CS 111: Review Quiz: Answer key

1. $$A^T = \begin{pmatrix} 3 & 0 & 1 \\ -1 & 1 & 0 \\ 2 & 2 & -1 \end{pmatrix}, A^2 = \begin{pmatrix} 11 & -4 & 2 \\ 2 & 1 & 0 \\ 2 & -1 & 3 \end{pmatrix}, A^T A = \begin{pmatrix} 10 & -3 & 5 \\ -3 & 2 & 0 \\ 5 & 0 & 9 \end{pmatrix}.$$

2. $||(3, 1, 4, 1, 5)^T||_2 = \sqrt{52} \approx 7.2111$

3. $$\begin{pmatrix} 2 & -3 & 1 \\ 0 & 2 & 3 \\ 1 & 0 & 1 \end{pmatrix} \times \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 1 \\ 7 \\ 4 \end{pmatrix}$$

4. $$x = \begin{pmatrix} 3 \\ 2 \\ 1 \end{pmatrix}$$

5. There are many answers to this. Here's one: In Matlab notation, take $A = [1\ 2; 2\ 4]$ and $b = [3; 3]$. Explanation 1 (column view): Matrix A is singular, so the space spanned by its columns is only one-dimensional, and it consists of multiples of the vector $[1; 2]$, which do not inclue b. Explanation 2 (row view): The two lines described by the row form of $Ax = b$ are parallel and hence do not intersect. Explanation 3 (brute force view): No matter what x is, the second entry of Ax will be equal to twice the first entry of Ax, which rules out b.

6. There are many answers to this. Here's one: In Matlab notation, take $A = [1\ 2; 2\ 4]$ and $b = [3; 6]$. Two solutions are $x = [1; 1]$ and $x = [3; 0]$.

7. No, its not possible to have exactly two solutions to $Ax = b$. If x and y are two different solutions, then there are infinitely many solutions: $x + \alpha(y - x)$ is a solution for every α.

8. A has two eigenvalues, 3 and 5. Any multiple of $[1; 1]$ is an eigenvector corresponding to 3, and any multiple of $[1; -1]$ is an eigenvector corresponding to 5.

9. $f'(x) = 21x^2 - 4x + 4$.

10. $\partial z/\partial x = e^{y/2}$, and $\partial z/\partial y = (x/2)e^{y/2}$.

11. $f(x) = x^3/3 - \cos x + c$ for some constant c (any constant will do).

12. The height is maximum when the derivative dh/dt is zero. $dh/dt = 1280 - 32t$, which is zero when $t = 40$, at which time the height is $h = 25600$ feet. The bullet hits the ground when $h = 0$, which means $1280t = 16t^2$, which means $t = 1280/16 = 80$ seconds after firing. (The other solution to $h = 0$ is of course $t = 0$.)

13. $y = e^{x^2/2}$.