Single Step Methods

$$
\begin{aligned}
y_{n+1} &= y_n + hf(t_n, y_n) \\
t_{n+1} &= t_n + h
\end{aligned}
$$

$$
\begin{aligned}
s_1 &= f(t_n, y_n) \\
s_2 &= f\left(t_n + \frac{h}{2}, y_n + \frac{h}{2} s_1\right) \\
y_{n+1} &= y_n + h s_2 \\
t_{n+1} &= t_n + h
\end{aligned}
$$

$$
\begin{aligned}
s_1 &= f(t_n, y_n) \\
s_2 &= f(t_n + h, y_n + h s_1) \\
y_{n+1} &= y_n + h \frac{s_1 + s_2}{2} \\
t_{n+1} &= t_n + h
\end{aligned}
$$

Classical Runge-Kutta

$$
\begin{aligned}
s_1 &= f(t_n, y_n) \\
s_2 &= f(t_n + \frac{h}{2}, y_n + \frac{h}{2} s_1) \\
s_3 &= f(t_n + \frac{h}{2}, y_n + \frac{h}{2} s_2) \\
s_4 &= f(t_n + h, y_n + h s_3) \\
y_{n+1} &= y_n + \frac{h}{6}(s_1 + 2s_2 + 2s_3 + s_4) \\
t_{n+1} &= t_n + h
\end{aligned}
$$

$$s_i = f(t_n + \alpha_i h, y_n + h \sum_{j=1}^{i-1} \beta_{i,j} s_j)$$

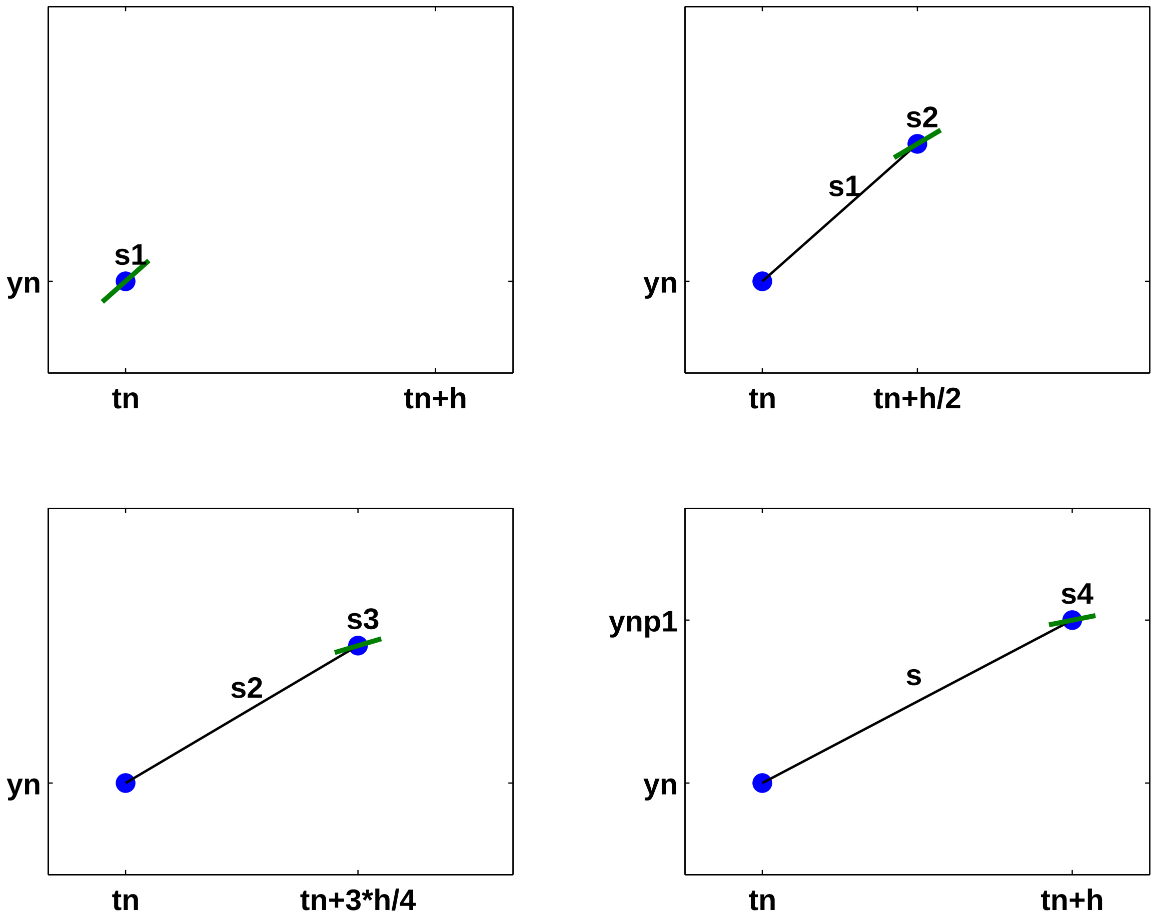
$$i = 1, \ldots, k$$

$$y_{n+1} = y_n + h \sum_{i=1}^{k} \gamma_i s_i$$

$$e_{n+1} = h \sum_{i=1}^{k} \delta_i s_i$$

The BS23 algorithm

$$\begin{aligned}
s_1 &= f(t_n, y_n) \\
s_2 &= f(t_n + \frac{h}{2}, y_n + \frac{h}{2} s_1) \\
s_3 &= f(t_n + \frac{3}{4} h, y_n + \frac{3}{4} h s_2) \\
t_{n+1} &= t_n + h \\
y_{n+1} &= y_n + \frac{h}{9}(2s_1 + 3s_2 + 4s_3) \\
s_4 &= f(t_{n+1}, y_{n+1}) \\
e_{n+1} &= \frac{h}{72}(-5s_1 + 6s_2 + 8s_3 - 9s_4)
\end{aligned}$$


s1
yn
tn
tn+h
s2
s1
yn
tn
tn+h/2
s3
s2
yn
tn
tn+3*h/4
s4
ynp1
s
yn
tn
tn+h