Early Detection of Business Rule Violations

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PhD Committee

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Business Rules

- Organizational goals
- Resource limits
- Safety regulations

• ...





 \rightarrow

Request(user u)@x, Approval(user u)@y, $x+1 \le y$



 \rightarrow

Request(user u)@x, Approval(user u)@y, $x+1 \le y$











Request(user u)@x, Approval(user u)@y, $x+1 \le y$

Request(user u)@x, Approval(user u)@y, $x+1 \le y$

 \rightarrow Payment(user v)@z, Receipt(user v)@w, $z \le y+10$, $z \le w \le z+2$

Request(Alice)@1 Approval(Alice)@2

Request(user u)@x, Approval(user u)@y, $x+1 \le y$



Request(user u)@x, Approval(user u)@y, $x+1 \le y$



Request(user u)@x, Approval(user u)@y, $x+1 \le y$



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An event stream *e* is a *violation* of a rule (set of rules) *R* if no extension of *e* satisfies (each rule in) *R*.

Request(user u)@x, Approval(user u)@y, $x+1 \le y$

 \rightarrow Payment(user v)@z, Receipt(user v)@w, $z \le y+10$, $z \le w \le z+2$



An event stream *e* is a *violation* of a rule (set of rules) *R* if no extension of *e* satisfies (each rule in) *R*.

Can we detect violations at the earliest possible time?

Outline



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Theorem. (Kamp, thesis, 1968): Given any dataless rule, there is an equivalent LTL formula.

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• For each singly-linked, acyclic, dataless rule r, an equivalent LTL formula whose size is *single-exponential* in |r|.

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(Mackey & Su, Info. Sys., 2023) provides translations...

- For each singly-linked, acyclic, dataless rule r, an equivalent LTL formula whose size is *single-exponential* in |r|.
- For each singly-linked, dataless rule r, an equivalent LTL formula whose size is *double-exponential* in |r|.

Outline


- A: Request(u)@x \rightarrow Payment(u)@y, x \leq yB: Request(u)@x, Payment(u)@y, x+5 \leq y \rightarrow Approval(v)@z, z < x</td>

- A:
- A: Request(u)@x → Payment(u)@y, x ≤ y B: Request(u)@x, Payment(u)@y, x+5 ≤ y → Approval(v)@z, z < x

Request(Alice)@10

- A:
- Request(u)@x \rightarrow Payment(u)@y, x \leq y Request(u)@x, Payment(u)@y, x+5 \leq y \rightarrow Approval(v)@z, z < x B:

Request(Alice)@10

Payment(Alice)@y, 10 ≤ y

A: Request(u)@x \rightarrow Payment(u)@y, x \leq y



A: Request(u)@ $x \rightarrow Payment(u)@y, x \le y$



A: Request(u)@ $x \rightarrow Payment(u)@y, x \le y$

B: Request(u)@x, Payment(u)@y, $x+5 \le y \rightarrow Approval(v)@z, z \le x$



• No violation of B at *t* =15

A: Request(u)@ $x \rightarrow Payment(u)@y, x \le y$

B: Request(u)@x, Payment(u)@y, $x+5 \le y \rightarrow Approval(v)@z, z \le x$



... but { A, B } is violated after t =15

- $\begin{cases} A: & \text{Request(u)}@x \rightarrow \text{Payment(u)}@y, x \le y \\ B: & \text{Request(u)}@x, \text{Payment(u)}@y, x+5 \le y \rightarrow \text{Approval(v)}@z, z < x \end{cases}$

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- A: Request(u)@x \rightarrow Payment(u)@y, x \leq y B: Request(u)@x, Payment(u)@y, x+5 \leq y \rightarrow Approval(v)@z, z < x



- A:
- $\begin{aligned} \text{Request(u)} @x & \rightarrow \text{Payment(u)} @y, \ x \leq y \\ \text{Request(u)} @x, \ \text{Payment(u)} @y, \ x+5 \leq y & \rightarrow \text{Approval(v)} @z, \ z < x \end{aligned}$ B:



- **A**:
- $\begin{aligned} \text{Request(u)} @x & \rightarrow \text{Payment(u)} @y, \ x \leq y \\ \text{Request(u)} @x, \ \text{Payment(u)} @y, \ x+5 \leq y & \rightarrow \text{Approval(v)} @z, \ z < x \end{aligned}$ B:



- **A**:
- $\begin{aligned} & \text{Request(u)}@x \rightarrow \text{Payment(u)}@y, \ x \leq y \\ & \text{Request(u)}@x, \ \text{Payment(u)}@y, \ x+5 \leq y \rightarrow \text{Approval(v)}@z, \ z < x \end{aligned}$ B:



- **A**:
- A:Request(u)@x → Payment(u)@y, x ≤ yB:Request(u)@x, Payment(u)@y, x+5 ≤ y → Approval(v)@z, z < x</th>



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- $\begin{aligned} \text{Request(u)} @x & \rightarrow \text{Payment(u)} @y, x \leq y \\ \text{Request(u)} @x, \text{Payment(u)} @y, x+5 \leq y & \rightarrow \text{Approval(v)} @z, z < x \end{aligned}$ B:



- A:
- A: Request(u)@x → Payment(u)@y, x ≤ y B: Request(u)@x, Payment(u)@y, x+5 ≤ y → Approval(v)@z, z < x



Chasing acyclic rules always terminates (Kolatis et al., PODS 2006)

Body assignments								
id	u	X	у	gaps				

Body assignments						He	ead a	assignm	ents
id	U	X	У	gaps	id	V	Z	gaps	deadline

	Body	ass	signi	ments		He	ad a	assignments		Extensions		ons	
id	U	X	У	gaps	id	V	Z	gaps	deadline	body	head	gaps	deadline

	Body	ass	signi	ments		Head assignments			ents		Extensio	ons
id	U	X	У	gaps	id	V	Z	gaps	deadline	body head	gaps	deadline

Request(u)@x, Payment(u)@y, $x+5 \le y \rightarrow Approval(v)@z, z \le x$

	Body	Body assignments				Head assignments			E	Extensio	ons	
id	U	X	У	gaps	id	V	Z	gaps	deadline	body head	gaps	deadline

Approval(Bob)@9

Request(u)@x, Payment(u)@y, $x+5 \le y \rightarrow Approval(v)@z, z < x$

	Body assignments									
id	u	X	У	gaps						

Head assignments									
id	V	Z	gaps	deadline					
β ₂	Bob	9	9 < x	-					

Extensions							
body head	gaps	deadline					

Approval(Bob)@9

Request(u)@x, Payment(u)@y, $x+5 \le y \rightarrow Approval(v)@z, z < x$

	Body assignments									
id	u	X	У	gaps						

	Head assignments									
id	V	Z	gaps	deadline						
β ₂	Bob	9	9 < x	-						

Extensions							
body I	head	gaps	deadline				

Approval(Bob)@9

Request(Alice)@10

Request(u)@x, Payment(u)@y, $x+5 \le y \rightarrow Approval(v)@z, z < x$

	Body assignments									
id	U	X	у	gaps						
α ₁	Alice	10	-	10 ≤ y						

	Head assignments							
Γ	id	V	Z	gaps	deadline			
	β ₂	Bob	9	9 < x	-			

Extensions					
body ł	nead	gaps	deadline		

Approval(Bob)@9

Request(Alice)@10

Request(u)@x, Payment(u)@y, $x+5 \le y \rightarrow Approval(v)@z, z < x$

Body assignments					
id	u	X	у	gaps	
α ₁	Alice	10	-	10 ≤ y	

	Head assignments						
id	V	Z	gaps	deadline			
β ₂	Bob	9	9 < x	-			

Extensions					
body head	gaps	deadline			

Approval(Bob)@9

Request(Alice)@10

Request(u)@x, Payment(u)@y, $x+5 \le y \rightarrow Approval(v)@z, z \le x$

	Body assignments						
id	u	X	У	gaps			
α ₁	Alice	10	-	10 ≤ y			
α ₂	Alice	-	16	x ≤ 16			

	Head assignments						
id	V	Z	gaps	deadline			
β ₂	Bob	9	9 < x	-			

Extensions					
body head	gaps	deadline			

Approval(Bob)@9

Request(Alice)@10

Request(u)@x, Payment(u)@y, $x+5 \le y \rightarrow Approval(v)@z, z \le x$

Body assignments					
id	U	X	У	gaps	
α ₁	Alice	10	-	10 ≤ y	
α ₂	Alice	-	16	x ≤ 16	
α ₃	Alice	10	16	10 ≤ 16	

	Head assignments						
id	V	Z	gaps	deadline			
β ₂	Bob	9	9 < x	-			

Extensions						
body head	gaps	deadline				

Approval(Bob)@9

Request(Alice)@10

Request(u)@x, Payment(u)@y, $x+5 \le y \rightarrow Approval(v)@z, z \le x$

Body assignments						
id	u	x	У	gaps		
α ₁	Alice	10	-	10 ≤ y		
α ₂	Alice	-	16	x ≤ 16		
α ₃	Alice	10	16	10 ≤ 16		

Head assignments				
id	V	Z	gaps	deadline
β ₂	Bob	9	9 < x	-

Extensions			
body	head	gaps	deadline
α ₃	-	z < 10	10

Approval(Bob)@9

Request(Alice)@10

Request(u)@x, Payment(u)@y, $x+5 \le y \rightarrow Approval(v)@z, z < x$

Body assignments				
id	u	x	У	gaps
α ₁	Alice	10	-	10 ≤ <mark>y</mark>
α ₂	Alice	-	16	x ≤ 16
α ₃	Alice	10	16	10 ≤ 16

Head assignments				
id	V	Z	gaps	deadline
β ₂	Bob	9	9 < x	-

Extensions			
body	head	gaps	deadline
α ₃	-	z < 10	10
α ₃	β ₂	-	-

Approval(Bob)@9

Request(Alice)@10

Payment(Alice)@16
- $\begin{cases} A: & \text{Request(u)}@x \rightarrow \text{Payment(u)}@y, x \le y \\ B: & \text{Request(u)}@x, \text{Payment(u)}@y, x+5 \le y \rightarrow \text{Approval(v)}@z, z < x \end{cases}$

- $\begin{cases} A: & \text{Request(u)}@x \rightarrow \text{Payment(u)}@y, x \le y \\ B: & \text{Request(u)}@x, \text{Payment(u)}@y, x+5 \le y \rightarrow \text{Approval(v)}@z, z < x \end{cases}$

	B: Body	' assi	gnme	ents
id	U	X	У	gaps

	B: ⊦	lead	assignn	nents
id	V	Z	gaps	deadline

		B	Extens	sions
	body	head	gaps	deadline
1	α3	-	z < 10	10

- $\begin{cases} A: & \text{Request(u)}@x \rightarrow \text{Payment(u)}@y, x \le y \\ B: & \text{Request(u)}@x, \text{Payment(u)}@y, x+5 \le y \rightarrow \text{Approval(v)}@z, z < x \end{cases}$

	B: B	ody a	assigr	nments			B: I	lead	assignr	ments		B	Extens	sions
id	u	У	gaps	Chased	id v z gaps deadline					body	head	gaps	deadline	
											α ₃	-	z < 10	10

- $\begin{array}{ll} \mathsf{A:} & \mathsf{Request}(\mathsf{u})@\mathsf{x} \to \mathsf{Payment}(\mathsf{u})@\mathsf{y}, \, \mathsf{x} \leq \mathsf{y} \\ & \mathsf{B:} & \mathsf{Request}(\mathsf{u})@\mathsf{x}, \, \mathsf{Payment}(\mathsf{u})@\mathsf{y}, \, \mathsf{x+5} \leq \mathsf{y} \to \mathsf{Approval}(\mathsf{v})@\mathsf{z}, \, \mathsf{z} < \mathsf{x} \end{array}$

	B: B	ody a	assigr	nments			B: ⊦	lead	assignr	ments		В	: Extens	sions
id	u	У	gaps	Chased	id v z gaps deadline					body	head	gaps	deadline	
										α ₃	-	z < 10	10	

- A: Request(u)@x → Payment(u)@y, x ≤ y
 B: Request(u)@x, Payment(u)@y, x+5 ≤ y → Approval(v)@z, z < x

	B: B	ody a	assigr	nments			B: ⊦	lead	assignr	nents		B	: Extens	ions
id	U	X	У	gaps	Chased	id	V	Z	gaps	deadline	body	head	gaps	deadline
α ₁	Alice	10	_	10 ≤ y	-						α ₃	-	z < 10	10
α ₂	Alice	-	У'	x ≤ y'	-									
α ₃	Alice	10	У'	10 ≤ y'	-									

Request(Alice)@10

Payment(<mark>Alice</mark>)@y', 10 ≤ y'

- $\begin{cases} A: & \text{Request(u)}@x \rightarrow \text{Payment(u)}@y, x \le y \\ B: & \text{Request(u)}@x, \text{Payment(u)}@y, x+5 \le y \rightarrow \text{Approval(v)}@z, z < x \end{cases}$

	B: B	ody a	assigr	nments			B: ⊦	lead	assignr	nents		В	: Extens	sions
id	u	X	У	gaps	Chased	id	V	Z	gaps	deadline	body	head	gaps	deadline
α ₁	Alice	10	_	10 ≤ y	-						α ₃	-	z < 10	10
α ₂	Alice	-	У'	x ≤ y'	-									
α ₃	Alice	10	У'	10 ≤ y'	Yes									
	Approv 15 ≤ y'	al(<mark>Alic</mark>	ce)@z	z', z' < 10	Re	equest	(Alice	e)@1	0		Paym	ent(<mark>Ali</mark>	<mark>ce)@y'</mark> ,	10 ≤ y'

_ _ _ _ _ _ _ _ _ _ _ _ _ _

 $\begin{cases} A: & \text{Request(u)}@x \rightarrow \text{Payment(u)}@y, x \le y \\ B: & \text{Request(u)}@x, \text{Payment(u)}@y, x+5 \le y \rightarrow \text{Approval(v)}@z, z < x \end{cases}$

	B: B	ody a	assigr	nments			B: ⊦	lead	assignr	nents		В	: Extens	sions
id	u	X	у	gaps	Chased	id	V	Z	gaps	deadline	body	head	gaps	deadline
α ₁	Alice	10	_	10 ≤ y		β ₂	Bob	Z'		_	α ₃	-	z < 10	10
α ₂	Alice	-	у'	x ≤ y'	-									
α ₃	Alice	10	У'	10 ≤ y'	Yes									
	$\begin{cases} \text{Approval}(\text{Alice})@z', z' < 10 \\ 15 \le v' \end{cases} \text{Request}(\text{Alice})@10 \\ \end{cases}$													

- $\begin{cases} A: & \text{Request(u)}@x \rightarrow \text{Payment(u)}@y, x \le y \\ B: & \text{Request(u)}@x, \text{Payment(u)}@y, x+5 \le y \rightarrow \text{Approval(v)}@z, z < x \end{cases}$

	B: B	ody a	issigr	nments			B: ⊢	lead	assignr	nents		В	: Extens	sions
id	u	X	У	gaps	Chased	id	V	Ζ	gaps	deadline	body	head	gaps	deadline
α ₁	Alice	10	-	10 ≤ y	-	β ₂	Bob	Ζ'	-	_	α ₃	-	z < 10	10
α ₂	Alice	-	у'	x ≤ y'	-						α ₃	β ₂	z' < 10 15 < v'	10
α ₃	Alice	10	У'	10 ≤ y'	Yes								10 – y	
	Approv 15 ≤ y'	al(<mark>Alic</mark>	ce)@2	z', z' < 10	Re	equest	(Alice)@1	0		Paym	ent(<mark>Ali</mark>	<mark>ce)@y'</mark> ,	10 ≤ y' _

- A: Request(u)@ $x \rightarrow$ Payment(u)@ $y, x \leq y$
- B: Request(u)@x, Payment(u)@y, $x+5 \le y \rightarrow Approval(v)@z, z \le x$



No violation at time t iff (when α_3 exists at t, it can match with β_2)

- $\begin{cases} A: & \text{Request(u)}@x \rightarrow \text{Payment(u)}@y, x \le y \\ B: & \text{Request(u)}@x, \text{Payment(u)}@y, x+5 \le y \rightarrow \text{Approval(v)}@z, z < x \end{cases}$

	B: B	ody a	assigr	nments			B: H	lead	assignr	nents		В	: Extens	sions
id	u	X	У	gaps	Chased	id	V	Ζ	gaps	deadline	body	head	gaps	deadline
α ₁	Alice	10	_	10 ≤ y	-	β ₂	Bob	Z'	-	_	α ₃	-	z < 10	10
α ₂	Alice	-	у'	x ≤ y'	-						α ₃	β ₂	z' < 10	10
α ₃	Alice	10	У'	10 ≤ y'	<mark>Yes</mark>								10 <u>-</u> y	
$ \left[\begin{array}{c} \text{Approval}(\text{Alice})@z', z' < 10 \\ 15 \le y' \end{array} \right] \left[\begin{array}{c} \text{Request}(\text{Alice})@10 \end{array} \right] $											ent(<mark>Ali</mark>	<mark>ce)@y'</mark> ,	10 ≤ y'	

No violation at time t iff (when α_3 exists at t, it can match with β_2)

No violation at time t iff SAT($(15 \le y' \rightarrow z' \le 10)$

- $\begin{cases} A: & \text{Request(u)}@x \rightarrow \text{Payment(u)}@y, x \le y \\ B: & \text{Request(u)}@x, \text{Payment(u)}@y, x+5 \le y \rightarrow \text{Approval(v)}@z, z < x \end{cases}$

	B: B	ody a	assigr	nments			B: ⊢	lead	assignr	nents		В	: Extens	sions
id	u	X	У	gaps	Chased	id	V	Ζ	gaps	deadline	body	head	gaps	deadline
α ₁	Alice	10	-	10 ≤ y	-	β ₂	Bob	Ζ'	_	_	α ₃	-	z < 10	10
α ₂	Alice	-	у'	x ≤ y'	-						α ₃	β ₂	z' < 10 15 < v'	10
α ₃	Alice	10	У'	10 ≤ y'	<mark>Yes</mark>								10 <u>-</u> y	
	Approv 15 ≤ y'	al(Alio	ce)@z	z', z' < 10	Re	equest	(Alice)@1	0		Paym	ent(<mark>Ali</mark>	<mark>ce)@y'</mark> ,	10 ≤ <mark>y</mark> '

No violation at time t iff (when α_3 exists at t, it can match with β_2)

No violation at time t iff SAT($(15 \le y' \rightarrow z' < 10) \land (t < y') \land (t < z')$)

- A: Request(u)@ $x \rightarrow$ Payment(u)@ $y, x \leq y$
- B: Request(u)@x, Payment(u)@y, $x+5 \le y \rightarrow Approval(v)@z, z \le x$



No violation at time t iff (when α_3 exists at t, it can match with β_2)

No violation at time t iff SAT($(15 \le y' \rightarrow z' \le 10) \land (t \le y') \land (t \le z')$)

Outline

















Aggregation functions: sum, max, min, count, countu

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Aggregation functions: sum, max, min, count, countu



Aggregation functions: sum, max, min, count, countu

```
SlidingMax( m = max(a), t)@t+2
```



Aggregation functions: sum, max, min, count, countu

```
SlidingMax( m = max(a), t)@t+2
```



For each (open) window and aggregation function (*sum*, *max*, *min*, *count*, *countu*), there is an equivalent Presburger arithmetic constraint.

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SlidingSum(s = sum(a), t)@t+2

```
SlidingMax( m = max(a), t)@t+2
```



SlidingSum(s', 1)@3, s' = 80 + 60 + a

For each (open) window and aggregation function (*sum*, *max*, *min*, *count*, *countu*), there is an equivalent Presburger arithmetic constraint.

SlidingSum(s = sum(a), t)@t+2

SlidingMax(*m* = max(a), t)@t+2

SlidingSum(s', 1)@3, s' = 80 + 60 + a

SlidingMax(m', 1)@3, ((m' = 80)
$$\lor$$
 (m' = 60) \lor (m' = a))
 \land
((80 ≤ m') \land (60 ≤ m') \land (a ≤ m'))
SlidingSum(s = sum(a), t)@t+2 $\rightarrow s \le 200$ SlidingMax(m = max(a), t)@t+2 $\rightarrow m \ge 100$

SlidingSum(s = sum(a), t)@t+2 $\rightarrow s \le 200$ SlidingMax(m = max(a), t)@t+2 $\rightarrow m \ge 100$

SlidingSum(s = sum(a), t)@t+2 $\rightarrow s \le 200$ SlidingMax(m = max(a), t)@t+2 $\rightarrow m \ge 100$

SlidingSum(s', 1)@3, s' = 80 + 60 + a

SlidingMax(m', 1)@3, ((m' = 80)
$$\lor$$
 (m' = 60) \lor (m' = a))
 \land
((80 ≤ m') \land (60 ≤ m') \land (a ≤ m'))

Theorem. The earliest violation for an acyclic set of rules with aggregation can be computed.

```
SlidingSum(s = sum(a), t)@t+2 \rightarrow s \le 200
SlidingMax(m = max(a), t)@t+2 \rightarrow m \ge 100
```

SlidingSum(s', 1)@3, s' = 80 + 60 + a

SlidingMax(m', 1)@3, ((m' = 80)
$$\lor$$
 (m' = 60) \lor (m' = a))
 \land
((80 ≤ m') \land (60 ≤ m') \land (a ≤ m'))

Theorem. The earliest violation for an acyclic set of rules with aggregation can be computed.

```
SlidingSum(s = sum(a), t)@t+2 \rightarrow s \le 200
SlidingMax(m = max(a), t)@t+2 \rightarrow m \ge 100
```

SlidingSum(s', 1)@3, s' = 80 + 60 + a

SlidingMax(m', 1)@3, ((m' = 80)
$$\lor$$
 (m' = 60) \lor (m' = a))
 \land
((80 ≤ m') \land (60 ≤ m') \land (a ≤ m'))

Sliding	SlidingSum Head assignments					
id t s gaps						
α ₁	1	s'	s' = 80 + 60 + a s' ≤ 200			

Theorem. The earliest violation for an acyclic set of rules with aggregation can be computed.

SlidingSum(
$$s = sum(a), t$$
)@t+2 $\rightarrow s \le 200$ SlidingMax($m = max(a), t$)@t+2 $\rightarrow m \ge 100$ idtsgapsPay(\$80)@1Pay(\$60)@2Pay(a)@3SlidingSum Head assignmentsidtmgapsSlidingSum(s', 1)@3, s' = 80 + 60 + aSlidingSum(s', 1)@3, s' = 80 + 60 + aSlidingSum(s', 1)@3, (m' = 80) \lor (m' = 60) \lor (m' = a))SlidingMax(m', 1)@3, ((m' = 80) \lor (m' = 60) \lor (m' = a))SlidingSum(s', 1)@3, ((m' = 80) \lor (m' = 60) \lor (m' = a))NSlidingMax(m', 1)@3, ((m' = 80) \lor (m' = 60) \lor (m' = a))NNNNSlidingMax(m', 1)@3, ((m' = 80) \lor (m' = 60) \lor (m' = a))NNNNSlidingMax(m', 1)@3, ((m' = 80) \lor (m' = 60) \lor (m' = a))NNNNSlidingMax(m', 1)@3, ((m' = 80) \lor (m' = 60) \lor (m' = a))NNNNSlidingMax(m', 1)@3, ((m' = 80) \lor (m' = 60) \lor (m' = a))NNNNSlidingMax(m', 1)@3, ((m' = 80) \lor (m' = 60) \lor (m' = a))NNN

Theorem. The earliest violation for an acyclic set of rules with aggregation can be computed.



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Outline



Theorem. Early violation detection for a set of rules is impossible.

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Proof Idea: We introduce *finite satisfiability* for a set of rules,

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Theorem. Early violation detection for a set of rules is impossible.

Proof Idea: We introduce *finite satisfiability* for a set of rules,

• Finite Satisfiability: given a set of rules R, is there a finite event stream that satisfies R?

which reduces to early violation detection,

and we show finite satisfiability is undecidable by a reduction from the empty-tape Turing machine halting problem.

configurations



configurations



Config(0, #, -) Config(1, __, s0) Config(2, #, -)

Config				
index	tape	state		
0	#	-		
1		sO		
2	#	-		

configurations



Config(0, #, -) Config(1, __, s0) Config(2, #, -)

Config				
index	tape	state		
0	#	-		
1		sO		
2	#	-		

configurations









Config				
index	tape	state		
0	#	-		
1		sO		
2	#	-		
3	С	-		
4		s2		
5	#	-		

configurations







s1

Config				
index	tape	state		
0	#	-		
1]	sO		
2	#	-		
3	С	-		
4]	s2		
5	#	-		
6	С	-		
7	0	-		
8		s1		
9	#	-		

configurations





s2



Config			
index	tape	state	



Config					
index	tape	state			
0	#	-			
1]	sO			
2	#	-			



Config					
index	tape	state			
0	#	-			
1]	sO			
2	#	-			



If M starts in s_0 , then R_M has:

true

 \rightarrow

Config(0, #, -), Config(1, __, s0), Config(2, #, -)

Next			Config	
index	next	index	tape	state
0	2	0	#	-
1	3	1		sO
2	5	2	#	-



If M starts in s0, then R_M has:

true

 \rightarrow

Config(0, #, -), Config(1, __, s0), Config(2, #, -) Next(0, 2), Next(1, 3), Next(2, 5),

Encoding transitions of TM with Config, Next, and rules

Next			Config	
index	next	index	tape	state
0	2	0	#	-
1	3	1]	sO
2	5	2	#	-



If M has $\delta(s0, _) = (c, s2, R)$, then R_M has:

Encoding transitions of TM with Config, Next, and rules

Next			Config	
index	next	index	tape	state
0	2	0	#	-
1	3	1		s0
2	5	2	#	-
3	6	3	С	-
4	7	4		s2
5	9	5	#	-



If M has $\delta(s0, _) = (c, s2, R)$, then R_M has:

Next(x-1, y-1), Config(x-1, #, -), Config(x, _, s0), Config(x+1, #, -)

```
\rightarrow
```

Next(x+2, y+3), Config(y-1, #, -), Config(y, c, -), Config(y+1, __, s2), Config(x+2, #, -)

Encoding transitions of TM with Config, Next, and rules

Nex	t	Config		
index	next	index	tape	state
0	2	0	#	-
1	3	1		sO
2	5	2	#	-
3	6	3	С	-
4	7	4		s2
5	9	5	#	-
6	10	6	С	-
7	11	7	0	-
8	12	8	<u> </u>	s1
9	13	9	#	-



Next(x-1, y-1), Config(x-1, #, -), Config(x, _, s2), Config(x+1, #, -)

 \rightarrow

Next(x+2, y+3), Config(y-1, #, -), Config(y, 0, -), Config(y+1, __, s1), Config(x+2, #, -)

Next		Config			
index	next	index	tape	state	
0	2	0	#	-	
1	3	1		sO	
2	5	2	#	-	
3	6	3	С	-	
4	7	3	l	s2	
5	9	4		-	
6	10	5	#	-	
7	11	6	С	-	
8	12	7	0	-	
9	14	8		s1	

Next		Config			
index	next	index	tape	state	
0	2	0	#	-	
1	3	1		sO	
2	5	2	#	-	
3	6	3	С	-	
4	7	3		s2	
5	9	4		-	
6	10	5	#	-	
7	11	6	С	-	
8	12	7	0	-	
9	14	8		s1	

Next		Config			Error
index	next	index	tape	state	x
0	2	0	#	-	
1	3	1	<u> </u>	sO	
2	5	2	#	-	
3	6	3	С	-	
4	7	3		s2	
5	9	4		-	
6	10	5	#	-	
7	11	6	С	-	
8	12	7	0	-	
9	14	8		s1	

Next		Config			Error
index	next	index	tape	state	x
0	2	0	#	-	0
1	3	1	<u> </u>	sO	
2	5	2	#	-	
3	6	3	С	-	
4	7	3	l	s2	
5	9	4		-	
6	10	5	#	-	
7	11	6	С	-	
8	12	7	0	-	
9	14	8		s1	

Don't allow malformed configurations:

Config(x, a, s), Config(x, b, s'), $a \neq b$ \rightarrow Error(0)
Detect non-valid computations with Error rules

Next		Config			Error
index	next	index	tape	state	x
0	2	0	#	-	0
1	3	1		sO	
2	5	2	#	-	
3	6	3	С	-	
4	7	3	l	s2	
5	9	4		-	
6	10	5	#	-	
7	11	6	С	-	
8	12	7	0	-	
9	14	8		s1	

Don't allow malformed configurations:

Config(x, a, s), Config(x, b, s'), $a \neq b$ \rightarrow Error(0)

Propagate Errors infinitely:

 $Error(x) \rightarrow Error(x+1)$

Detect non-valid computations with Error rules

Next		Config			Erro
index	next	index	tape	state	x
0	2	0	#	-	0
1	3	1		sO	1
2	5	2	#	-	2
3	6	3	С	-	3
4	7	3		s2	
5	9	4		-	
6	10	5	#	-	
7	11	6	С	-	
8	12	7	0	-	
9	14	8		s1	

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Detect non-valid computations with Error rules

Next		Config			Error
index	next	index	tape	state	x
0	2	0	#	-	0
1	3	1	<u> </u>	sO	1
2	5	2	#	-	2
3	6	3	С	-	3
4	7	3	l	s2	
5	9	4		-	
6	10	5	#	-	
7	11	6	С	-	
8	12	7	0	-	
9	14	8		s1	

Don't allow malformed configurations:

Config(x, a, s), Config(x, b, s'), $a \neq b$ \rightarrow Error(0)

Propagate Errors infinitely:

Error(x) \rightarrow Error(x+1)

For a Turing machine M, the set R_M is finitely satisfiable *iff* M halts on empty tape.

Outline



Thesis Organization



- We improve the size complexity of translations from two subclases of dataless rules to LTL.

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Future Directions

- We improve the size complexity of translations from two subclases of dataless rules to LTL.
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Future Directions

- Can violations of more complex time constraints be detected early?

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- We show early violation detection for an arbitrary set of rules is impossible.

Future Directions

- Can violations of more complex time constraints be detected early?
- Do richer sets of rules have (efficient) algorithms, e.g., negation, disjunction?

Thank you! Questions?