Private Information Retrieval in Large Scale Public Data Repositories

Ishtiyaque Ahmad, Divyakant Agrawal, Amr El Abbadi, and Trinabh Gupta

University of California, Santa Barbara



The problem of protecting *private data repositories* stored remotely is well-studied



Encryption hides file contents from an attacker.

Encryption does not hide data access patterns

The access patterns leaks:

- Which file is being accessed?
- When was it last accessed?
- Is it being accessed for a read or a write?
- Is it being accessed sequentially or randomly?

• ...

ORAM (STOC '87) hides data access patterns for private files



- → Whether the access is a read or write
- → When was the file accessed last

 $\mathbf{x}_{i} \in \mathbf{x}_{i}$

We can extend protection to *private relational databases* stored remotely

CryptDB SOSP '11, MONOMI VLDB '13, ...



Adjustable query-based encryption (onion)

What is common to all of these cases?



The user owns the data!

But, much of the content on the Internet is in *public* data repositories



User

I want to stream "The Godfather"



Remote server



User

Show me the latest post by Elon Musk



Remote server

But, much of the content on the Internet is in *public* data repositories



- Encryption
- ORAM
- CryptDB-like solution

How can we hide access patterns (queries) over public data repositories?

Both users and service providers want to hide access patterns over public repositories



- 1. Brian Fung. Analysis: There is now some public evidence that China viewed TikTok data. CNN, 2023.
- 2. Sapna Maheshwari and Ryan Mac. *Driver's Licenses, Addresses, Photos: Inside How TikTok Shares User Data.* New York Times, 2023

This tutorial:

Discuss a cryptographic method to privately retrieve data from public data repositories, thus making server *opaque* to data access patterns

Private retrieval from public databases can be abstracted into the key-value store model



Client retrieves:

• v, if (k,v) at Server

k

• Ø, otherwise

k ₀	V ₀	
k ₁	V ₁	
k ₂	V ₂	
k _{n-1}	V _{n-1}	
		•

Untrusted Server

Focus on performance, scalability, and practicality

This tutorial is in two parts

Part 1: Retrieval by location



Untrusted Server

Part 1: How can the client privately retrieve the value corresponding to a given location?

This tutorial is in two parts

Part 2: Retrieval by key



Part 2: How can the client privately retrieve the value corresponding to a given key?

This tutorial is in two parts

Part 1: Retrieval by location



Untrusted Server

Part 1: How can the client privately retrieve the value corresponding to a given location?

This problem can be solved using **Private Information Retrieval** (PIR) (Chor et al. FOCS '95)

PIR: Query, Answer, Decode



Untrusted Server

PIR has two key requirements

Correctness

Query for db[i] returns db[i] to the user Decode(Answer(db, Query(*i*))) = db[i]

Privacy

Server learns "nothing" about the location i

For all locations i, j,

{View of the server in answering Query(i)} ≈

{View of the server in answering Query(j)}

We are also interested in performance considerations

Network cost

Request size: |Query(i)|

Response size: |Answer(db, Query(i))|

Compute cost

Time to compute Answer(db, Query(i))

One solution to private information retrieval in Trivial PIR

db



Answer(db, q): db

Decode(i, ans): select the i-th item from ans

Performance characteristics of trivial PIR

Network cost

Request size: 1 bit

Response size: n x |db[i]|

Compute cost

Time to compute Answer(db, Query(i))

Can we do better than sending the entire database? If so, how?

Warmup for (non-trivial) PIR

Assume that we do not care about privacy yet; only correctness



Untrusted Server

Retrieval is equivalent to computing a dot product

Warmup for (non-trivial) PIR in more detail

Multiply component-wise



- → Multiplications (8 x 0, 5 x 1, etc.)
- → Additions (e.g., 0 + 5 + ...)

Detour: Introduction to Homomorphic Encryption

A form of encryption which allows computations over encrypted data

Two classes of homomorphic encryption

Fully Homomorphic Encryption [Gentry'09]

- Supports computations for any arbitrary function
- Challenge: Can be Quite inefficient

Partially Homomorphic Encryption

Supports a particular type of operation



 $Enc(4) \bigoplus Enc(8) = Enc(4 + 8) = Enc(12)$

Multiplicative Homomorphic encryption

$$Enc(4) \otimes Enc(8) = Enc(4 \times 8) = Enc(32)$$

Detour: Introduction to Homomorphic Encryption

A form of encryption which allows computations over encrypted data

Two classes of homomorphic encryption

Fully Homomorphic Encryption [Gentry'09]

- Supports computations for any arbitrary function
- Challenge: Can be Quite inefficient

Partially Homomorphic Encryption

Supports a particular type of operation



 $Enc(4) \oplus Enc(8) = Enc(4 + 8) = Enc(12)$

Multiplicative Homomorphic encryption

Example: El Gamal additive homomorphic encryption

We have a message m which we want to encrypt

Encryption key: (g, h)

Encryption procedure:

Pick a random number r

 $Enc(m, r) = (g^r, g^m h^r)$

Example: El Gamal additive homomorphic encryption

 $Enc(m, r) = (g^r, g^m h^r)$

Given two messages m1 and m2

 $Enc(m1, r1) = (g^{r1}, g^{m1}h^{r1})$

 $Enc(m2, r2) = (g^{r2}, g^{m2}h^{r2})$

 $Enc(m1) \times Enc(m2) = Enc(m1 + m2)$

The product of the encryptions of two messages is *an* encryption of the sum of the two messages.

 $Enc(m1, r1) \times Enc(m2, r2) = (g^{r1}, g^{m1}h^{r1}) \times (g^{r2}, g^{m2}h^{r2})$

$$= (g^{r1+r2}, g^{m1+m2}h^{r1+r2})$$

= Enc(
$$m_1 + m_2, r_1 + r_2$$
)

Example: El Gamal additive homomorphic encryption

 $Enc(m1) \times Enc(m2) = Enc(m1+m2)$

Additive Homomorphic Encryption supports multiplying an encrypted value with a plaintext value

We have a message m, encrypted as Enc(m)

We have another message k (not encrypted)

$$[Enc(m)]^{k} = Enc(m) \times Enc(m) \times \dots \times Enc(m)$$

= Enc(m + m + ... + m)

= Enc(m * k)

$$Enc(m)^{k} = Enc(m * k)$$

We only need additive homomorphic encryption for PIR

Homomorphic addition

$$Enc(m_1) \times Enc(m_2) = Enc(m_1 + m_2)$$

Homomorphic plaintext multiplication

 $Enc(m)^{k} = Enc(m * k)$

Multiply component-wise



- → Multiplications (8 x 0, 5 x 1, etc.)
- → Additions (e.g., 0 + 5 + ...)

Multiply component-wise



- → Multiplications (8 x 0, 5 x 1, etc.)
- → Additions (e.g., 0 + 5 + ...)



- → Multiplications (8 x 0, 5 x 1, etc.)
- → Additions (e.g., 0 + 5 + ...)



- → Multiplications (8 x 0, 5 x 1, etc.)
- → Additions (e.g., 0 + 5 + ...)

 $Enc(m)^{k} = Enc(m * k)$ Homomorphically multiply component-wise db Query(1) 8 Enc Enc 0 0 0 5 5 Enc 1 Enc 1 Enc 0 2 0 Enc 0 5 Homomorphically Enc Enc ٠ add components n - 1 2 Enc Enc 0 0 $Enc(m_1) \times Enc(m_2) = Enc(m_1 + m_2)$

- → Multiplications (8 x 0, 5 x 1, etc.)
- → Additions (e.g., 0 + 5 + ...)

 $Enc(m)^{k} = Enc(m * k)$ Homomorphically multiply component-wise db Query(1) 8 Enc Enc 0 0 0 5 5 Enc 1 Enc 1 Enc 0 2 0 Enc 0 5 Homomorphically Enc Enc ٠ add components n - 1 2 Enc Enc 0 0 $Enc(m_1) \times Enc(m_2) = Enc(m_1 + m_2)$

- → Multiplications (8 x 0, 5 x 1, etc.)
- → Additions (e.g., 0 + 5 + ...)

Putting it all together: A PIR protocol



db[1] = Decode(ans) = Decrypt(ans)

Retrieval is equivalent to computing a secure dot product

What is the size of the PIR response?

Response is a ciphertext: Enc(db[i])

Recall:

 $Enc(m, r) = (g^r, g^m h^r)$

Encrypting 1 message yields 2 components

Expansion factor, *f* = *size of ciphertext / size of plaintext*

Expansion factor for El Gammal = 2

Performance characteristics of additively HE-based PIR

Network cost

Request size: n x |ciphertext|

Response size: |ciphertext|

Expansion factor: f = |ciphertext| / |db[i]|

Compute cost

Time to compute Answer(db, Query(i)) is O(n) homomorphic ops

This linear compute overhead is a fundamental lower bound (Beimel et al. CRYPTO '00)

Much of the research on PIR is on reducing request size and server-side compute overhead

Overhead	High-level technique
Request size	 Recursion (Stern 1998) Cryptographic query compression (SealPIR '18)
Server-side compute	 PIR with preprocessing (Beimel et al. '00, SimplePIR '23) Lattice-based cryptography (FastPIR '21)
How to reduce query size?

а

b

С

d

е

f

g

h

. . .

р

0	
0	
0	
0	
0	
0	
1	
0	
0	

Instead of 1 dim database, view it in 2 dims. Instead of 1 query, use 2 queries.

0	0	1	0
---	---	---	---

	а	b	С	d
	е	f	g	h
	i	j	k	I
	m	n	0	р

Two-stage query execution



Add rows



In first pass, extract the row of interest

Two-stage query execution



_	_	-	
0	0	g	0

Add columns

g

So, query size is down from n to $2\sqrt{n}$.

But result is double encrypted

- After first stage, each element is a ciphertext, size is *f* * *plaintext size*
- After second stage, result size is
 f² * plaintext size
- The efficient homomorphic encryption schemes can have $f \ge 8$

Trade-off between query and response size Stern (1998) recursion scheme

- Reduce query size to $d^{*d}\sqrt{n}$
- Expand result size by f^{d}
- Used in XPIR (2016)

Much of the research on PIR is on reducing request size and server-side compute overhead

Overhead	High-level technique
Request size	 Recursion (Stern 1998) Cryptographic query compression (SealPIR '18)
Server-side compute	 PIR with preprocessing (Beimel et al. '00, SimplePIR '23) Lattice-based cryptography (FastPIR '21)

SealPIR (Microsoft Research - 2018)

- Compress query by a large factor (2¹¹)
- Trade-off: query expansion at the server requires high compute cost

How to reduce server-side compute overhead?

PIR with preprocessing (Beimel et al CRYPTO '00, SimplePIR '23)



Untrusted Server

db

Does not violate the linear compute lower bound (Beimel et al. CRYPTO '00)

How to reduce server-side overhead?

Another option is to pay linear overhead but improve the constant

Key techniques in FastPIR (OSDI '21)

- Use lattice-based additively homomorphic encryption scheme
- Single-input multiple data (SIMD) capabilities
- Query and response compression using homomorphic rotation operations

FastPIR has lower processing time than all other variants (that do not use preprocessing)

Experiment results (c5.12x large in AWS; 1M values, 256 bytes each)

PIR Scheme	Processing time (ms)	Response size (KB)
FastPIR	947	64
XPIR-1	3,389	32
XPIR-2	1,894	288
SealPIR-1	76,216	32
SealPIR-2	2,556	320

This tutorial is in two parts

Part 1: Retrieval by location



Untrusted Server

Part 1: How can the client privately retrieve the value corresponding to a given location?

This tutorial is in two parts

Part 2: Retrieval by key



Part 2: How can the client privately retrieve the value corresponding to a given key?

This area originated as Private retrieval by keywords in 1998 (Chor et al. TOC '98)

Private Keyword retrieval can be performed by two stages:

Stage 1: Retrieve the key location



Give me the *i*-th value



Stage 2: Perform PIR with location

Has (key, location)

mapping

 $\begin{array}{c|cccc} 0 & v_0 & & \\ 1 & v_1 & & \\ 2 & v_2 & & \\ \dots & \dots & & \\ n-1 & v_{n-1} & & \\ \end{array}$

PIR-by-keywords has two requirements

Correctness

Query for k returns v iff (k, v) is in db

Privacy

Server learns "nothing" about the key k

For any two possible keys k_i, k_j

{View of the server in answering $Query(k_i)$ } \approx

{View of the server in answering Query(k_j)}

We are also interested in performance considerations

Network cost

Request size, Response size

Number of round trips between user and server

Compute cost

Time to compute the response

This area originated as Private retrieval by keywords in 1998 (Chor et al. TOC '98)

Private Keyword retrieval can be performed by two stages:

Stage 1: Retrieve the key location



Give me the location for key k



 V_0

 V_2

 V_{n-1}



What is the location of 17? Assume keys are integers and arranged in a BST

K = {1, 5, 6, 10, 17, 19, 20}



What is the

User

location of 17?

Assume keys are integers and arranged in a BST

K = {1, 5, 6, 10, 17, 19, 20}



What is the

Assume keys are integers and arranged in a BST

K = {1, 5, 6, 10, 17, 19, 20}



What is the location of 17?

User

Assume keys are integers and arranged in a BST

K = {1, 5, 6, 10, 17, 19, 20}



Level 3: Retrieve element at index 2 using PIR-by-index

17 = 17 (found it!)

Path from root to leaf is index of k in keyset K

This area originated as Private retrieval by keywords in 1998 (Chor et al. TOC '98)

Private Keyword retrieval can be performed by two stages:



Performance of BST-based PIR-by-keywords Stage 1 + Stage 2

Network cost: 0 < level < log(n)

Request size: $\sum PIR$ -request-size(2^{level}) + PIR-request-size(n)

Response size: \sum PIR-response-size(2^{level}) + PIR-response-size(n)

Number of round trips between user and server: log(n) + 1

Compute cost: 0 < level < log(n)

Time to compute response: $\sum PIR$ -compute-time(2^{level}) + PIR-compute-time(n)

BST-based solution is also not database-updates friendly

- Client must know *n*, the total number of keys
- Server cannot insert / delete keys while a client is executing

the log(n) + 1 rounds

Current research on PIR-by-keywords is on reducing the number of round trips and dynamic keyset issues

Overhead	High-level technique	
Round trips	 Constant-weight equality operator (SEC '22) Pantheon (tomorrow at H3 — 10:30 AM session) 	
Dynamic keyset	• Pantheon (tomorrow at H3 — 10:30 AM session)	



Untrusted Server

- Can we retrieve the location in single-round?
- Can we make the query independent of the number of keys (n)?



Untrusted Server

• Can we compose the two stages without involving the client in between?









Warmup for oblivious equality checking

Assume that we do not care about privacy yet; only correctness





Warmup for oblivious equality checking

Assume that we do not care about privacy yet; only correctness

Step 1: Subtraction

Step 2: Binarization

Step 3: Complement





Fermat's little theorem

if *p* is a prime number and *a* is a number not divisible by *p*, then,

 $a^{(p-1)} \equiv 1 \pmod{p}$

Example:

```
Let, p =17. Then for any 0 < a < 17,
a<sup>16</sup> % p = 1
2<sup>16</sup> % 17 = 65536 % 17 = 1
3<sup>16</sup> % 17 = 43046721 % 17 = 1
```

However, if a = 0, then $0^{16} \% p = 0$

Fermat's little theorem enables distinction between zero and non-zero value!



Recall the warmup for oblivious equality checking

Assume that we do not care about privacy yet; only correctness



Pantheon: An efficient and scalable solution



For more details, please attend the paper presentation:

Wednesday 10:30—noon session (H3)

Current research on PIR-by-keywords is on reducing the number of round trips and dynamic keyset issues

Overhead	High-level technique	
Round trips	 Constant-weight equality operator (SEC '22) Pantheon (tomorrow at H3 — 10:30 AM session) 	
Dynamic keyset	 Pantheon (tomorrow at H3 — 10:30 AM session) 	

This tutorial is in two parts

Part 2: Retrieval by key

k ₁	1
k ₂	2
k ₃	3
k _n	n



Client retrieves:

- v, if (k,v) at Server
- Ø, otherwise

Give me value for key k



Untrusted Server

Part 2: How can the client privately retrieve the value corresponding to a given key?



key-based retrieval

Looking ahead — Private retrieval over public repositories

Private GET for key k

But overheads still high

Untrusted Server



Latency ~ 1 second



Needs high compute resources


Looking ahead — Private retrieval over public repositories

Query interface is narrow

- PIR-by-location (Chor et al. FOCS '95)
- PIR-by-keywords (Chor et al. TOC '98)
- Private top-K queries?
 - Retrieve price for 5 stocks similar to AAPL
- Private range queries?
 - Retrieve daily price of AAPL between a start and end date
- Private aggregation queries?
 - Calculate the average price of AAPL within a date range

Coeus: Oblivious top-K ranking & retrieval (SOSP '21)



"red apple"



tf-idf matrix						
	apple	bat	red			
Doc1	0.5	0.2	0			
Doc2	0.8	0.1	0.1			
Doc3	0	0	0.6			
-	-					



	tf-idf matrix						
	apple	bat	red				
Doc1	0.5	0.2	0				
Doc2	0.8	0.1	0.1				
Doc3	0	0	0.6				











Client reads relevant document

D[idx*]



 $D[idx_1],..., D[idx_K]$

Document Provider (D)				
doc1				
doc2				
doc3				
doc4				

Coeus: A novel 3 round protocol for oblivious top-K

- Ranks documents using scores computed against tf-idf matrix
- A new large-scale secure matrix-vector multiplication protocol
- Composes secure multiplication with PIR to retrieve documents
- End-to-end latency of 3.9 seconds for 5M documents in English Wikipedia

How can we expand the query interface beyond point queries?

• Private top-K queries?

Coeus SOSP '21

- Retrieve price for 5 stocks similar to AAPL
- Private range queries?

. . .

- Retrieve daily price of AAPL between a start and end date
- Private aggregation queries?
 - Calculate the average price of AAPL within a date range

Summary and takeaway points

- Private access over public data repositories is underserved
- This area derives from private information retrieval (PIR)
 - PIR-by-location, PIR-by-keywords
 - Applications of homomorphic encryption, secure dot-product
- Much research focuses on reducing overhead (compute, network) or improving suitability for dynamic databases
- An exciting area for future research
 - How can we further improve performance?
 - How can we expand to a full-fledged key-value database?

Thank You!

https://github.com/ishtiyaque/

