CS240A: Measurements of Graphs

slides under construction –
see the Matlab transcript for
what I actually did in class

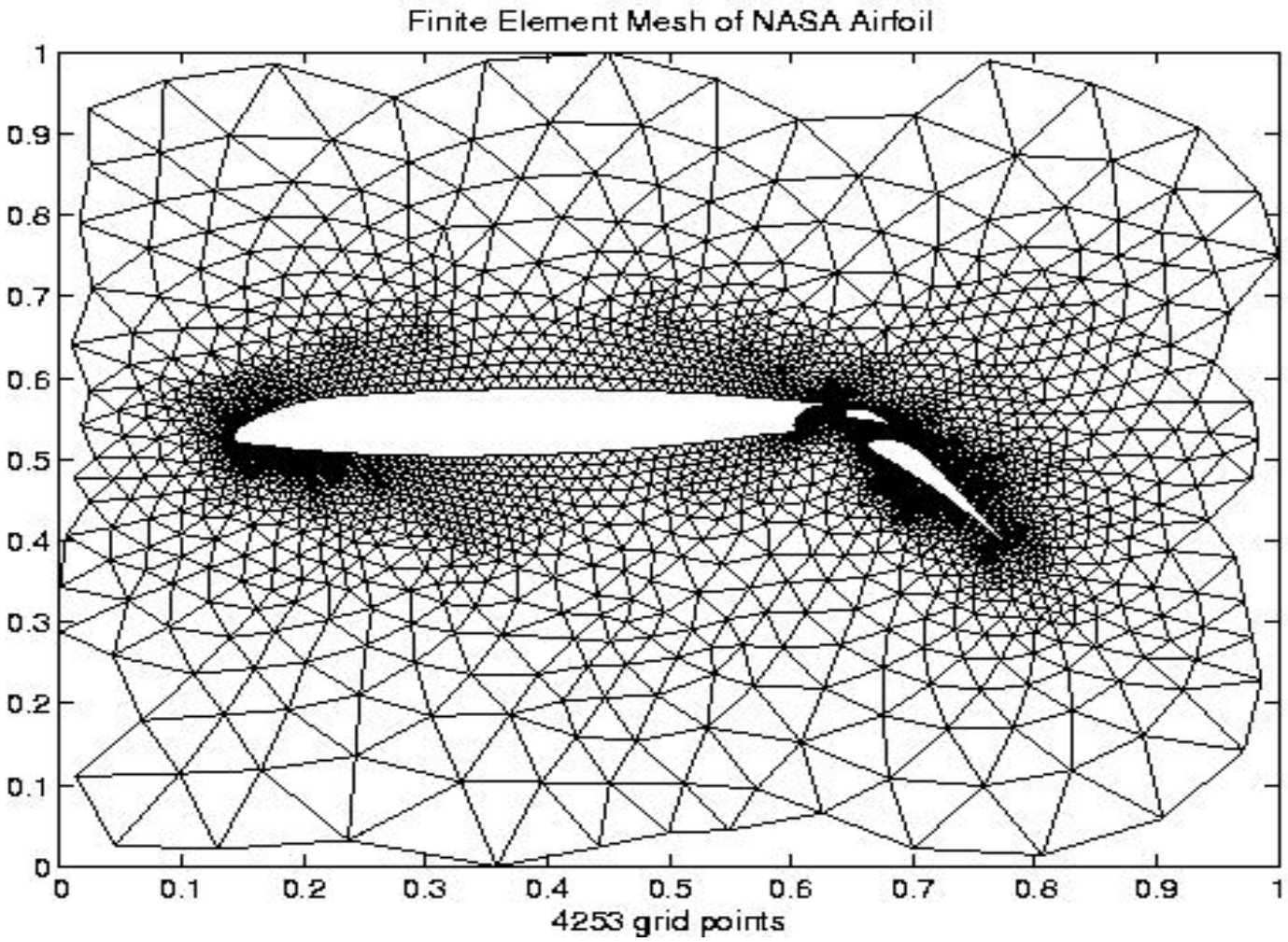
Some classes of graphs

- Classifications of graphs:
  - Planar graphs (drawable without crossing edges)
  - Overlap graphs (physical locality)
  - Power-law graphs (hist(vtx degree d) ~ $d^{-\beta}$ for some $\beta > 0$)
  - Small-world graphs (small diameter, large cluster coefficient)

- Generators for classes of graphs:
  - Erdos-Renyi (flat) random graphs: **`sprandsym.m`**
  - RMAT random graph generator: **`rmat.m`**
  - 2-D and 3-D mesh generators: **`grid5.m`** etc. in **`meshpart`** toolbox

- Graphs observed in the wild (see Florida collection for many examples):
  - Finite element meshes: **`meshes.m`**
  - Circuit simulation graphs: **`circuit_3.mat`**
  - Relationship networks: **`coAuthorsDBLP.mat`**, **`PGPgiantcompo.mat`**
  - … many others!

Planar graphs



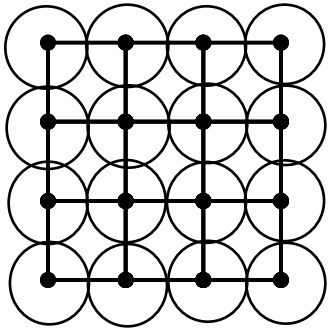
Overlap Graphs *[Miller, Teng, Thurston, Vavasis]*

- A k-ply neighborhood system in d dimensions is a set $\{D_1,\ldots,D_n\}$ of closed disks in R^d such that no point in R^d is interior to more than k disks

- An (α,k) overlap graph (for $\alpha >= 1$) has vertices at the centers of the disks $\{D_1,\ldots,D_n\}$ of a k-ply neighborhood system, with an edge (i, j) if expanding the smaller disk (D_i or D_j) by α makes them overlap

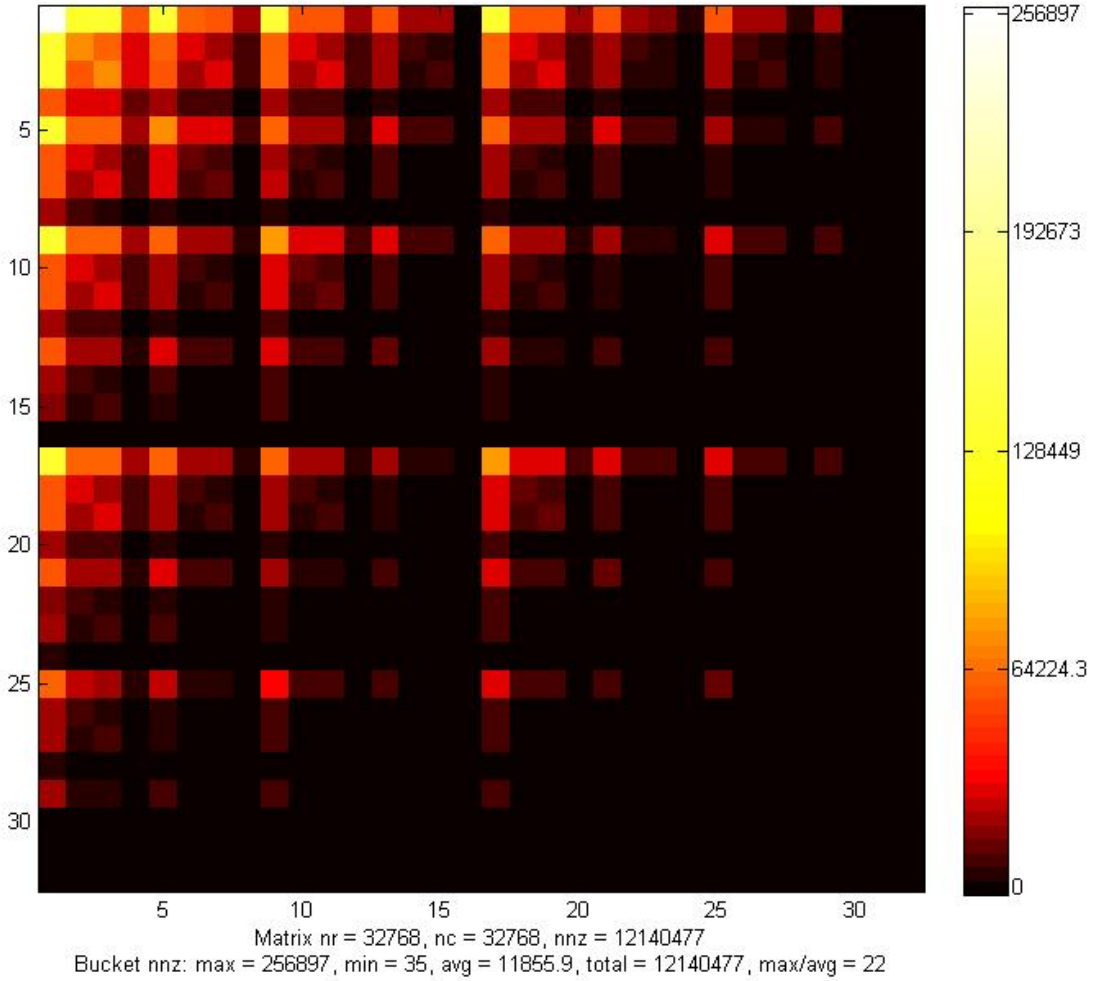
An n-by-n mesh is a (1,1) overlap graph

Every planar graph is (α,k) overlap

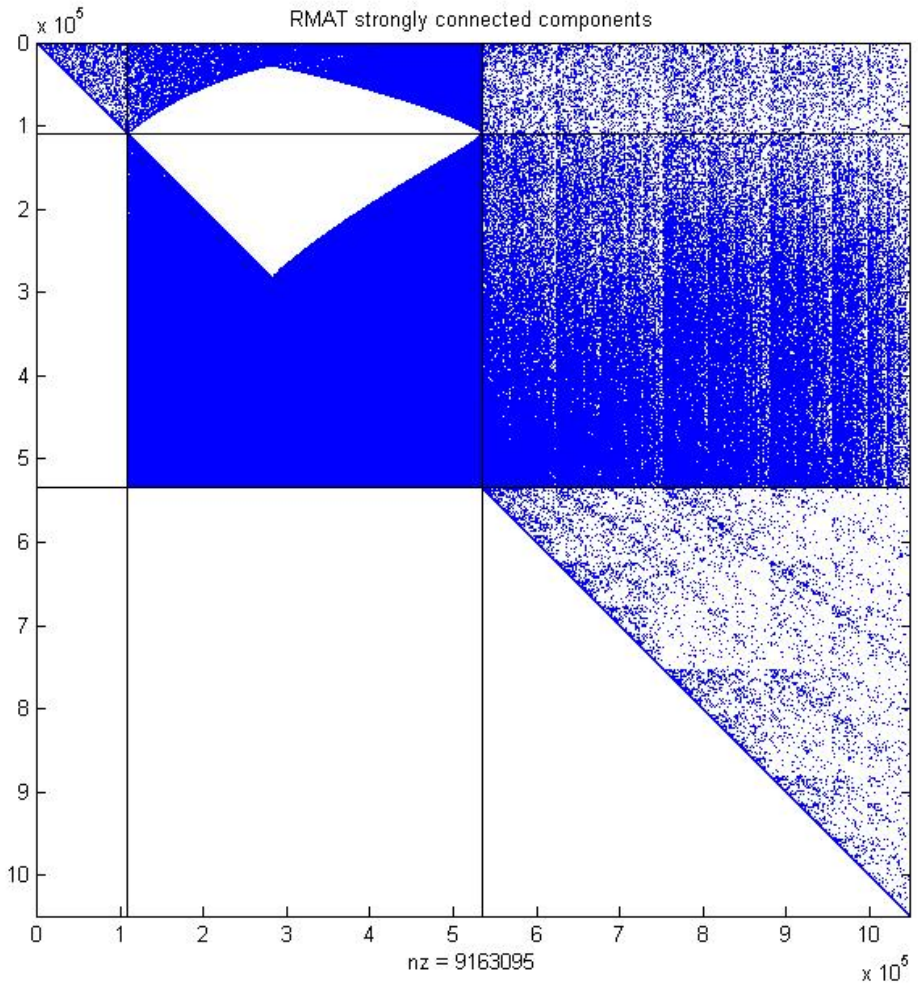


2D Mesh is (1,1) overlap graph

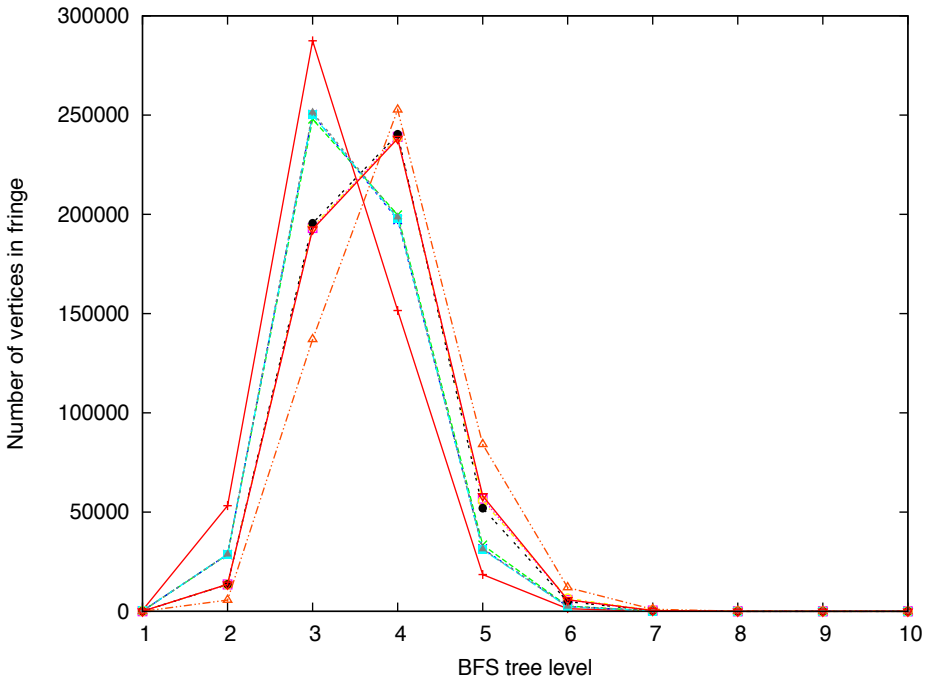
RMAT Approximate Power-Law Graph



Strongly connected components of an RMAT graph



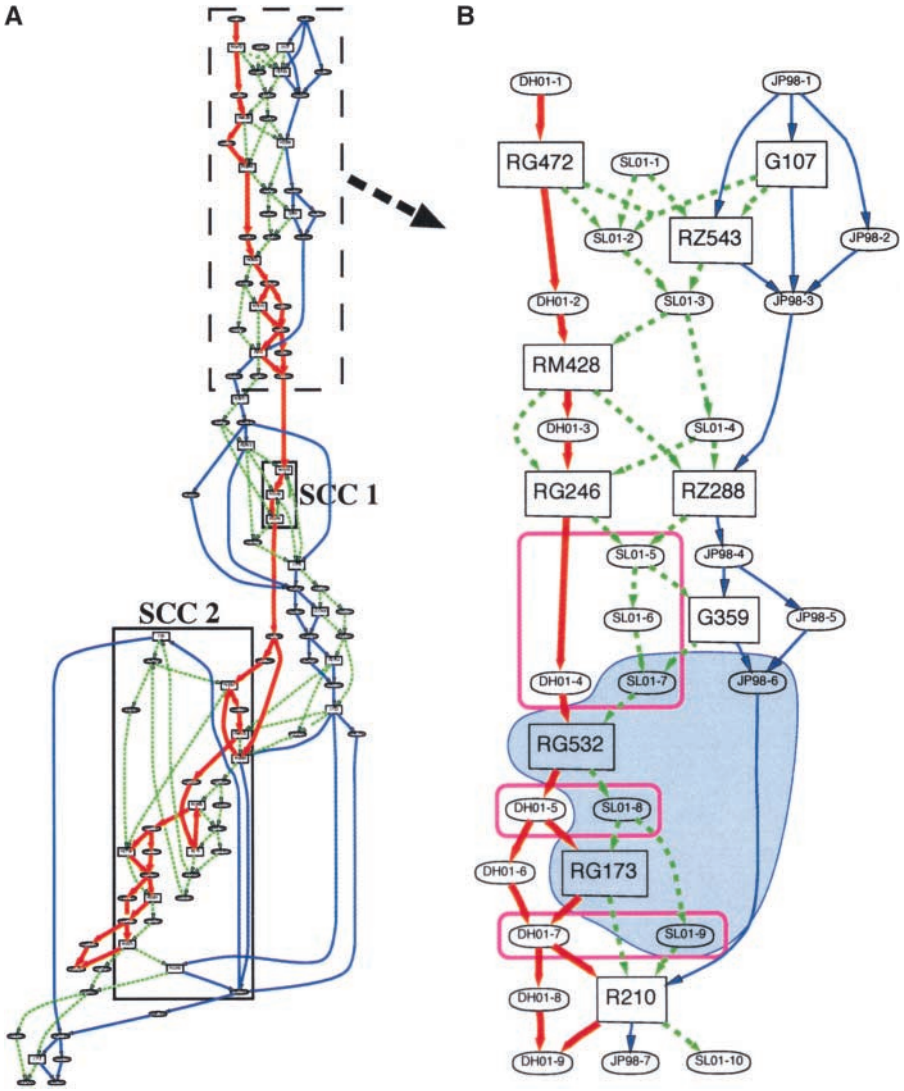
Diversity of graphs in the wild…



Low diameter graph (R-MAT)

vs.

Long skinny graph (genomics)



Gene linkage map, courtesy Yan et al.

Some graph statistics (and Matlab tools)

- Vertex degree histogram: **`dhist.m`**
- BFS level profile, gives a feeling for avg shortest paths :
  `bfslevels.m`
- Clustering coefficient: **`ccoeff.m`**
  - c = 3*(# triangles) / (# connected triples)
- Laplacian eigenvalues (and vectors): **`meshpart`** toolbox,
  `eigs(laplacian(A),5,'lm')`
- Separator size: **`meshpart`** toolbox
- Fill (chordal completion size): **`analyze.m`** and **`amd.m`**