CS 266 - Formal Specification and Verification
Winter 2009

Homework #1 -- Review of Propositional and Predicate Calculus

Due: 13 JAN 09, 11:00am

This problem set is intended as a "tune-up". You should be able to do all of the problems herein. If you are unfamiliar with the concepts illustrated in a given problem, you should consult the appropriate books in the library.

PROPOSITIONAL LOGIC

The following BNF grammar defines the syntax for fully parenthesized propositions:

<proposition> ::=
T | F | <identifier>
| (~<proposition>)
| (<proposition> & <proposition>)
| (<proposition> | <proposition>)
| (<proposition> → <proposition>)
| (<proposition> ↔ <proposition>)

In order to omit parenthesis we assume the following operator precedence from high to low:
~, &, |, →, ↔
with sequences of the same operator evaluated left to write.

1. In the table below each line contains a proposition and two states s1 and s2. Evaluate the proposition in both states.

proposition	s1	s2
	m n p q	m n p q
(a) ~(m | q)	T F T T	F T T T
(b) ~m | q	T F T T	F T T T
(c) ~(m & q)	T F T T	F T T T
(d) ~m & q	T F T T	F T T T
(e) (m | q) → p	T F T T	T T F T
(f) m | (n → p)	T F T T	T T F T
(g) m ↔ (n & (p ↔ q))	F F T F	T F T F
(h) (m ↔ n) & (p ↔ q)	F F T F	T F T F
(i) m ↔ (n & p ↔ q)	F F T F	T F T F
(j) (m ↔ n & p) → q	F T F T	T T F F
(k) (m ↔ n) & (p → q)	F T F T	T T F F
(l) (m → n) → (p → q)	F F F F	T T T T
(m) (m → (n → p)) → q	F F F F	T T T T

2. For each of the English sentences below introduce identifiers to represent each atomic part (eg. "it's raining cats and dogs") and translate the sentence into a proposition.

(a) Whether or not it's raining, I'm going swimming.
(b) If it's raining I'm not going swimming.
(c) It's raining cats and dogs.
(d) If it rains cats and dogs I'll eat my hat, but I wont go swimming.
(e) If it rains cats and dogs while I'm going swimming I'll eat my hat.

The following are twelve basic laws of equivalence. The identifiers E1, E2, and E3 each represent a proposition.

1. Commutative Laws
   $(E1 \,\&\, E2) \leftrightarrow (E2 \,\&\, E1)$
   $(E1 \mid E2) \leftrightarrow (E2 \mid E1)$
   $(E1 \leftrightarrow E2) \leftrightarrow (E2 \leftrightarrow E1)$

2. Associative Laws
   $E1 \,\&\, (E2 \,\&\, E3) \leftrightarrow (E1 \,\&\, E2) \,\&\, E3$
   $E1 \mid (E2 \mid E3) \leftrightarrow (E1 \mid E2) \mid E3$

3. Distributive Laws
   $E1 \mid (E2 \,\&\, E3) \leftrightarrow (E1 \mid E2) \,\&\, (E1 \mid E3)$
   $E1 \,\&\, (E2 \mid E3) \leftrightarrow (E1 \,\&\, E2) \mid (E1 \,\&\, E3)$

4. De Morgan's Laws
   $\sim(E1 \,\&\, E2) \leftrightarrow \sim E1 \mid \sim E2$
   $\sim(E1 \mid E2) \leftrightarrow \sim E1 \,\&\, \sim E2$

5. Law of Negation
   $\sim(\sim E1) \leftrightarrow E1$

6. Law of Excluded Middle
   $E1 \mid \sim E1 \leftrightarrow T$

7. Law of Contradiction
   $E1 \,\&\, \sim E1 \leftrightarrow F$

8. Law of Implication
   $E1 \rightarrow E2 \leftrightarrow \sim E1 \mid E2$

9. Law of Equality
   $(E1 \leftrightarrow E2) \leftrightarrow (E1 \rightarrow E2) \,\&\, (E2 \rightarrow E1)$

10. Laws of or-Simplification
   $E1 \mid E1 \leftrightarrow E1$
   $E1 \mid T \leftrightarrow T$
   $E1 \mid F \leftrightarrow E1$
   $E1 \mid (E1 \,\&\, E2) \leftrightarrow E1$

11. Laws of and-Simplification
    E1 & E1 ↔ E1
    E1 & T ↔ E1
    E1 & F ↔ F
    E1 & (E1 | E2) ↔ E1

12. Law of Identity
    E1 ↔ E1

There is also the Rule of Substitution: Let e1 ↔ e2 be an equivalence and E(p) be a proposition, written as a function of one of its identifiers p. Then E(e1) ↔ E(e2) and E(e2) ↔ E(e1). That is, equals may be substituted for equals.

Finally there is the Rule of Transitivity: If e1 ↔ e2 and e2 ↔ e3 are equivalences, then so is e1 ↔ e3.

3. Verify that rules 1, 3, and 10 are equivalences by building truth tables for them.

4. Prove that ~T ↔ F is an equivalence by using the Rules of Substitution and Transitivity and the laws 1-12.

5. Each of the propositions below can be simplified to one of the six propositions F, T, x, y, x & y, and x | y. Simplify them using the Rules of Substitution and Transitivity and the laws 1-12.

(a) x | (y | x) | ~y
(b) x | y | ~x
(c) (x & y) | (x & ~y) | (~x & y) | (~x & ~y)
(d) ~x → (x & y)
(e) x → (y → (x & y))
(f) ~y → y

6. Explain how to translate an ordinary propositional statement into one that uses only *and* (&) and *not* (~) operators.

PREDICATE CALCULUS

In predicate calculus we use the existential quantifier (read "there exists") denoted by

$\exists$

and the universal quantifier (read "for all") denoted by

$\forall$

Some rules of inference are given on the last two pages of this homework.

7. Define *bound variable* and *free variable.*

8. Argue (not necessary to prove formally) why the following Quantifier Negation Rules should hold.

˜∀xF(x)	∃y˜F(y)
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∃x˜F(x)	˜∀yF(y)

∀x˜F(x)	˜∃yF(y)
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˜∃xF(x)	∀y˜F(y)