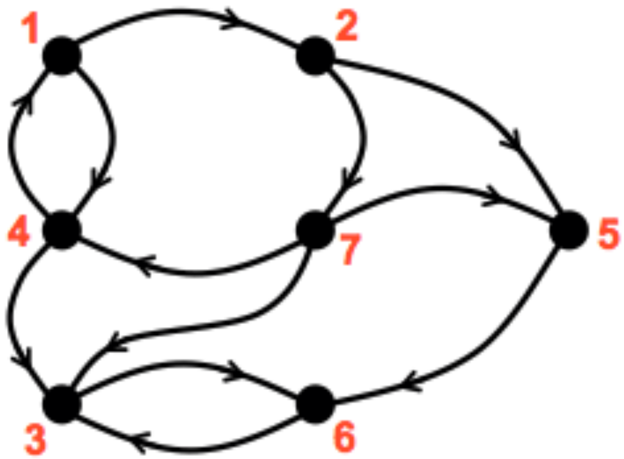
About 3*nz memory

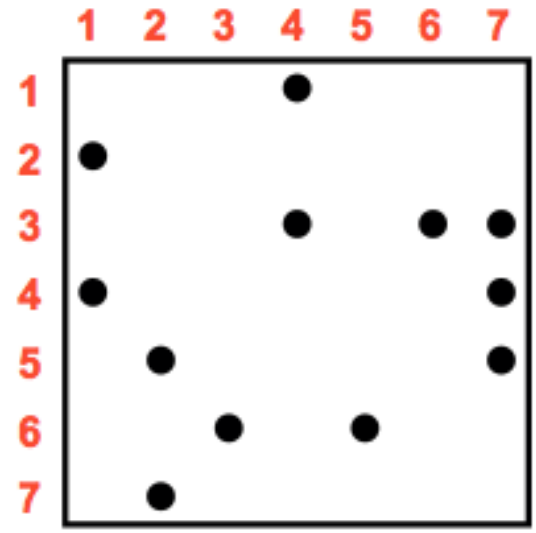
Other formats

- List of Lists
  - List of rows, each element is a list of column indices
    - values stored similarly
- Dictionary of Keys
  - (row, column) → value
- Compressed Sparse Blocks
  - Dense matrix of blocks
    - Each block has elements in Morton Z order
  - $A * X$ and $A^T * X$ takes same time

Graphs and Matrices

Graph



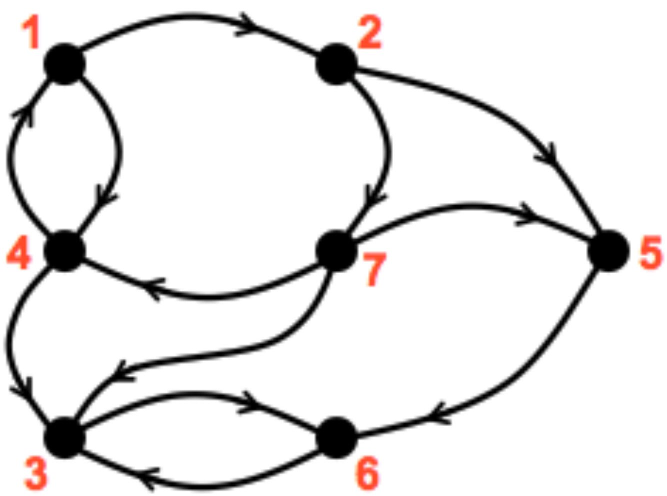


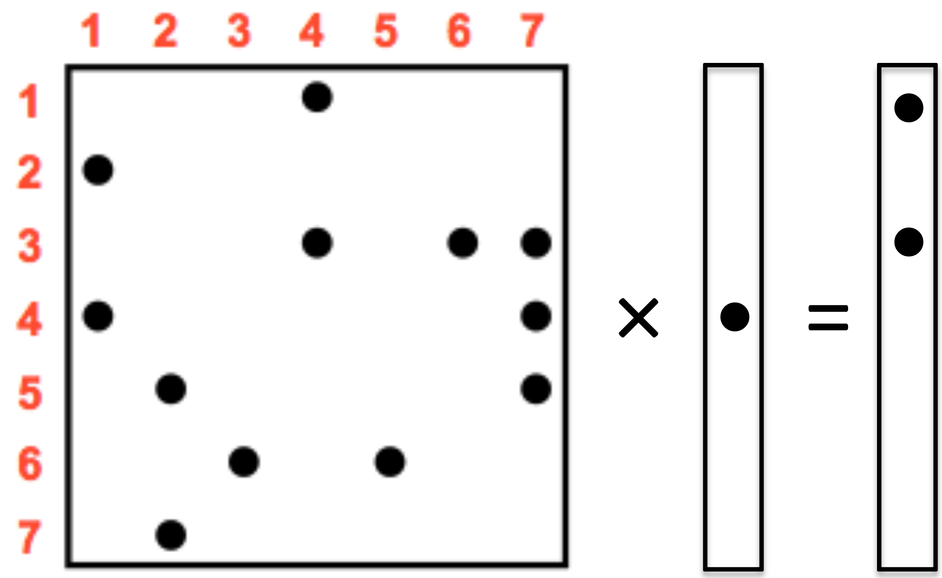
Matrix

- Starting with the matrix:
  - One graph vertex for each row (or column) of the matrix
  - One graph edge (i,j) for each nonzero A(j,i) in the matrix
  - (Some people point the edges the opposite way, from rows to columns; either way is ok as long as it's consistent.)
- Or, starting with the graph:
  - The adjacency matrix has A(j,i)=1 if (i,j) is an edge.

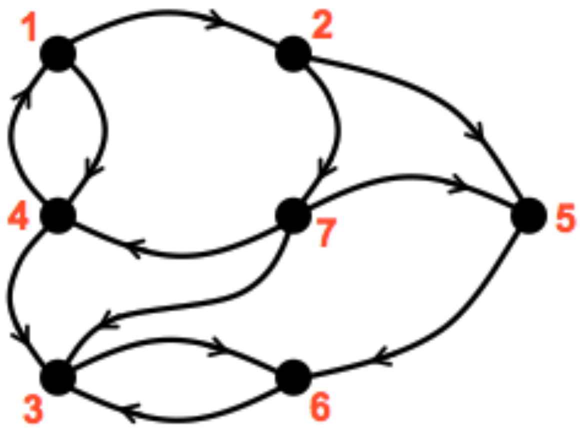
- What if you interpret a CSC as a CSR?
- What does a matrix permutation mean?
- What does matrix-vector multiplication mean?

SpMV – Matrix-Vector multiply





Breadth First Search - levels



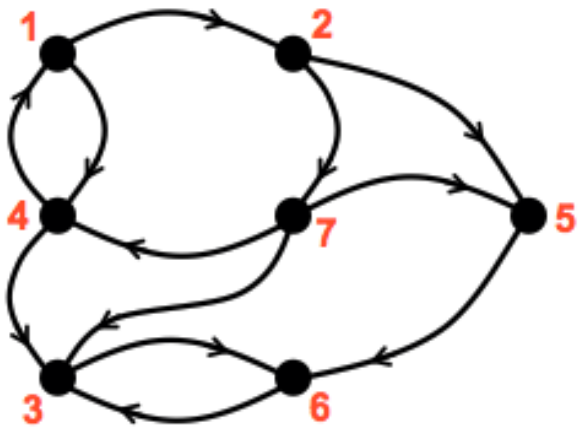
$$\begin{bmatrix} \mathbf{0} & \infty & \infty & \mathbf{1} & \infty & \infty & \infty \\ 1 & 0 & \infty & \infty & \infty & \infty & \infty \\ \infty & \infty & 0 & 1 & \infty & 1 & 1 \\ 1 & \infty & \infty & 0 & \infty & \infty & 1 \\ \infty & 1 & \infty & \infty & 0 & \infty & 1 \\ \infty & \infty & 1 & \infty & 1 & 0 & \infty \\ \infty & 1 & \infty & \infty & \infty & \infty & 0 \end{bmatrix} \times \begin{bmatrix} \ \\ \ \\ \ \\ 0 \\ \ \\ \ \\ \ \end{bmatrix} = \begin{bmatrix} \mathbf{1} \\ \ \\ 1 \\ 0 \\ \ \\ \ \\ \ \end{bmatrix}$$

SpMV over Min-Plus Semiring

Min-Plus Semiring:
addition => min
multiplication => plus

Starting vertex has distance zero
Each SpMV finds the next frontier

Breadth First Search – parents



$$
\overset{A}{\begin{bmatrix}
 & & & \mathbf{1} & & & \\
1 & & & & & & \\
 & & & 1 & & 1 & 1 \\
1 & & & & & & 1 \\
 & 1 & & & & & 1 \\
 & & 1 & & 1 & & \\
 & 1 & & & & &
\end{bmatrix}}
\times
\overset{x}{\begin{bmatrix} \\ \\ \\ 4 \\ \\ \\ \\ \end{bmatrix}}
=
\overset{b}{\begin{bmatrix} \mathbf{4} \\ \\ 4 \\ \\ \\ \\ \\ \end{bmatrix}}
$$

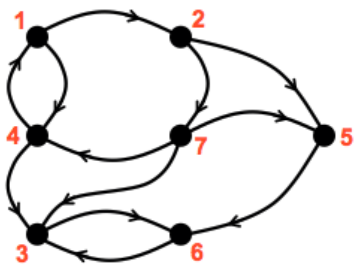
SpMV over Select-Max Semiring

Select-Max Semiring:
addition => max
multiplication => max

nonzero values in x are starting frontier
value=index

After SpMV, a nonzero in b contains index of corresponding vertex' parent

Breadth First Search – parents



1st Frontier:

$$\overset{A}{\begin{bmatrix} & & & 1 & & & \\ 1 & & & & & & \\ & & & 1 & & 1 & 1 \\ 1 & & & & & & 1 \\ & 1 & & & & & 1 \\ & & 1 & & 1 & & \\ & 1 & & & & & \end{bmatrix}} \times \overset{x}{\begin{bmatrix} \ \\ \ \\ \ \\ 4 \\ \ \\ \ \\ \ \end{bmatrix}} = \overset{b}{\begin{bmatrix} \mathbf{4} \\ \ \\ 4 \\ \ \\ \ \\ \ \\ \ \end{bmatrix}} \qquad \overset{parent}{\begin{bmatrix} 4 \\ \ \\ 4 \\ \bullet \\ \ \\ \ \\ \ \end{bmatrix}}$$

2nd Frontier:

$$\begin{bmatrix} & & & 1 & & & \\ 1 & & & & & & \\ & & & 1 & & 1 & 1 \\ 1 & & & & & & 1 \\ & 1 & & & & & 1 \\ & & 1 & & 1 & & \\ & 1 & & & & & \end{bmatrix} \times \begin{bmatrix} \mathbf{1} \\ \ \\ 3 \\ \ \\ \ \\ \ \\ \ \end{bmatrix} = \begin{bmatrix} \ \\ 1 \\ \ \\ 1 \\ \ \\ 3 \\ \ \end{bmatrix} \qquad \begin{bmatrix} \mathbf{4} \\ 1 \\ 4 \\ \bullet \\ \ \\ 3 \\ \ \end{bmatrix}$$