# A General Framework for Wireless Spectrum Auctions

Sorabh Gandhi, Chiranjeeb Buragohain*, Lili Cao, Haitao Zheng, Subhash Suri
Department of Computer Science
University of California, Santa Barbara, CA 93106 U.S.A
* Amazon.com, Seattle, WA 98106 U.S.A

***Abstract*— We propose a real-time spectrum auction framework to distribute spectrum among a large number wireless users under interference constraints. Our approach achieves conflict-free spectrum allocations that maximize auction revenue and spectrum utilization. Our design includes a compact and yet highly expressive bidding language, various pricing models to control tradeoffs between revenue and fairness, and fast auction clearing algorithms to compute revenue-maximizing prices and allocations. Both analytical and experimental results verify the efficiency of the proposed approach. We conclude that bidding behaviors and pricing models have significant impact on auction outcomes. A spectrum auction system must consider local demand and spectrum availability in order to maximize revenue and utilization.**

## I. Introduction

Reliable and efficient spectrum access is vital for the growth and innovation of wireless technologies. Unfortunately, historical (and current) spectrum regulations assign different technologies with static spectrum in long-term leases to prevent interference among them. Over time, this has led to significant over-allocation and under-utilization of spectrum, slowing down wireless deployments. To realize efficient spectrum usage, we must migrate from the current static spectrum access to dynamic spectrum access.

One promising solution is spectrum trading that applies pricing based incentives to stimulate users to sell and lease under-utilized spectrum. One particular form of trading is *auctions*, widely known for providing efficient allocation of scarce resources [3], [10]. Sellers use auctions to improve revenue by dynamically pricing based on buyer demands. Buyers benefit since auctions assign resources to buyers who value them the most. Hence, many systems use auction based allocation models, including energy markets [3], treasury bonds [2] and commercial goods [10].

In this paper, we consider the problem of how to efficiently auction spectrum to satisfy user demands while maximizing system revenue. Figure 1 illustrates a general spectrum auction scenario where $n$ buyers (wireless service providers) bid for spectrum from a seller (government agencies or spectrum owners) who auctions its spectrum periodically, *i.e.* every hour.

Because of the requirement to minimize radio interference, spectrum auction systems are significantly different from traditional auction systems, and face a number of new challenges:

**(1) Radio interference constraints.** To provide conflict-free spectrum usage, spectrum auctions are constrained by radio interference. Buyers in close proximity interfere with each other and can not use the same spectrum; while well-separated buyers can reuse the same spectrum. Hence, spectrum auctions need to explicitly account for the impact of interference when determining allocations and prices. In this paper, we model the interference constraints using the widely-used *protocol interference model* [16], a succinct model to formulate the impact of interference within resource allocation problems. Using this model, we can represent interference constraints as a conflict graph, shown in Figure 1. In Section VI we describe practical considerations on how to improve this model for more realistic characterization of interference.

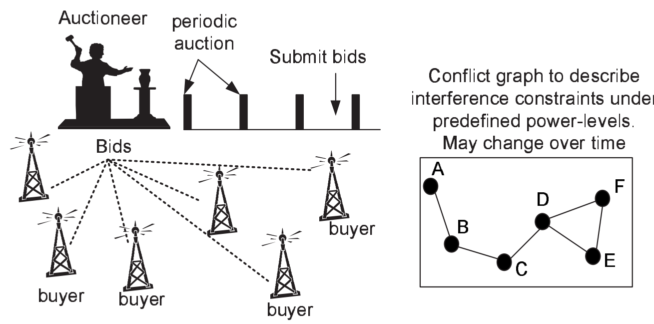



Fig. 1. A dynamic auction scenario. (left) An auctioneer performs periodic auctions of spectrum to buyers. (right) A conflict graph illustrates the interference constraints among buyers.

**(2) Supporting diverse demands.** To realize the potential of dynamic spectrum access and improve spectrum utilization, spectrum auctions need to accommodate diverse demands. These include both traditional long-term spectrum usage using, and short-term spontaneous spectrum usage to support bursty traffic. For example, occasional events like sports and conferences will create demand spikes at a specific location for a short-period of time. It is important for these users to obtain and pay for what they need.

**(3) Online multi-unit allocations –** Spectrum auctions are multi-unit auctions where multiple identical copies of goods are for sale – spectrum is divided into a number of channels. Users wish to obtain different amount of spectrum at their desired power level, and may be willing to pay differently depending on the assignment. Hence, we need a new *bidding language* to allow buyers conveniently express their desire, and do it so compactly. To support dynamic spectrum access, we need an efficient *allocation* algorithm to distribute resource in real-time. However, existing solutions for multi-unit auctions apply combinatorial auctions as the most general framework [8]. These auctions require complex bid expression that grows exponentially with the size of goods, and apply complex allocation and pricing process that requires solving NP-hard problems. Hence, they are in general intractable and not suitable for real-time dynamic *hourly* auctions.

We also make the following assumptions on the spectrum auction system. First, we assume each buyer bids spectrum with specific but fixed power requirements, and hence focus solely on channel allocation[1]. The seller divides its spectrum into a large number of *homogeneous* channels with equal power limit and transmission bandwidth. We assume centralized auctions where the seller collects bids and auctions spectrum in single rounds periodically. In Section VI we discuss extensions to heterogeneous channels, decentralized systems and iterative auctions, as well as practical mechanisms to acquire the knowledge of interference constraints.

### A. Our Contributions

We consider the problem of real-time dynamic spectrum auctions to distribute spectrum among a large number of buyers in a large geographic area. We focus on computational-efficient channel allocation/pricing algorithms to support large scale networks with real-time spectrum trading. While the problem is NP-hard, we show that by restricting bids and radio interference constraints judiciously, we can design a practical and efficient auction system that is simple, scalable and yet provides powerful performance guarantee. Our work differs significantly from prior works on spectrum auctions [13], [15], [21] which assume small scale networks. We also perform extensive experiments to understand the impact of pricing models and bidding behaviors on spectrum utilization and revenue. This paper makes four key contributions:

**(1) A compact and highly expressive bidding language — piecewise linear price-quantity (PLPQ).** Each buyer expresses its demand as the amount of spectrum desired at each particular per-unit price. PLPQ can approximate a very broad class of demand curves with high accuracy. It allows bidders to express fairly sophisticated valuations in a single bid, and do so very compactly.

**(2) Different pricing models to explore tradeoffs of revenue and fairness.** We investigate two pricing models, a simple uniform pricing model where all winners pay the same per-unit price, and a discriminatory pricing model where winners' per-unit prices are different. While the decision of pricing model depends on the tradeoff between revenue and fairness, we focus on designing allocation algorithms for both models and exploring their impact on auction outcomes and user allocations.

[1]Extensions to joint channel and power allocations are beyond the scope of this paper, and will be addressed in a future study.

**(3) Low-complexity allocation algorithms with analytical bounds.** While the revenue-maximizing auction problem is NP-hard, we propose low-complexity approximation algorithms to derive prices and allocations. Our algorithms are supported by strong theoretical bounds on performance and complexity. Our algorithms run in polynomial time (1 min for 3500 nodes using a $3.0$ GHz processor with $1$ GB RAM.) while the optimal solution takes exponential run time (4 hours for 80 nodes).

**(4) Extensive experiments and evaluations.** We perform extensive experiments to examine the proposed system, and explore the impact of bidding behavior, network topology and pricing model. Results show our algorithms run in real-time and produce near optimal solutions. We conclude that to maximize revenue and spectrum utilization, prices must be determined based on local demand and spectrum availability.

The rest of the paper is organized as follows. In Section II we describe the general model of spectrum auction, the impact of wireless interference and discuss some related work. In Section III we propose the auction framework and introduce our bidding language and pricing models. Section IV describes auction clearing algorithms for both pricing models and their theoretical bounds. We discuss experimental results in Section V. We discuss in Section VI several practical issues related to the proposed framework and conclude in Section VII.

## II. Preliminaries and Related Work

This section briefly describe multi-unit auctions, existing solutions of spectrum auctions, and challenges on the problem of spectrum allocation under interference constraints.

### A. Multi-unit Auctions

Auctions have been widely used to provide efficient allocation of scare resources, including the sale of single-item indivisible goods (*e.g.* a painting), single-item in multi-unit bundles [9], [23] and multi-item, multi-unit bundles [8] (*e.g.* bonds).

A successful auction system must not only produce financial efficiency [17], but also provide efficient bidding process and fast execution. Bids express user's preference for various outcomes. There is often an inverse relation between the "expressiveness of the auction" and the computational complexity of determining the winners in the auction. Combinatorial auctions allow users to express their bids over arbitrary subsets of the goods, but are known to be intractable to solve optimally, or even approximately [22].

Given bids, auctioneers use *auction-clearing* algorithms to compute the revenue-maximizing prices and auctions. Clearing is simple is single-item single-unit auction: assign the item to bidders with the highest bid. However, auctioning multi-unit items can be much more complex since multiple winners split the items. The complexity of clearing algorithms also depends on the complexity of bidding language. A comprehensive study of market clearing algorithms for single item, multiple-unit auctions were given by Sandholm and Suri [23].

Multi-unit auctions have two pricing models:

**- Uniform pricing** The auctioneer determines a per-unit price and applies it to all winning bidders. The auction clearing problem here is to determine a market-clearing price that maximizes the auctioneer's revenue. Ebay multi-unit auctions [9] have been using this model.

**- Discriminatory pricing** The auctioneer charges different prices to different bidders. While producing higher financial revenue, this model is also perceived as less "fair" to bidders than the uniform pricing model.

The various issues that arise in uniform pricing versus discriminatory pricing models have been studied in diverse markets such as US treasury security auction [20], government bonds auction in UK [2], and electricity auctions in California [3], [14]. For one time auctions, discriminatory pricing always generates more revenue. On the other hand, uniform pricing is simple, and provides "fairness" to bidders and promotes aggressive bidding [20]. However, uniform pricing is suspect to collusion among the bidders [4] and for an unsettled market, it might be more dangerous with respect to the amount of revenue it generates [20]. Because of these complex factors, we leave the choice of pricing model to auctioneers, and focus on designing efficient bidding language and fast clearing-algorithms for both models.

### B. Related Work on Spectrum Auctions

There are multiple complementary ways to design spectrum auctions, each applicable to different scenarios. First, the system can allocate/auction transmit power to minimize interference [13], while all buyers use the same spectrum band. Second, the system can allocate conflicting users with orthogonal channels to avoid interference, and compute appropriate prices and allocations to maximize system utility. Prior work in this category uses cellular network model. The work in [15] uses a demand responsive pricing framework, and applies iterative bidding to maximize social welfare for small scale networks. In [5], the authors propose the general problem in cellular systems and centralized heuristics for small scale networks. Ryan *et al.* [21] proposed a hybrid pricing model to reduce the frequency of auctions – use simple auctions during peak period while applying a uniform price to all buyers during off-peak.

### C. Interference Constraints in Spectrum Auctions

Spectrum auction differs from conventional auctions because it has to address radio interference. Given bids, the problem of auction-clearing becomes the problem of interference-constrained resource allocation. Next, we briefly discuss the impact of interference and the corresponding spectrum allocation problem.

We start from a sample scenario in Figure 1 where nodes $A$ to $F$ are wireless access points that provide network access for their associated users. Since $A$ and $B$ are located closely to each other, their associated users will receive signals from both nodes. Signals from non-associated access points become interference and could disrupt communications. To avoid interference, $A$ and $B$ should not use the same spectrum frequencies. Assuming spectrum consists of $M$ channels, we use $F_A$ and $F_B$ to represent the spectrum assigned to $A$ and $B$, $F_A = \{s_1^A, s_2^A, ...s_M^A\}$ where $s_k^A = 1$ if the $k$th channel is assigned to $A$, and otherwise $0$. We can represent the interference constraint between $A$ and $B$ as

**Interference Constraints:** $F_A \cap F_B = \emptyset$, *i.e.* $s_k^A s_k^B = 0,\ \forall k \in [1, M]$.

In this case, $f_A + f_B \leq 1$, where $f_A = |F_A|/M$, $f_B = |F_B|/M$ represent the *normalized* spectrum assigned to $A$ and $B$, respectively. Figure. 1 shows the graphic interpretation of the constraints as a *Conflict Graph*. Vertices represent access points, and an edge exists between any two vertices if they conflict.

Under interference constraints, we define the auction clearing problem as:

$$\text{Maximize} \sum_{i \in \text{bidders}} f_i p_i(f_i), \qquad \text{subject to} \tag{1}$$

$$f_i \leq 1 \tag{2}$$

$$\textit{Interference Constraints} \tag{3}$$

where $p_i(f_i)$ represents the per-unit price that the bidder $i$ pays if he obtains $f_i$ unit of spectrum.

This problem is a special case of non-linear integer programming and is known to be NP-hard. Jain et al. [16] were the first to study a class of related optimization problems and proposed an exponential time algorithm to solve it optimally. The works of [1], [6], [18] have provided polynomial time approximation algorithms with provable performance guarantees for the same throughput maximization problem. Our work builds on existing work of [6], [23] to solve spectrum auction problems that maximize revenue under interference constraints.

## III. Spectrum Auction Framework

To support real-time dynamic spectrum trading, we propose a computational-efficient auction framework with simple and effective bidding and fast auction clearing algorithms. Specifically, buyers use a compact and yet expressive bidding format to express their desired spectrum usage and willingness to pay, while sellers execute fast clearing algorithms to derive prices and allocations under different pricing models. Next, we present the proposed bidding formats and the corresponding optimization problems under different pricing models. We will describe fast auction clearing algorithms in Section IV.

### A. Piecewise Linear Price-Demand (PLPD) Bids

A good bidding language should provide expressive but concise bids. At the same time, it also needs to be compact, preventing complicated auction-clearing process. We propose to use piecewise linear price demand (PLPD) curves that not only satisfy both requirements, but also lead to low-complexity clearing algorithms.

With PLPD, a bidder $i$ expresses the desired quantity of spectrum $f_i$ at each per-unit price $p_i$ using a continuous

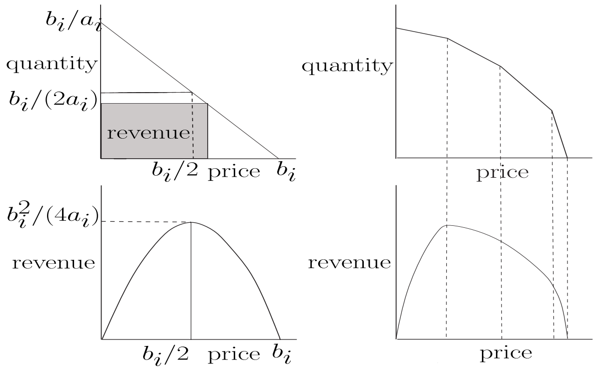



Fig. 2. On the left, linear demand curve (top) and the corresponding revenue generated (bottom) and on the right a concave piecewise linear demand curve (top) and the corresponding piecewise quadratic revenue function.

concave piecewise linear demand curve. A simple example is linear demand curves

$$p_i(f_i) = -a_i f_i + b_i, \quad a_i \geq 0, b_i > 0, \tag{4}$$

where the negative slope represents *price sensitivity* at buyers – as the per-unit price decreases, demands in general increase. Any PLPD curve can be expressed as a conglomeration of a set of individual linear pieces (see Figure 2). For ease of explanation, we will use linear demand curves to describe auction problems and solutions. However, our algorithms and proofs easily generalize to concave piecewise linear demand curves.

When $a_i > 0$, the revenue produced by each bidder is a *piecewise quadratic* function of the price. Figure 2 shows the quantity $f_i(p_i)$, and the revenue generated $R_i(p_i)$ as a function of the price $p_i$:

$$f_i(p_i) = \frac{b_i - p_i}{a_i}, \quad 0 \leq p_i \leq b_i \tag{5}$$

$$R_i(p_i) = f_i(p_i)p_i = \frac{b_i p_i - {p_i}^2}{a_i} \tag{6}$$

For linear demand curves, the revenue is a quadratic function of price, with a unique maximum at $p_i = b_i/2$. Further, if $p_i \to 0$, $R_i(p_i) \to 0$; and if $p_i \to b_i$, $R_i(p_i) \to 0$.

PLPD has several attractive advantages. First, it is simple and yet highly expressive. PLPD can approximate any arbitrary continuous concave functions, and hence support a broad class of demands. Bidders express their preferences privately, eliminating complex bid signaling and collusive strategies. Second, each single bid covers different pricing options, eliminating the need for auctioneers to collect bids iteratively. Finally, PLPD produces (piecewise) quadratic revenue functions which significantly simplify the auction-clearing problem.

Although auction revenue and efficiency depend on buyer's social and financial strategy and their PLPD formats, we do not address mechanisms to compute the optimal PLPD curves. Instead, we assume that each buyer has its own curve, and focus on how to solve the auction-clearing problem given the

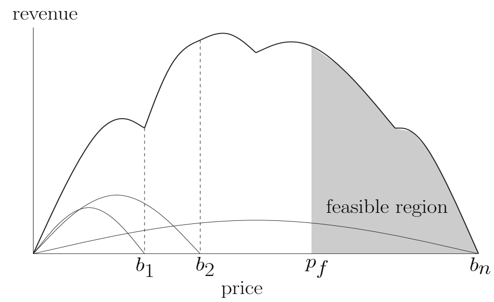



Fig. 3. The revenue as a function of clearing price $p$ in the uniform pricing model.

bids. We perform experiments in Section V to explore the impact of various bidding behaviors, particularly aggressive versus conservative bidding. In Section VI, we also discuss extensions to iterative auctions where buyers adjust their PLPD bids iteratively based on market feedback.

### B. Pricing Models and Auction-Clearing Problems

We now describe the auction clearing problem under both uniform and discriminatory pricing models. Note that when $a_i = 0$, the clearing problem becomes a classical weighted throughput maximization problems with good solutions [6], [7], [16]. Hence in this paper, we assume the general cases where $a_i > 0$.

**Uniform pricing** – The auctioneer sets a clearing price $p$. Each bidder obtains a fraction of spectrum $f_i(p) = (b_i - p)/a_i$ and produces a revenue of $R_i(p) = (b_i p - p^2)/a_i$. Any bidder $i$ with $b_i \leq p$ gets zero assignment. In this case, the optimization problem is to search for the revenue-maximizing price $p$.

Without loss of generality, we assume that bidders 1 to $n$ are labeled in increasing order of $b_i$, *i.e.* $b_1 \leq b_2 \leq b_3 \leq \ldots \leq b_n$. And $b_0 = 0$. For a given price $p$, we compute the revenue $R(p)$ as:

$R(p) = \sum_{i \in [1,n],\ b_i > p} R_i(p) = \sum_{i,\ b_i > p} \frac{b_i p - p^2}{a_i}$

Since each $R_i(p)$ is a quadratic function of $p$, the total revenue is a *continuous piece-wise quadratic* function as shown in Figure 3. Each of the quadratic piece has a parabolic shape.

The overall auction clearing problem becomes

$$\text{Maximize} \sum_{i \in [1,n],\ b_i > p} \frac{b_i p - p^2}{a_i} \qquad \text{subject to}$$

$$\textit{Interference Constraints} \tag{7}$$

$$f_i = \frac{b_i - p}{a_i}. \tag{8}$$

**Discriminatory pricing** – Next we consider the case when the clearing prices are non-uniform and vary across $i$. Clearly the problem of uniform clearing is a special case. The optimization problem becomes

$$\text{Maximize} \sum_{i=1}^{n} (-a_i f_i^2 + b_i f_i), \qquad \text{subject to}$$

$$\textit{Interference Constraints} \tag{9}$$

$$-a_i f_i + b_i \geq 0, \quad f_i \geq 0 \tag{10}$$

### C. The Optimal Clearing Algorithm

Both clearing problems are NP-hard. Next, we briefly describe an optimal solution with exponential run time complexity and will use it in this paper as a benchmark for evaluating approximation algorithms.

Consider a single channel of the wireless spectrum. If we allocate this channel to any bidder, none of his neighbors in the conflict graph can be allocated this channel. Thus if we consider a maximal independent set of the conflict graph, then all bidders corresponding to the independent set can use the same channel simultaneously. Based upon this observation, Jain et al. [16] proposed an optimal algorithm to resolve interference conflicts: their approach results in a linear programming (LP) problem with an exponentially large number of constraints. Clearly solving such an LP requires exponentially large amount of time and hence not feasible for large number of bidders. We use a variant of this algorithm in our experiments to produce the optimal solution in order to compare the quality of our approximations. Next, we propose fast approximation algorithms to solve these problems in polynomial time.

## IV. Fast Auction-Clearing Algorithms

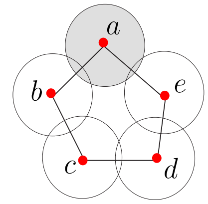
In this section, we show that by judiciously restricting the interference constraints, we can develop fast approximations to the original NP-hard clearing problems in polynomial time. Note that in this paper, we assume the auctioneer has global information on interference constraints and bids. We will discuss extensions to decentralized auction systems in Section VI.

### A. Linearizing the Interference Constraints

The auction clearing problem is complex because the discrete interference constraints grow exponentially with the number of buyers. We propose to restrict the interference constraints and reduce them into a number of constraints that grow linearly with the number of buyers. The new constraints are stricter and hence lead to a feasible but sub-optimal solution. We show that analytically this sub-optimal solution can never be too far off from the optimal one.

To *linearize* the constraints, we assume that the spectrum is finely partitioned into a large number of channels. Each buyer $i$ obtains a normalized allocation of $\{f_i : i = 1, 2, \ldots, n\}$ where $f_i \leq 1.0$. For example, a 1MHz spectrum band is divided into 100 channels of 10kHz each. A buyer $i$ with $f_i = 0.143$ will obtain $\lfloor 0.143 \times 100 \rfloor = 14$ channels. In practice this rounding down will lead to some loss of revenue. However, if the number of channels is significantly larger than the highest node degree in the conflict graph, the loss will not lead to undue reduction in revenue. Hence, in the following, $f_i$ behaves as a continuous variable.

In the following, we refer to each buyer as a node in the conflict graph. We define a neighbor of a node $i$ as any node that interferes with $i$ and hence connects to $i$ in the conflict graph.




| $Node$ | $NI$ | $NLI$ | $OPT$ |
|---|---|---|---|
| $a$ | $\{1\}$ | $\{2,3\}$ | $\{1,4\}$ |
| $b$ | $\{2\}$ | $\{4,5\}$ | $\{2,5\}$ |
| $c$ | $\{3\}$ | $\{1,2,3\}$ | $\{1,3\}$ |
| $d$ | $\{4\}$ | $\{4,5\}$ | $\{4,2\}$ |
| $e$ | $\{5\}$ | $\{1\}$ | $\{3,5\}$ |

Fig. 4. Example network, the conflict graph and the channel allocations by NI (Node-Interference), NLI (Node-L-Interference), and OPT (Optimal). There are a total of 5 channels.

**Node-ALL Interference Constraints (NI)** The simplest constraint is to restrict $i$ and every neighbor of $i$ to use different spectrum channels, *i.e.*

$$f_i + \sum_{j \in N(i)} f_j \leq 1, \quad i = 1, 2, \ldots, n \tag{11}$$

where $N(i)$ represents the set of neighbors of $i$ and $n$ represents the total number of nodes.

While leading to simple interference free allocations, this constraint is more restrictive than necessary. Using a sample topology, Figure 4 illustrates the channel allocation using *NI* where each node gets only one channel, although node $a$ and $d$ do not conflict with each other and can both use channel $4$. Clearly, we need better approximations.

**Node-L Interference Constraints (NLI)** We introduce a less restrictive constraint by imposing an order among nodes. By integrating the order in the allocation process, we can achieve much more efficient allocations than that using the NI constraints.

We define the notion of *left of*. Let two nodes $i$ and $j$ locate at coordinates $(x_i, y_i)$ and $(x_j, y_j)$. Node $i$ is to the left of node $j$ if $x_i < x_j$. If $x_i = x_j$, then the node with the smaller index is considered to be to the node to the left. The constraint becomes: every neighbor of $i$ to the *left* of $i$, and $i$ itself should be assigned with different channels:

$$f_i + \sum_{j \in N_L(i)} f_j \leq 1, \quad i = 1, 2, \ldots, n \tag{12}$$

where $N_L(i)$ is the set of neighbors of $i$ lying to its left. Figure 4 compares the allocation results using *NLI* and *NI*, and the original constraints *(OPT)*. We see that *NLI* achieves a more efficient channel allocation than *NI*.

In the following, we apply *NLI* constraint to develop approximation algorithms. We show that while it is still more restrictive than the original one, in both theory and practice, algorithms based on *NLI* produce near-optimal channel allocations in polynomial time. Further, *NLI* leads to the optimal solution when the conflict graph is a tree.

### B. A Toy Example: Fixed Per-Unit Price Auctions

To illustrate our algorithm, we start from a simple model where each buyer pays a fixed per-unit price regardless of the allocated amount, *i.e.* $p_i(f_i) = b_i$, $a_i = 0, \forall i$. We approximate this problem by using *NLI* as:

$$\text{Maximize} \sum_i f_i b_i, \qquad \text{subject to}$$

$$f_i + \sum_{j \in N_L(i)} f_j \leq 1 \tag{13}$$

$$0 \leq f_i \leq 1 \tag{14}$$

This is an optimization problem with linear constraints and a linear objective function and hence can be solved easily using linear programming (LP) in polynomial time. The quality of the solution produced by this LP is bounded by the following worst case error guarantee, proved by [6].

*Lemma 1:*

$$R_{\text{LP}} \geq \frac{1}{3} R_{\text{OPT}}, \tag{15}$$

*where $R_{\text{LP}}$ is the revenue generated by solving the LP and $R_{\text{OPT}}$ is the optimum possible revenue.*

Simulation results reveal that this worse-case bound is almost never realized, and the LP solution is very close to the optimal [6].

The above simple example can be solved using linear programming because of its linear objective function. However, the general auction clearing problems are non-linear. Next, we design approximation algorithms for the general auction problems and derive theoretical bounds on the performance and complexity.

### C. Clearing algorithm for uniform pricing (CAUP)

Under NLI, the optimization problem under uniform pricing model becomes

$$\text{Maximize } R(p) = \sum_{i \in [1,n],\ b_i > p} \frac{b_i p - p^2}{a_i} \qquad \text{subject to}$$

$$f_i + \sum_{j \in N_L(i)} f_j \leq 1, \tag{16}$$

$$f_i = \frac{b_i - p}{a_i}. \tag{17}$$

The optimization is to find the optimal price $p$, which is an one-dimension search process. We propose a two-step solution: first find the feasible values of $p$ subject to interference constraints and then search for the revenue-maximizing $p$.

**Step I: find the feasible region of $p$ subject to interference constraints.** We use the following Lemma to simplify the search:

*Lemma 2: There exists a unique price $p^T$ where for any $p$, $p \geq p^T$, the channel allocation according to (17) will satisfy the constraints defined by (16), and for any $p$, $p < p^T$ results in allocations that violate the constraints.*

*Proof:* Assume that the buyers (1 to $n$) are sorted by $b_i$, $b_1 \leq b_2 ... \leq b_n$. When $p = b_n$, then $f_i = 0, \forall i$. Obviously this allocation is feasible. From (17), as the price decreases, buyers obtain more spectrum and could violate the constraints. If there is a price for which the constraints are violated, reducing the price further will only increase allocations and continue to violate the constraints. ∎

Therefore, the feasibility region of $p$ is $[p^T, b_n]$. To find $p^T$, we use binary search over all possible values of $p$ ranging from 0 to $b_n$. Let $b_{j-1} \leq p^T < b_j$.

**Step II: search for the revenue-maximizing $p$.** We divide the feasible region of $p$ into intervals $(p^T, b_j], (b_j, b_{j+1}], \ldots, (b_{n-1}, b_n]$. Within each interval the revenue $R(p)$ is a quadratic function, as explained in Section III-A. Since every quadratic function has a single maximum, finding the optimal $p$ that maximizes the revenue function in a interval $[b_k, b_{k+1}]$ is straightforward. Hence, by finding the maximum of the revenue function over all feasible intervals we can find the optimal $p$.

The following theorem provides theoretical bounds on the proposed algorithm.

*Theorem 1: CAUP solves the revenue maximization problem with concave piecewise linear demand curves and uniform clearing price, within an approximation factor of 3 ($R_{CAUP} \geq \frac{1}{3} R_{OPT}$), in time $O(n \log n + n \log U)$. $U$ represents the search range $b_n$.*

The proof is omitted due to space limit but can be found in [11]. When the conflict graph is a tree graph, CAUP produces the optimal solution to the revenue maximization problem.

### D. Clearing algorithm for discriminatory pricing (CADP)

Using NLI, the problem becomes

$$\text{Maximize} \sum_{i=1}^{n} (-a_i f_i^2 + b_i f_i), \qquad \text{subject to}$$

$$f_i + \sum_{j \in N_L(i)} f_j \leq 1, \tag{18}$$

$$-a_i f_i + b_i \geq 0, \quad f_i \geq 0 \tag{19}$$

We propose an approximation algorithm using separable programming [12], a special case of semi-definite programming. This method allows one to approximately solve a special class of non-linear programs using linear programming. Since the discussion is fairly technical, we only provide the main result as theorem. Additional details on algorithms and proofs are in [11].

*Theorem 2: CADP solves the revenue maximization problem with concave piecewise linear demand curves and discriminatory clearing price, within an approximation factor $3(1 + 1/n)$, in polynomial time (depends on time required to solve the linear program).*

Similarly, when the conflict graph is a tree graph, CADP produces the optimal solution to the revenue maximization problem under discriminatory pricing.

### E. Scheduling Spectrum Usages

Given spectrum allocations $\{f_i\}$, we need to schedule the actual usage patterns, *i.e.* the index of channels assigned to each buyer. We follow the *left of* order in the NLI constraints. We start from the leftmost node in the network and assign to it the initial portion of the spectrum. For every next node $i$, we examine the rightmost node lying to the left of $i$, referred to $\mathcal{R}_i$, and assign to $i$ the portion of its allocated spectrum starting from where the assignment of $\mathcal{R}_i$ finishes. This schedule is *always feasible* because the constraint (16) – no node and its left neighbors can consume all the spectrum. This conclusion can be proved by induction, but in the interest of space its proof is omitted. We would like to note that this schedule in general assigns a continuous block of spectrum to each bidder, however, there are cases where a bidder may be allocated with two separate blocks of spectrum when the allocated spectrum falls on the boundary of the total spectrum range.

## V. Experimental results

In this section, we conduct experiments to investigate the performance of the proposed auction framework. We consider the scenario described by Figure 1, where wireless service providers deploy their access points to serve their associated users (each access point is a buyer). We simulate this by randomly deploying these access points in a unit square (normalized area). We plan to examine the performance of our system in planned networks in future work. We include the results using linear demand curves while piecewise linear curves lead to similar conclusions.

We use the fixed power model and assume that every buyer wants to support users within a fixed radius ($0.05$ in our simulations). To produce the conflict graph, we use a simple distance-based interference model - any two access points conflict with each other if they are within $0.1$ (twice the radius) distance of each other. While this assumption is used to produce the conflict graph, it does not limit the application of our approach to other general interference conditions. The maximum spectrum available at any location in the network is normalized to $1$. All results shown are averaged over $5$ random seeds. All the simulations are run in C++ on a $3.0$ GHz processor with $1$ GB of RAM.

We consider three types of bidding curves:

| behavior | spec. vs. unit price | unit price vs. spec. |
|---|---|---|
| normal | $f(p) = -p + 1$ | $p(f) = -f + 1$ |
| conservative | $f(p) = -2p + 1$ | $p(f) = 1/2(-f + 1)$ |
| aggressive | $f(p) = -p/2 + 1$ | $p(f) = 2(-f + 1)$ |

Note that the maximum per unit prices are 1, $1/2$ and 2 for normal, conservative, and aggressive bidders respectively. Unless mentioned, all buyers are normal bidders for all experiments.

We use the following performance metrics:

- Revenue, $R = \sum f_i(p_i)p_i$.
- Spectrum utilization $U = \sum f_i(p_i)$.
- Buyer $i$'s price $p_i$ and channel assignment $f_i(p_i)$.
- Complexity in terms of algorithm execution time.

Using our experiments, we examine the performance of two pricing models, the performance of the proposed approximation algorithms regarding to the optimal solutions, the impact of bidding behavior and node density, and finally the algorithm execution time.

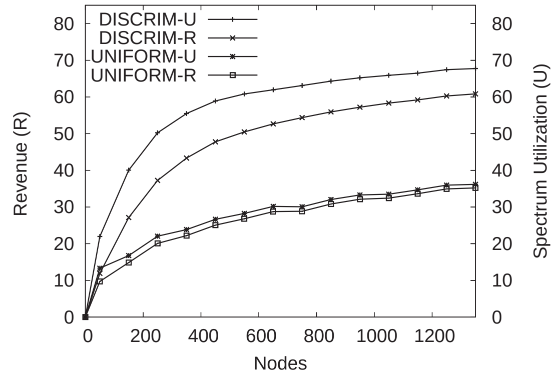



Fig. 5. Revenue and spectrum utilization for varying network sizes for both pricing models.

### A. Uniform vs. Discriminatory Pricing

We start by examining the performance of the proposed auction-clearing algorithms under two pricing models. We vary the network size from $0$ to $1300$, increasing the average conflict degree from $0$ to $10$. Results in Figure 5 show that both revenue $R$ and spectrum utilization $U$ grow with the network size, but the growth rate decreases with the network size since spectrum usages saturate at high node density.

At small network sizes ($< 20$), the difference between the revenue produced by the uniform and discriminatory pricing is small. As network size increases beyond $40$, the discriminatory pricing model leads to nearly twice the revenue (and spectrum utilization) compared to the uniform pricing model. Under the uniformed pricing model, the market-clearing price depends on the maximum level of conflict in the network, *i.e.* the maximum node degree in the conflict graph. As the network size increases, the market-clearing price moves towards $1$. Under the discriminatory pricing model, the seller charges buyers based on local conflict condition, and hence leads to higher spectrum utilization and revenue.

### B. Optimal vs. Approximation Algorithms

Using the discriminatory pricing model, we compare the performance of the approximation algorithm to the optimal solution. We use the randomized algorithm proposed in [16] to generate maximal independent sets and solve the linear programming problem. We run the randomized algorithm for $200000$ iterations for network sizes of $20 - 100$ to compute the optimal revenue. Figure 6 compares the revenue produced by the optimal solution and the proposed approximation algorithm. The approximation is always within $10\%$ of the optimal solution for all network sizes. However, the optimal randomized algorithm requires 4 hours of computation time for a network of $100$ nodes, more than $20000$ times slower than our proposed algorithm.

### C. Impact of Bidding Behaviors

We examine the impact of bidding behaviors on prices and allocations, for both pricing models. In this experiment, buyers

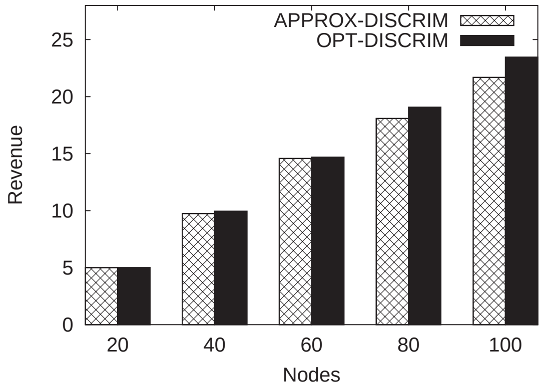



Fig. 6. Comparison of the optimal solution and the approximation algorithm under the discriminatory pricing model.

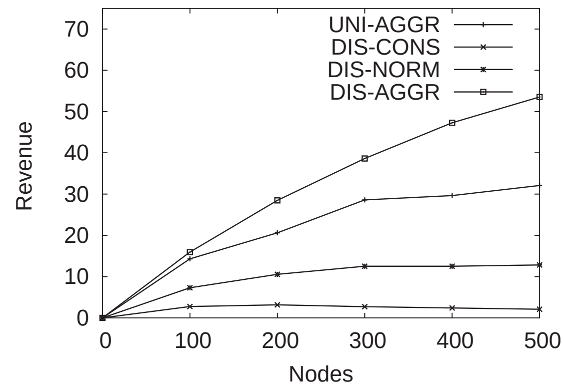



Fig. 8. Revenue generated by different category bidders for uniform and discriminatory pricing algorithms.

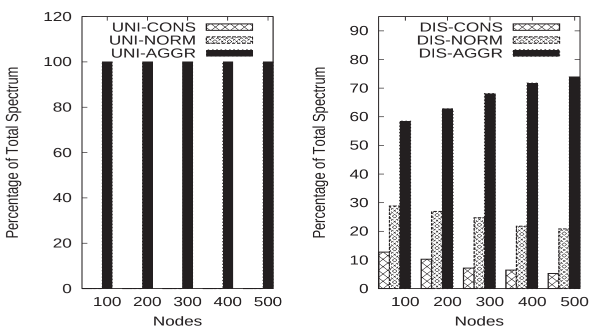



Fig. 7. Percentage of spectrum allocated to different bidder categories for uniform (left) and discriminatory (right) pricing models. UNI-CONS/NORM/AGGR: conservative, normal and aggressive bidders in uniform pricing model; DIS-CONS/NORM/AGGR: conservative, normal and aggressive bidders in discriminatory pricing model.

randomly choose their bidding curves as conservative, normal or aggressive, with equal probability. Figure 7 shows the percentage of spectrum allocated to different bidding categories. Under the uniform pricing model, aggressive buyers take over all the spectrum. Since the market-clearing price is high ($p > 1$), conservative and normal buyers are completely cut of. On the other hand, under the discriminatory pricing model, aggressive buyers obtain a large portion of the spectrum, and their allocation increases with the network size. At small network sizes (low node density), there are not enough aggressive bidders to consume all the spectrum, hence conservative and normal users obtain a small portion of spectrum. As the network size increases, the level of contention increases and so does the price charged to individual buyers. Conservative and normal users are slowly cut off from the auction while aggressive users start to dominate.

Figure 8 compares the total revenue generated by different bidders under both pricing models. Using the uniform pricing model, we only show the revenue from aggressive bidders (UNI-AGGR) since they obtain all the spectrum. We observe that aggressive bidders under the discriminatory pricing model produce higher revenue than those under uniform pricing. Normal and conservative bidders contribute to a small portion of the revenue.

### *D. Impact of Node Clustering*

In practice, wireless service providers might not position access points randomly over the area. They deploy many access points in areas with dense user populations, known as hotspots. We simulate a hotspot scenario by deploying a clustered network, illustrated by the leftmost figure in Figure 9. We initially deploy 200 nodes randomly on a unit square area, and then deploy the next $k$ ($0 \leq k \leq 150$) nodes in a clustered region.

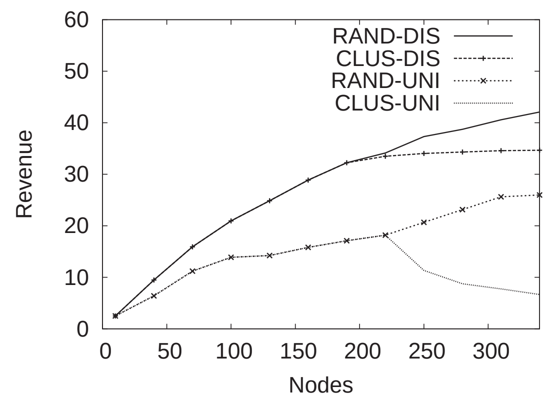



Fig. 10. Effect of clustering on the revenue. RAND and CLUS represent random deployments and clustered deployments.

To examine the impact of clustering, we first compare the revenue of random deployments and clustered deployments assuming normal bidders. Figure 10 shows the revenue under both pricing models, at varying network sizes. For network sizes of $200$ of less, random and clustered deployments produce the exactly same topology, and their revenue curves overlap.

An interesting observation is that the uniform and discriminatory pricing models respond very differently to clustering.

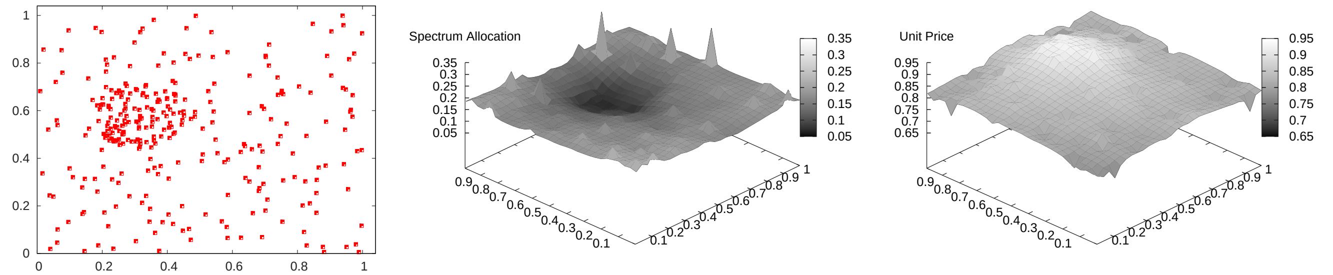



Fig. 9. A clustered network (leftmost). We initially deploy 200 nodes randomly on a unit square area, and then deploy the next $k$ ($0 \leq k \leq 150$) nodes on a clustered area which is $(1/40)$-th of the total area. The channel allocation (middle) and price (rightmost) in a clustered network (when the size is 310), assuming discriminatory pricing.

Under the discriminatory pricing model, the revenue converges very fast to a constant value, corresponding to a full utilization of spectrum inside the cluster. In contrast, under the uniform pricing model, the revenue drops with the clustering. This is because that the market-clearing price now is being governed by the maximum level of contention, *i.e.* the node density in the cluster. As $k$ increases, the market-clearing price quickly rises to $0.99$ ($k = 140$, network size = $340$). Therefore, spectrum allocations (and revenue) at non-clustered regions drop drastically, and the degradation overweighs the improvement inside the cluster. Note that under random deployments, the market-clearing price under uniform pricing also increases with the node density. However, the increase is not drastic and fully compensated for by the increase in network size.

To further examine the impact of clustering on buyer performance, in Figure 9 we plot the allocation and price for each buyer under the discriminatory pricing model, for the same cluster topology, with $k = 100$ (total $300$ nodes). We see that buyers in the cluster have significantly lower allocations and higher prices. This shows that, to maximize revenue and spectrum utilization, pricing should depend on the conflict condition – price should be high at places with high demand and scarce resources. Note that there are a number of allocation spikes, which correspond to the nodes at sparse area with minimal conflicting neighbors – price should be low at places with small demand.

The above observation triggers an interesting question: *how can a node in a clustered area obtain more spectrum?* In order to answer this question, we investigate the impact of bidding behavior on individual buyer's performance using the discriminatory pricing model.

We monitor a particular buyer's spectrum allocation while varying his bidding behavior. We consider the same clustering scenario, and pick a particular buyer $i$ from the clustering area when $k = 0$. Next, we randomly add $k$ nodes to the cluster. As $k$ increases, the level of competition around buyer $i$ increases. We model $i$'s bidding behavior using

$f_i(p_i) = -p_i/c_i + 1$, or equivalently, $p_i(f_i) = -c_i f_i + c_i$,

where $c_i$ represents the bidding aggressiveness. The rest of the network nodes are normal bidders with $c = 1$.

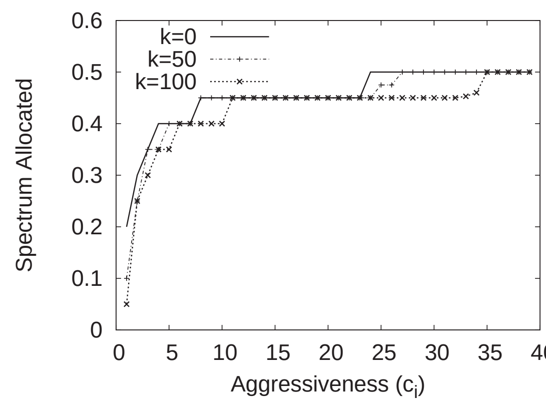



Fig. 11. The impact of bidding aggressiveness on individual buyers with clustering.

Figure 11 shows buyer $i$'s allocation $f_i$ for various aggressiveness levels, for $k = 0$ (no clustering), $k = 50$ (mild clustering) and $k = 100$ (heavy clustering). We see that bidding aggressively ($c_i > 1$) as compared to neighboring nodes does bring in extra allocations (at higher prices). However, the benefit drops with the level of aggressiveness and the allocation curves flatten out. In order to have monopoly of the spectrum, the buyer has to pay significantly more per unit (depending on the density of the cluster) to obtain that last fraction of the spectrum.

### E. Algorithm Complexity

In Figure 12 we compare the algorithm run times for varying network sizes. We see that the approximation algorithm under the uniform pricing model runs extremely fast ($0.05$ seconds for $3500$ nodes), while the discriminatory pricing approximation algorithm requires less than $80$ seconds for up to $3500$ nodes. Earlier we had mentioned that the optimal solution runs $20000$ slower (for a much smaller network size $20 - 100$).

## VI. Practical Considerations

In this section, we discuss the practical issues when implementing the proposed auction system.

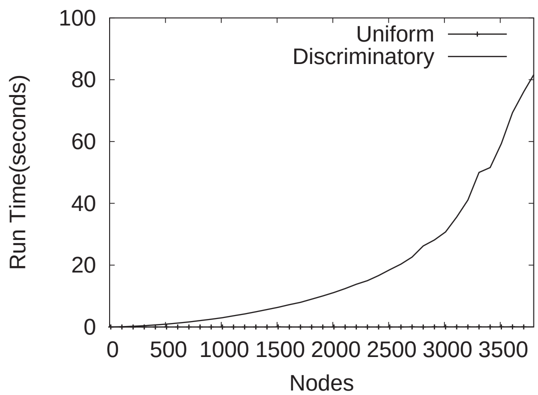



Fig. 12. Run-time of the approximation algorithms under different pricing models.

### A. Identifying interference constraints

The proposed auction system requires information on the interference constraints among buyers. There are multiple mechanisms to obtain this information. Using the scenario of access points based buyers, we list three complementary mechanisms.

(1) The auctioneer (seller) perform network interference measurements to collect interference constraints. A similar mechanism is used in cellular networks to examine interference conditions among base stations.

(2) Individual access points scan radio signals to find interfering access points and report their findings to the auctioneer.

(3) Clients associated with access points sense radio signals and provide feedback on findings of interfering access points [19]. This mechanism has been shown to help refine the interference map.

To minimize the overhead in building conflict graphs, auctioneers can collect conflict information on all the candidate access points. In each round, the auctioneer constructs a conflict graph on the current buyers.

### B. Decentralized auction systems

CAUP and CADP require a centralized server, which in practice might not always be available. In such a case, buyers send bids to local service points who coordinate among themselves to derive allocations and prices. Decentralized systems have the advantage of allowing simple and scalable deployment and providing resilience against point failures.

To build a decentralized auction system, we apply the same bidding language and pricing models, and design coordination [7] based approximation algorithms. The basic concept is to let service points coordinate to apply local adjustments of allocations and prices to their associated buyers, and perform them recursively to improve the total revenue. The complexity of this approach depends on the algorithm complexity and the cost/delay of communications between local service points. We plan to study this system in a future paper.

### C. Iterative bidding and heterogeneous channels

In iterative auctions, buyers submit bids in multiple rounds, and adjust bids based on market feedbacks. Auctioneers use clearing algorithms to derive prices and allocations and provide feedbacks. The challenge lies in simulating feedback and adjusting the bids accordingly. Also, in case of heterogeneous channels with different propagation properties and power limitations, the key issue is to define a standard price-quantity relationship. “Good” spectrum bands should cost more.

Both issues are important for practical spectrum auctions, and can be addressed by combining computational and non-computational (social behavior based) approaches. These are interesting problems in themselves, but we limit ourselves to homogeneous channels and single round bidding to investigate the absolute performance of the auction algorithms.

## VII. Conclusion and Future Work

We propose a spectrum auction framework to provide fast and efficient allocations of spectrum to wireless users. We propose a compact and expressive bidding language using piecewise linear price-quantity curves, two pricing models to address revenue and fairness, and low-complexity market-clearing algorithms to derive prices and allocations in real-time. We perform extensive experiments to verify the performance of the proposed system, and to explore the impact of bidding behaviors, pricing models and node clustering. We conclude that to maximize revenue and spectrum utilization, pricing must be determined based on local demand and availability of resources.

We summarize several practical issues and open problems in Section VI. We are currently working on extending our framework to address these issues. We are also working on extending the proposed framework to maximize other system utility functions.

## Acknowledgement

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