# Private Information Retrieval in Large Scale Public Data Repositories 

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The problem of protecting private data repositories stored remotely is well-studied

Private Files


Encrypted files



Remote file storage

Encryption hides file contents from an attacker.

## Encryption does not hide data access patterns

The access patterns leaks:

- Which file is being accessed?
- When was it last accessed?
- Is it being accessed for a read or a write?
- Is it being accessed sequentially or randomly?


## ORAM (STOC ‘87) hides data access patterns for private files

Private Files


Oblivious RAM
(Goldreich STOC ‘87, Path ORAM JACM ‘18, SCORAM CCS ' $14, \ldots$ )

## Encryption +

 randomized data accesses

Remote file storage

Dropbox


User

Hidden:
$\rightarrow$ Which file is being accessed?
$\rightarrow$ Whether the access is a read or write
$\rightarrow$ When was the file accessed last

## We can extend protection to private relational databases stored remotely

CryptDB SOSP ‘11, MONOMI VLDB ‘13, ...


Adjustable query-based encryption (onion)

## What is common to all of these cases?

Private Files


The user owns the data!

## But, much of the content on the Internet is in public data repositories


User

I want to stream "The Godfather"
$\qquad$


Show me the latest post by Elon Musk


NETFLIX YouTube

Remote server

facebook

## But, much of the content on the Internet is in public data repositories



How can we hide access patterns (queries) over public data repositories?

## Both users and service providers want to hide access patterns over public repositories



User may:

- Consider queries private
- Belong to a vulnerable population or a minority group

Server can be:

- Hacked by an outsider
- Compromised by an insider
- Coerced by a nation state [1, 2]

1. Brian Fung. Analysis: There is now some public evidence that China viewed TikTok data. CNN, 2023.
2. Sapna Maheshwari and Ryan Mac. Driver's Licenses, Addresses, Photos: Inside How TikTok Shares User Data. New York Times, 2023

## This tutorial:

Discuss a cryptographic method to privately retrieve data from public data repositories, thus making server opaque to data access patterns

Private retrieval from public databases can be abstracted into the key-value store model


Client retrieves:

- $v$, if $(k, v)$ at Server
- $\varnothing$, otherwise


Untrusted Server

Focus on performance, scalability, and practicality

## This tutorial is in two parts

Part 1: Retrieval by location

| key | location |
| :---: | :---: |
| $\mathrm{k}_{0}$ | 0 |
| $\mathrm{k}_{1}$ | 1 |
| $\ldots$ | $\cdots$ |
| $\mathrm{k}_{\mathrm{n}-1}$ | $\mathrm{n}-1$ |



Give me the $i$-th value
Has (key, location) mapping

|  | $\mathrm{v}_{0}$ |
| :---: | :---: |
|  | $\mathrm{v}_{1}$ |
|  | $\mathrm{v}_{2}$ |
| -1 | $\ldots$ |
|  | $\mathrm{v}_{\mathrm{n}-1}$ |
|  |  |

Untrusted Server

Part 1: How can the client privately retrieve the value corresponding to a given location?

## This tutorial is in two parts

## Part 2: Retrieval by key



Client retrieves:

- v , if ( $\mathrm{k}, \mathrm{v}$ ) at Server
- $\emptyset$, otherwise

| $\mathrm{k}_{0}$ | $\mathrm{v}_{0}$ |
| :---: | :---: |
| $\mathrm{k}_{1}$ | $\mathrm{v}_{1}$ |
| $\mathrm{k}_{2}$ | $\mathrm{v}_{2}$ |
| $\ldots$ | $\ldots$ |
| $\mathrm{k}_{\mathrm{n}-1}$ | $\mathrm{v}_{\mathrm{n}-1}$ |

Untrusted Server

Part 2: How can the client privately retrieve the value corresponding to a given key?

## This tutorial is in two parts

## Part 1: Retrieval by location

| key | location |
| :---: | :---: |
| $\mathrm{k}_{0}$ | 0 |
| $\mathrm{k}_{1}$ | 1 |
| $\cdots$ | $\cdots$ |
| $\mathrm{k}_{\mathrm{n}-1}$ | $\mathrm{n}-1$ |



Has (key, location)
Give me the $i$-th value mapping

|  | $\mathrm{v}_{0}$ |
| ---: | :---: |
|  | $\mathrm{v}_{1}$ |
|  | $\mathrm{v}_{2}$ |
|  | $\ldots$ |
| -1 | $\mathrm{v}_{\mathrm{n}-1}$ |
|  |  |

Untrusted Server

Part 1: How can the client privately retrieve the value corresponding to a given location?

## This problem can be solved using Private Information Retrieval (PIR) (Chor et al. FOCS ‘95)

PIR: Query, Answer, Decode



## PIR has two key requirements

## Correctness

Query for $\mathrm{db}[\mathrm{i}]$ returns $\mathrm{db}[\mathrm{i}]$ to the user
Decode(Answer(db, Query $(i)))=d b[i]$

## Privacy

Server learns "nothing" about the location i
For all locations i, j,
$\{$ View of the server in answering Query $(\mathrm{i})\} \approx$
\{View of the server in answering Query(j)\}

## We are also interested in performance considerations

## Network cost

Request size: |Query(i)|
Response size: |Answer(db, Query(i))|
Compute cost
Time to compute Answer(db, Query(i))

## One solution to private information retrieval in Trivial PIR



## Performance characteristics of trivial PIR

## Network cost

Request size: 1 bit
Response size: $\mathrm{n} \times|\mathrm{db}[\mathrm{i}]|$

## Compute cost

Time to compute Answer(db, Query(i))

Can we do better than sending the entire database? If so, how?

## Warmup for (non-trivial) PIR

Assume that we do not care about privacy yet; only correctness


Untrusted Server
Retrieval is equivalent to computing a dot product

## Warmup for (non-trivial) PIR in more detail

Multiply component-wise


Dot product requires two types of operations:
$\rightarrow$ Multiplications ( $8 \times 0,5 \times 1$, etc.)
$\rightarrow$ Additions (e.g., $0+5+\ldots$ )

## Detour: Introduction to Homomorphic Encryption

A form of encryption which allows computations over encrypted data
Two classes of homomorphic encryption

## Fully Homomorphic Encryption [Gentry'09]

- Supports computations for any arbitrary function
- Challenge: Can be Quite inefficient


## Partially Homomorphic Encryption

- Supports a particular type of operation

Additive Homomorphic encryption
$\operatorname{Enc}(4) \oplus \operatorname{Enc}(8)=\operatorname{Enc}(4+8)=\operatorname{Enc}(12)$

Multiplicative Homomorphic encryption
$\operatorname{Enc}(4) \otimes \operatorname{Enc}(8)=\operatorname{Enc}(4 \times 8)=\operatorname{Enc}(32)$

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## Example: El Gamal additive homomorphic encryption

We have a message $m$ which we want to encrypt
Encryption key: (g, h)

Encryption procedure:
Pick a random number r
$\operatorname{Enc}(m, r)=\left(g^{r}, g^{m} h^{r}\right)$

## Example: El Gamal additive homomorphic encryption

$\operatorname{Enc}(m, r)=\left(g^{r}, g^{m} h^{r}\right)$
Given two messages m1 and m2

$$
\operatorname{Enc}(m 1, r 1)=\left(g^{r 1}, g^{m 11} h^{r 1}\right)
$$

$\operatorname{Enc}(\mathrm{m} 1) \times \operatorname{Enc}(\mathrm{m} 2)=\operatorname{Enc}(\mathrm{m} 1+\mathrm{m} 2)$

The product of the encryptions of two messages is an encryption of the sum of the two messages.
$\operatorname{Enc}(m 2, r 2)=\left(g^{r 2}, g^{m 2} h^{r 2}\right)$
$\operatorname{Enc}(m 1, r 1) \times \operatorname{Enc}(m 2, r 2)=\left(g^{r 1}, g^{m 1} h^{r 1}\right) \times\left(g^{r 2}, g^{m 2} h^{r 2}\right)$

$$
\begin{aligned}
& =\left(g^{r 1+r 2}, g^{m 1+m 2} h^{r 1+r 2}\right) \\
& =\operatorname{Enc}\left(m_{1}+m_{2}, r_{1}+r_{2}\right)
\end{aligned}
$$

## Example: El Gamal additive homomorphic encryption

$$
E n c(m 1) \times E n c(m 2)=E n c(m 1+m 2)
$$

Additive Homomorphic Encryption supports multiplying an encrypted value with a plaintext value

We have a message m, encrypted as Enc(m)
We have another message k (not encrypted)

$$
\begin{array}{rlr}
{[\operatorname{Enc}(m)]^{\mathrm{k}}} & =\operatorname{Enc}(m) \times \operatorname{Enc}(m) \times \ldots \times \operatorname{Enc}(m) \\
& =\operatorname{Enc}(m+m+\ldots+m) \\
& =\operatorname{Enc}(m * k) & \operatorname{Enc}(m)^{\mathrm{k}}=\operatorname{Enc}\left(m^{*} k\right)
\end{array}
$$

## We only need additive homomorphic encryption for PIR

Homomorphic addition
$\operatorname{Enc}\left(m_{1}\right) \times \operatorname{Enc}\left(m_{2}\right)=\operatorname{Enc}\left(m_{1}+m_{2}\right)$

Homomorphic plaintext multiplication

```
Enc(m)}\mp@subsup{)}{}{k}=\operatorname{Enc}(m* k
```


## Recall the warmup for (non-trivial) PIR

Multiply component-wise


Dot product requires two types of operations:
$\rightarrow$ Multiplications ( $8 \times 0,5 \times 1$, etc.)
$\rightarrow$ Additions (e.g., $0+5+\ldots$ )

## Recall the warmup for (non-trivial) PIR

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## Recall the warmup for (non-trivial) PIR

Homomorphically multiply component-wise $\operatorname{Enc}(m)^{\mathrm{k}}=\operatorname{Enc}\left(\mathrm{m}^{*} k\right)$


Dot product requires two types of operations:
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$$
\operatorname{Enc}\left(m_{1}\right) \times \operatorname{Enc}\left(m_{2}\right)=\operatorname{Enc}\left(m_{1}+m_{2}\right)
$$

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$$

Dot product requires two types of operations:
$\rightarrow$ Multiplications ( $8 \times 0,5 \times 1$, etc.)
$\rightarrow$ Additions (e.g., $0+5+\ldots$ )

## Putting it all together: A PIR protocol

Step 1: Query(1)=

Step 3:
db[1] = Decode(ans) = Decrypt(ans)

Query(1)


Step 2:
Answer $(\mathrm{db}, \mathrm{q})$ is a secure dot product

Untrusted Server

Retrieval is equivalent to computing a secure dot product

## What is the size of the PIR response?

Response is a ciphertext: Enc(db[i])
Recall:

$$
\operatorname{Enc}(m, r)=\left(g^{r}, g^{m} h^{r}\right)
$$

Encrypting 1 message yields 2 components
Expansion factor, $f=$ size of ciphertext / size of plaintext
Expansion factor for El Gammal $=2$

## Performance characteristics of additively HE-based PIR

Network cost
Request size: nx |ciphertext|
Response size: |ciphertext|
Expansion factor: $\mathrm{f}=|\mathrm{ciphertext\mid} /|\mathrm{db}[\mathrm{i}]|$
Compute cost
Time to compute Answer(db, Query(i)) is $\mathbf{O}(\mathrm{n})$ homomorphic ops
This linear compute overhead is a fundamental lower bound (Beimel et al. CRYPTO '00)

## Much of the research on PIR is on reducing request size and server-side compute overhead

| Overhead | High-level technique |
| :--- | :--- |
| Request size | $\bullet$ <br> $\bullet$$\quad$ Recursion (Stern 1998) |
| Server-side compute | $\bullet$ <br> $\bullet$ <br> $\bullet$$\quad$ PIR with preprocessing (Beimel et al. ‘00, SimplePIR ‘23) |

## How to reduce query size?

| 0 |
| :--- |
| 0 |
| 0 |
| 0 |
| 0 |
| 0 |
| 1 |
| 0 |
| $\cdots$ |
| 0 |


| $a$ |
| :---: |
| $b$ |
| $c$ |
| $d$ |
| $e$ |
| $f$ |
| $g$ |
| $h$ |
| $\ldots$ |
| $p$ |

Instead of 1 dim database, view it in 2 dims. Instead of 1 query, use 2 queries.


| 0 |
| :---: |
| 1 |
| 0 |
| 0 |


| $a$ | $b$ | $c$ | $d$ |
| :---: | :---: | :---: | :---: |
| $e$ | $f$ | $g$ | $h$ |
| $i$ | $j$ | $k$ | l |
| $m$ | $n$ | $o$ | $p$ |

## Two-stage query execution



In first pass, extract the row of interest

## Two-stage query execution



Add columns

So, query size is down from $n$ to $2 \sqrt{ } n$.

## But result is double encrypted

- After first stage, each element is a ciphertext, size is $f^{*}$ plaintext size
- After second stage, result size is $f^{2}$ * plaintext size
- The efficient homomorphic encryption schemes can have $f \geq 8$

Trade-off between query and response size Stern (1998) recursion scheme

- Reduce query size to $d^{* d} \sqrt{ } n$
- Expand result size by $f^{\text {d }}$
- Used in XPIR (2016)


## Much of the research on PIR is on reducing request size and server-side compute overhead

| Overhead | High-level technique |
| :--- | :--- |
| Request size | $\bullet \quad$ Recursion (Stern 1998) |

SealPIR (Microsoft Research - 2018)

- Compress query by a large factor ( $2^{11}$ )
- Trade-off: query expansion at the server requires high compute cost


## How to reduce server-side compute overhead?

PIR with preprocessing (Beimel et al CRYPTO ‘00, SimplePIR ‘23)


Does not violate the linear compute lower bound (Beimel et al. CRYPTO '00)

## How to reduce server-side overhead?

Another option is to pay linear overhead but improve the constant

## Key techniques in FastPIR (OSDI '21)

- Use lattice-based additively homomorphic encryption scheme
- Single-input multiple data (SIMD) capabilities
- Query and response compression using homomorphic rotation operations


## FastPIR has lower processing time than all other variants (that do not use preprocessing)

Experiment results (c5.12x large in AWS; 1M values, 256 bytes each)

| PIR Scheme | Processing time (ms) | Response size (KB) |
| :---: | :---: | :---: |
| FastPIR | 947 | 64 |
| XPIR-1 | 3,389 | 32 |
| XPIR-2 | 1,894 | 288 |
| SealPIR-1 | 76,216 | 32 |
| SeaIPIR-2 | 2,556 | 320 |

## This tutorial is in two parts

Part 1: Retrieval by location

| $\mathrm{k}_{0}$ | 0 |
| :---: | :---: |
| $\mathrm{k}_{1}$ | 1 |
| $\mathrm{k}_{2}$ | 2 |
| $\cdots$ | $\cdots$ |
| $\mathrm{k}_{\mathrm{n}-}$ | $\mathrm{n}-1$ |
| 1 |  |



Has (key, location) mapping

Give me the $i$-th value

|  | $\mathrm{v}_{0}$ |
| ---: | :---: |
|  | $\mathrm{v}_{1}$ |
|  | $\mathrm{v}_{2}$ |
|  | $\ldots$ |
| -1 | $\mathrm{v}_{\mathrm{n}-1}$ |
|  |  |

Untrusted Server

Part 1: How can the client privately retrieve the value corresponding to a given location?

## This tutorial is in two parts

Part 2: Retrieval by key?


Part 2: How can the client privately retrieve the value corresponding to a given key?

This area originated as Private retrieval by keywords in 1998 (Chor et al. TOC ‘98)

Private Keyword retrieval can be performed by two stages:
Stage 1: Retrieve the key location


| $k_{0}$ | $v_{0}$ |
| :---: | :---: |
| $k_{1}$ | $v_{1}$ |
| $k_{2}$ | $v_{2}$ |
| $\ldots$ | $\cdots$ |
| $\mathrm{k}_{\mathrm{n}-1}$ | $\mathrm{v}_{\mathrm{n}-1}$ |

Stage 2: Perform PIR with location


## PIR-by-keywords has two requirements

## Correctness

Query for $k$ returns $v$ iff ( $k, v$ ) is in $d b$

## Privacy

Server learns "nothing" about the key k
For any two possible keys $\mathrm{k}_{\mathrm{i}}, \mathrm{k}_{\mathrm{j}}$
$\left\{\right.$ View of the server in answering Query $\left.\left(\mathrm{k}_{\mathrm{i}}\right)\right\} \approx$
$\left\{V i e w\right.$ of the server in answering Query $\left(\mathrm{K}_{\mathrm{j}}\right)$ \}

## We are also interested in performance considerations

## Network cost

Request size, Response size
Number of round trips between user and server

## Compute cost

Time to compute the response

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Stage 1: Retrieve the key location


| $k_{0}$ | $v_{0}$ |
| :---: | :---: |
| $\mathrm{k}_{1}$ | $\mathrm{v}_{1}$ |
| $\mathrm{k}_{2}$ | $\mathrm{v}_{2}$ |
| $\ldots$ | $\cdots$ |
| $\mathrm{k}_{\mathrm{n}-1}$ | $\mathrm{v}_{\mathrm{n}-1}$ |

## Stage 2: Perform PIR by index




## Key location can be retrieved using PIR-by-index

(Chor et al. TOC ‘98)


Assume keys are integers and arranged in a BST

$$
K=\{1,5,6,10,17,19,20\}
$$



Untrusted Server

## Key location can be retrieved using PIR-by-index

(Chor et al. TOC ‘98)


Assume keys are integers and arranged in a BST

$$
K=\{1,5,6,10,17,19,20\}
$$

Level 1: Retrieve element at index 0 (trivial)

$$
10<17
$$

Go right


Untrusted Server

## Key location can be retrieved using PIR-by-index

(Chor et al. TOC ‘98)

## Assume keys are integers and arranged in a BST



$$
K=\{1,5,6,10,17,19,20\}
$$

Level 2: Retrieve element at index 1 using PIR-by-index
$17<19$
Go left


Untrusted Server

## Key location can be retrieved using PIR-by-index

(Chor et al. TOC ‘98)


Level 3: Retrieve element at index 2 using PIR-by-index

$$
\begin{aligned}
& 17=17 \text { (found it!) } \\
& \text { Path from root to leaf is } \\
& \text { index of } k \text { in keyset K }
\end{aligned}
$$

Assume keys are integers and arranged in a BST

$$
K=\{1,5,6,10,17,19,20\}
$$



Untrusted Server

This area originated as Private retrieval by keywords in 1998 (Chor et al. TOC ‘98)

Private Keyword retrieval can be performed by two stages:
Stage 1: Retrieve the key location


Stage 2: Perform PIR by index


## Performance of BST-based PIR-by-keywords

 Stage $1+$ Stage 2Network cost: $0<$ level $<\log (\mathrm{n})$
Request size: $\sum$ PIR-request-size(2 $\left.2^{\text {level }}\right)+$ PIR-request-size( $n$ )
Response size: $\sum$ PIR-response-size(2level) + PIR-response-size(n)
Number of round trips between user and server: $\log (\mathbf{n})+1$
Compute cost: $0<$ level $<\log (\mathrm{n})$
Time to compute response: $\sum$ PIR-compute-time ( $\left.2^{\text {level }}\right)+$ PIR-compute-time( $n$ )

## BST-based solution is also not database-updates friendly

- Client must know $n$, the total number of keys
- Server cannot insert / delete keys while a client is executing
the $\log (n)+1$ rounds


## Current research on PIR-by-keywords is on reducing the number of round trips and dynamic keyset issues

| Overhead | High-level technique |
| :--- | :--- |
| Round trips | $\bullet$ Constant-weight equality operator (SEC ‘22) |
|  | $\bullet$ Pantheon (tomorrow at H3-10:30 AM session) |
| Dynamic keyset | $\bullet$ Pantheon (tomorrow at H3-10:30 AM session) |

## Pantheon: A single round approach for PIR-by-keywords



- Can we retrieve the location in single-round?
- Can we make the query independent of the number of keys (n)?


## Pantheon: A single round approach for PIR-by-keywords



- Can we compose the two stages without involving the client in between?


## Pantheon: A single round approach for PIR-by-keywords



| Key | Value |
| :---: | :---: |
| $\mathrm{k}_{0}$ | $\mathrm{v}_{0}$ |
| $\mathrm{k}_{1}$ | $\mathrm{v}_{1}$ |
| $\mathrm{k}_{2}$ | $\mathrm{v}_{2}$ |
| $\ldots$ | $\cdots$ |
| $\mathrm{k}_{\mathrm{n}-1}$ | $\mathrm{v}_{\mathrm{n}-1}$ |

Untrusted Server

## Pantheon: A single round approach for PIR-by-keywords



| Key | Value |
| :---: | :---: |
| $\mathrm{k}_{0}$ | $\mathrm{v}_{0}$ |
| $\mathrm{k}_{1}$ | $\mathrm{v}_{1}$ |
| $\mathrm{k}_{2}$ | $\mathrm{v}_{2}$ |
| $\ldots$ | $\ldots$ |
| $\mathrm{k}_{\mathrm{n}-1}$ | $\mathrm{v}_{\mathrm{n}-1}$ |

Untrusted Server $\quad$| $\operatorname{Enc}(0)$ |
| :---: |
| $\operatorname{Enc}(0)$ |
| $\operatorname{Enc}(1)$ |
| $\ldots$ |
| $\operatorname{Enc}(0)$ |

## Pantheon: A single round approach for PIR-by-keywords



## Pantheon: A single round approach for PIR-by-keywords



## Warmup for oblivious equality checking

Assume that we do not care about privacy yet; only correctness


## Warmup for oblivious equality checking

Assume that we do not care about privacy yet; only correctness

Step 1: Subtraction

Step 2: Binarization
Step 3: Complement


## Fermat's little theorem

if $p$ is a prime number and $a$ is a number not divisible by $p$, then,

$$
a^{(p-1)} \equiv 1(\bmod p)
$$

Example:

$$
\begin{aligned}
& \text { Let, } p=17 . \text { Then for any } 0<a<17 \text {, } \\
& a^{16} \% p=1 \\
& 2^{16} \% 17=65536 \% 17=1 \\
& 3^{16} \% 17=43046721 \% 17=1
\end{aligned}
$$

However, if $a=0$, then $0^{16} \% p=0$
Fermat's little theorem enables distinction between zero and non-zero value!

## Recall the warmup for oblivious equality checking

Assume that we do not care about privacy yet; only correctness

Step 1: Subtraction


Step 2: Binarization




## Pantheon: An efficient and scalable solution

For more details, please attend the paper presentation:

Wednesday 10:30—noon session (H3)

## Current research on PIR-by-keywords is on reducing the number of round trips and dynamic keyset issues

| Overhead | High-level technique |
| :--- | :--- |
| Round trips | -Constant-weight equality operator (SEC '22) <br> - Pantheon (tomorrow at H3 - 10:30 AM session) <br> Dynamic keyset - Pantheon (tomorrow at H3 - 10:30 AM session) |

## This tutorial is in two parts

Part 2: Retrieval by key


|  |  |  | Give me value for key k | $\mathrm{k}_{0}$ | $\mathrm{v}_{0}$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| $\mathrm{k}_{1}$ | 1 |  |  | $\mathrm{k}_{1}$ | $\mathrm{v}_{1}$ |
| $\mathrm{k}_{2}$ | 2 |  |  | $\mathrm{k}_{2}$ | $\mathrm{v}_{2}$ |
| $\mathrm{k}_{3}$ | 3 | Client retrieves: |  | $\ldots$ | $\ldots$ |
|  | $\ldots$ | - v , if ( $\mathrm{k}, \mathrm{v}$ ) at Server |  | $\mathrm{k}_{\mathrm{n}-1}$ | $v_{n-1}$ |
| k | n | - $\emptyset$, otherwise |  | Untrusted |  |

Part 2: How can the client privately retrieve the value corresponding to a given key?

## We have come a long way - Private retrieval from public repositories



## Looking ahead - Private retrieval over public repositories

But overheads still high
Untrusted Server


| $\mathrm{k}_{0}$ | $\mathrm{v}_{0}$ |
| :---: | :---: |
| $\mathrm{k}_{1}$ | $\mathrm{v}_{1}$ |
| $\mathrm{k}_{2}$ | $\mathrm{v}_{2}$ |
| $\ldots$ | $\cdots$ |
| $\mathrm{k}_{\mathrm{n}-1}$ | $\mathrm{v}_{\mathrm{n}-1}$ |

Needs high compute resources


## Looking ahead - Private retrieval over public repositories

## Query interface is narrow

- PIR-by-location (Chor et al. FOCS ‘95)
- PIR-by-keywords (Chor et al. TOC ‘98)
- Private top-K queries?
- Retrieve price for 5 stocks similar to AAPL
- Private range queries?
- Retrieve daily price of AAPL between a start and end date
- Private aggregation queries?
- Calculate the average price of AAPL within a date range


## Coeus: Oblivious top-K ranking \& retrieval (SOSP ‘21)

Search keyword:
"red apple"


Give me top-K matching documents


## Simple IR with ranking in one round of communication

"red apple"

| tf-idf matrix |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
|  apple bat red $\ldots$. <br> Doc1 0.5 0.2 0 $\ldots$ <br> Doc2 0.8 0.1 0.1 $\ldots$ <br> Doc3 0 0 0.6 $\ldots$ <br> $\ldots$ $\ldots$ $\cdots$ $\cdots$ $\cdots$ <br> $\ldots$ $\ldots$ $\cdots$ $\cdots$ $\cdots$ |  |  |  |  |

## Simple IR with ranking in one round of communication



| tf-idf matrix |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
|  apple bat red $\cdots$ <br> Doc1 0.5 0.2 0 $\ldots$ <br> Doc2 0.8 0.1 0.1 $\cdots$ <br> Doc3 0 0 0.6 $\cdots$ <br> $\cdots$ $\cdots$ $\cdots$ $\cdots$ $\cdots$ <br> $\cdots$ $\cdots$ $\cdots$ $\cdots$ $\ldots$ |  |  |  |  |

## Simple IR with ranking in one round of communication

## matrix-vector multiplication



| tf-idf matrix |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: |
|  apple bat red <br> $\cdots$ $\cdots$   <br> Doc1 0.5 0.2 0 <br> $\ldots$    <br> Doc2 0.8 0.1 0.1 <br> Doc3 0 0 0.6 <br> $\cdots$ $\cdots$ $\cdots$ $\cdots$ <br> $\cdots$ $\cdots$ $\cdots$ $\cdots$ <br>  $\cdots$   |  |  |  |  |

## Simple IR with ranking in one round of communication



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## Simple IR with ranking in one round of communication

Client reads relevant document
D[idx*]

$\mathrm{D}\left[\mathrm{idx}_{1}\right], \ldots ., \mathrm{D}\left[\mathrm{idx}_{\mathrm{k}}\right]$

| Document Provider (D) |
| :---: |
| doc1 <br> doc2 <br> doc3 <br> doc4 <br> $\ldots$ <br> $\ldots$ <br> $\ldots$ |

## Coeus: A novel 3 round protocol for oblivious top-K

- Ranks documents using scores computed against tf-idf matrix
- A new large-scale secure matrix-vector multiplication protocol
- Composes secure multiplication with PIR to retrieve documents
- End-to-end latency of 3.9 seconds for 5M documents in English Wikipedia


## How can we expand the query interface beyond point queries?

- Private top-K queries?


## Coeus SOSP ‘21

- Retrieve price for 5 stocks similar to AAPL
- Private range queries?
- Retrieve daily price of AAPL between a start and end date
- Private aggregation queries?
- Calculate the average price of AAPL within a date range


## Summary and takeaway points

- Private access over public data repositories is underserved
- This area derives from private information retrieval (PIR)
- PIR-by-location, PIR-by-keywords
- Applications of homomorphic encryption, secure dot-product
- Much research focuses on reducing overhead (compute, network) or improving suitability for dynamic databases
- An exciting area for future research
- How can we further improve performance?
- How can we expand to a full-fledged key-value database?


