Symmetric Weighted Matching and Application to Indefinite Multifrontal Solvers

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Introduction

We are interested in using multifrontal methods in the LDL^T factorization of symmetric indefinite matrices where L is a lower triangular matrix and D is a block diagonal matrix with 1x1 and 2x2 blocks.

For the LU factorization of a matrix A, `MC64` ([4]) can be applied to get a maximum weighted matching with large entries on the diagonal and row/column scaling factors in order to improve the numerical stability of the factorization. Then an ordering (for example [1]) can be computed on the permuted matrix to get a symmetric permutation in order to decrease the fill-in in the factors. After these two steps, we need to factorize $P^T D_r A D_c Q P$ where D_r and D_c are the scaling factors, Q is the permutation returned by `MC64`, and P is a symmetric permutation. If we apply these two steps to a symmetric matrix we lose the symmetry. That is why we are interested in the development of a robust multifrontal approach which has to address numerical problems and sparsity conservation in the symmetric case.

We started from three existing codes : two symmetric multifrontal codes `MA47` ([5]) and `MA57` ([3]) to validate our preprocessing heuristics on real test problems and `MC64`.

Firstly we will describe our `MC64` modifications to get a symmetric scaling and a set of 2x2 and 1x1 numerically acceptable pivots. In a second stage we modified the ordering and factorization phases of `MA47` and `MA57` to take into account the previous decisions. Finally we tested different strategies on three sets of matrices, structurally singular augmented systems, structurally nonsingular augmented systems and general indefinite matrices.

1 MC64SYM, a modification of MC64

After computing an `MC64` maximum weighted matching $\mathcal{M}$, we build a set $\mathcal{M}_{\mathbf{2x2}}$ of 2x2 pivots. We use the 2x2 pivot selection suggested in [2], but with a structural metric instead of a numerical criterion. Indeed, because of the scaling, all the entries in the paths of the permutation are 1, and we have to use another criterion to choose the 2x2 pivots. For each path in the permutation, we take the 2x2 pivots so that the sequence of 2x2 pivots (i_k, j_k) maximizes $\prod |Row_{i_k} \cap Row_{j_k}|$.

2 Analysis and factorization strategies

`MA47` and `MA57` are two symmetric indefinite multifrontal solvers. The main difference between these two codes is that `MA47` can manage the zeros blocks due to oxo or tile pivots during analysis and factorization.

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For each code we developed several approaches where we modified the analysis and/or factorization phases. During the analysis, we influenced the ordering to force it to take the 2x2 pivots of MC64SYM or to take pivots in a subset of the entries of the matrix. During the factorization we forced the solver to follow these pivots and to do static pivoting (if there are numerical problems) or we gave very high priority to these pivots but allowed some numerical pivoting or delayed pivots.

For example, on matrix $A = \begin{pmatrix} 2 & -1 & 1 & 0 \\ -1 & 2 & 0 & 0 \\ 1 & 0 & 0 & 2 \\ 0 & 0 & 2 & 0 \end{pmatrix}$ we detected a 2x2 pivot in rows/columns 3,4 and two 1x1 pivots on the diagonal. With MA57 we can test the following strategy :

- Build $A_r = \begin{pmatrix} X & X & X \\ X & X & 0 \\ X & 0 & X \end{pmatrix}$ where row/column 3 corresponds to the 2x2 pivot,
- Get the permutation 3,2,1 on A_r and convert this permutation to 3,4,2,1 on A,
- Apply the MA57 factorization and get L factor, $\begin{pmatrix} F & X & 0 & X \\ 0 & F & 0 & F \\ 0 & 0 & X & X \\ 0 & 0 & 0 & X \end{pmatrix}$, where F corresponds to fill-in or explicitly stored zeros.

Thanks to a better management of oxo pivots, MA47 with the same ordering does not store any fill-in in the factors.

3 Experiments and conclusions

We will present some conclusions from experiments on different approaches. We have identified the most robust strategies and will discuss the gain over already existing codes. We will also present some perspectives of our future work. Indeed some effects have still to be understood and some alternatives have not yet been tested.

References

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