

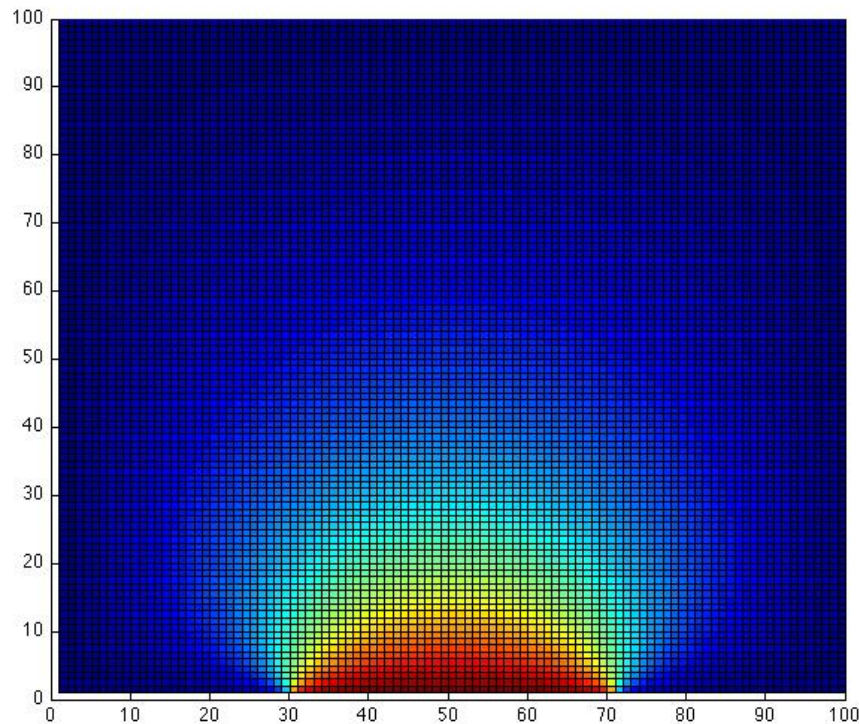
Example: The Temperature Problem

- A cabin in the snow
- Wall temperature is 0° , except for a radiator at 100°
- What is the temperature in the interior?



Example: The Temperature Problem

- A cabin in the snow (the unit square ☺)
- Wall temperature is 0° , except for a radiator at 100°
- What is the temperature in the interior?



The physics: Poisson's equation

$$\nabla^2 u(x, y) \equiv \frac{\partial^2 u}{\partial x^2}(x, y) + \frac{\partial^2 u}{\partial y^2}(x, y) = f(x, y)$$

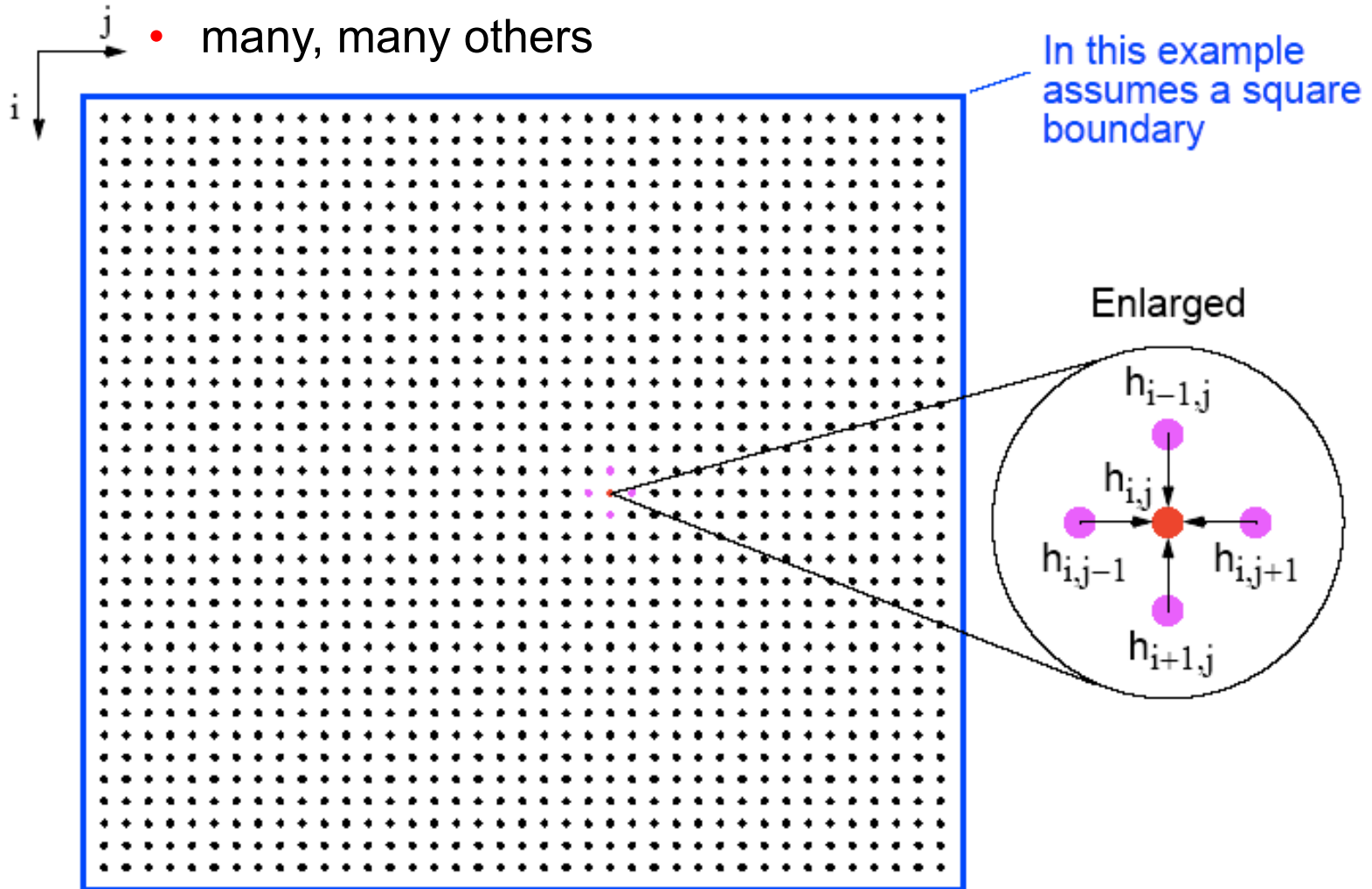
for $(x, y) \in R = \{ (x, y) \mid a < x < b, \ c < y < d \}$, and

$$u(x, y) = g(x, y)$$

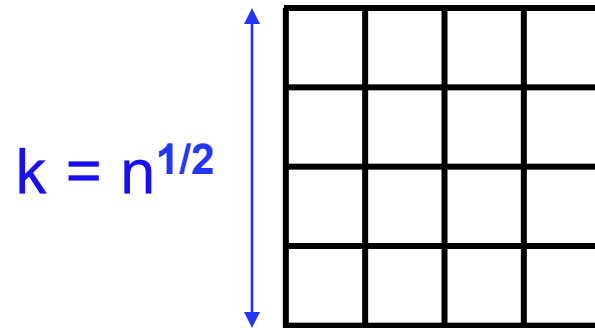
for (x, y) on the boundary of R .

Many Physical Models Use Stencil Computations

- PDE models of heat, fluids, structures, ...
- Weather, airplanes, bridges, bones, ...
- Game of Life
- many, many others



Model Problem: Solving Poisson's equation for temperature



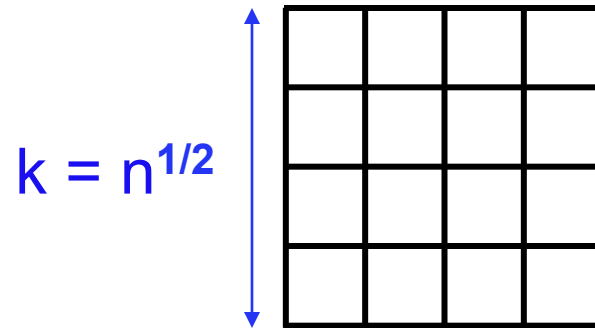
- Discrete approximation to Poisson's equation:

$$t(i) = \frac{1}{4} (t(i-k) + t(i-1) + t(i+1) + t(i+k))$$

- Intuitively:

Temperature at a point is the average
of the temperatures at surrounding points

Model Problem: Solving Poisson's equation for temperature



- For each i from 1 to n , except on the boundaries:
$$-t(i-k) - t(i-1) + 4t(i) - t(i+1) - t(i+k) = 0$$
- n equations in n unknowns: $A^*t = b$
- Each row of A has at most 5 nonzeros
- In three dimensions, $k = n^{1/3}$ and each row has at most 7 nzs

A Stencil Computation Solves a System of Linear Equations

- Solve $Ax = b$ for x
- Matrix A , right-hand side vector b , unknown vector x
- A is *sparse*: most of the entries are 0

The diagram illustrates the system of linear equations $Ax = b$ for a stencil computation. The matrix A is shown as a large vertical column with a dashed diagonal line representing the main diagonal. A specific row, labeled " i th equation" in purple, is highlighted with a box. The entries in this row are: 1 (above $a_{i,i-n}$), 1 (above $a_{i,i-1}$), -4 (above $a_{i,i}$), 1 (above $a_{i,i+1}$), and 1 (above $a_{i,i+n}$). The vector x is shown as a vertical column with entries $x_1, x_2, \dots, x_{N-1}, x_N$. The right-hand side vector b is shown as a vertical column with entries $0, 0, \dots, 0, 0$. Annotations include: "Those equations with a boundary point on diagonal unnecessary for solution" pointing to the top-left corner of A ; "To include boundary values and some zero entries (see text)" pointing to the top of x and b ; and a multiplication sign \times and equals sign $=$ between the vectors.