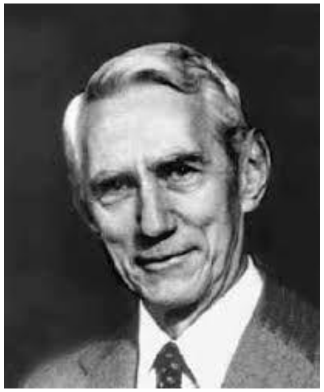
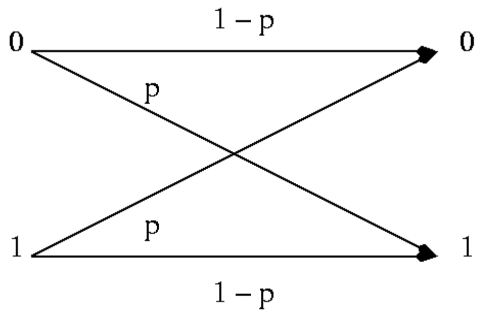
Creativity in Communications Theory – Claude Shannon[1]

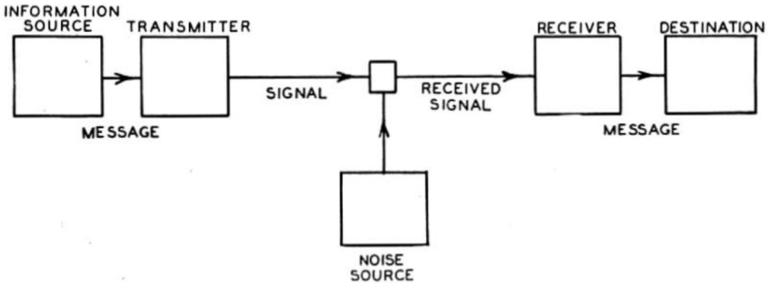
``The fundamental problem of communication is that of reproducing at one point either exactly or approximately a message selected [sent] at another point.”



(April 30, 1916 – February 24, 2001)



Binary Symmetric Channel (BSC)



<u>To Obtain Reliable Transmission of BSC Data</u>: Add redundancy to the data.

- Combine $\underline{d}= (d_0 , d_1 ,\ldots, d_{n-1})$with check bits $\underline{c} = (c_0 , c_1 ,\ldots, c_{k-1})$.
- Transmitted message: coding $\underline{d}\to\underline{x} = (\underline{d}, \underline{c})$; Received message: $\underline{y} = \underline{x} + error$
- Decoded message: $\underline{y}\to\underline{x}^* \to\underline{d}^*$
- Recover the original data $\underline{d}^* =\underline{d}$ with probability > 1- ε.

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[1]Claude Shannon ``A Mathematical Theory of Communication”, *Bell System Technical Journal,* (**27**)3, July 1948, pp. 379-423.

Repetition Code[2]

- Transmit odd number of copies *2j+1*of each of the *n*data bits.
- Majority decision decoding ($0 \leq p < ½$)

<u>*Mini-Shannon BSC Coding Theorem:*</u> For every data bit length *n* and ε > 0, there exists *j* (a repetition factor) such that the MAP probability of error is bounded above by ε.

- Number of check bits $k = 2jn$.
- Code word <u>*x*</u>= *(2j+1) copies of* <u>*d.*</u>
- Transmission rate $R = \frac{1}{2j+1} \rightarrow 0$ as n↑, ε↓
- Decoding Rule: MAP (maximize *a posteriori* probability)
  Maximum Likelihood Estimation (Bayes Theorem).

[2]``If at first you don't succeed, try, try again. Then quit. There's no point in being a damn fool about it.'' W.C. Fields

Reliable and Efficient Transmission of BSC Data

Shannon's BSC Coding Theorem I: For every ε, δ> 0, there exists an integer $n^* = n^*(\varepsilon,\delta,p)$ such that for data $\underline{d}$ of bit length $n>n^*$, a coding scheme $\underline{d}\rightarrow\underline{x}$ into code words of length m exists for which *i)* MAP decoding achieves an error probability smaller than ε and *ii)* the coding rate $R = n/m$ satisfies $C-\delta \leq R < C$

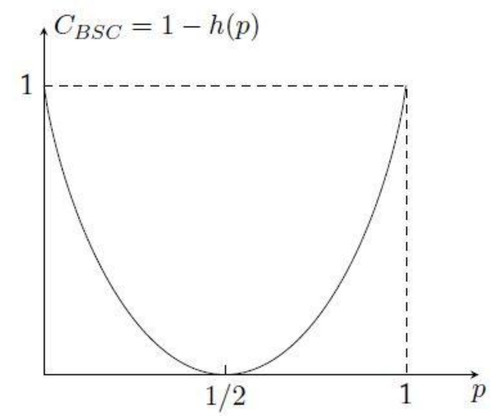
Weak Converse to Shannon's BSC Coding Theorem I: For any coding scheme, $\underline{d}\rightarrow x$, which has a rate greater than the channel capacity C, the MAP error probability is bounded away from zero as the data lengths $n\rightarrow\propto$. In other words, the maximum achievable reliable transmission rate is *C.*

- *Channel Capacity:* $C_{BSC} = 1-h(p)$

$h(p) = -p \log_2 p - q \log_2 q; \quad 0 \leq p < ½; \quad q = 1-p$

- *Value of p*: Signal power (S), Noise energy (N) and Bandwidth (B).

- *Shannon-Hartley Law*: $C = B \log_2 (1+S/N)$



Binary Symmetric Channel Capacity

Achieving Shannon Coding Theorem by Algebraic Coding

Shannon's paper cites results from R. W. Hamming's paper on

error-correcting and detecting codes.[3]

- Subset **C** of $\mathcal{Z}_{m,2}$ $\underline{z}_1, \underline{z}_2 \in \mathcal{Z}_{m,2}$

C is a (binary) *code* for the BSC consisting of *m*-bit vector code words.

- Hamming distance $d(\underline{z}_1, \underline{z}_2)$ (Bit difference between $\underline{z}_1$ and $\underline{z}_2$)
- $S_r[\underline{x}] = \{ \underline{z} \in \mathcal{Z}_{m,2} : d(\underline{z}, \underline{x}) \leq r, \underline{x} \in \mathbf{C} \}$ (*r*-Sphere about $\underline{x}$)

  Volume of $S_r[\underline{x}]$: $\| S_r[\underline{x}] \| = C(m,0) + C(m,1) + \ldots + C(m,r)$

- $2^n \| S_r[\underline{x}] \| \leq 2^{n+k}$ (*Hamming Bound*) $\Rightarrow$ *Rate* of code is $n/(n+k)$.

- Hamming (7,4) Code: $n = 4$, $m = 7$ (Single error-correcting, double error-detecting)
- Hamming $(2^r - 1, 2^r - r - 1)$ Code: $n = 2^r - r - 1$, $m = 2^r - 1$ $(r \geq 2)$

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[3] R. W. Hamming, ``Error Detecting and Error Correcting Codes, *Bell System Technical Journal,* (**29**)2, April 1950, pp. 147-160.

<u>Algebraic Coding Literature</u>

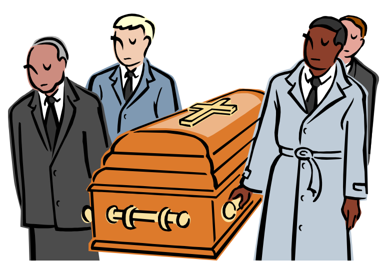
1. I. S. Reed, ``A Class of Multiple Error Correcting codes and the Decoding Scheme," *IEEE Transactions on Information Theory*, **IT-4**, pp. 38–49, September 1954.

Google search (1/1/15) *"<u>block algebraic coding books</u>"*yields>7,000,000 hits.

2. W. W. Peterson, ``Error-Correcting Codes,", MIT Press, 1961.
3. W. W. Peterson and E. J. Weldon, Jr.,``Error-Correcting Codes," MIT Press, 1972.
4. F. J. MacWilliams and N. J. A. Sloane, ``The Theory of Error-Correcting Codes," Elsevier, 1977.

(Talk at IEEE Communications Society Meeting 1971, St. Petersburg, Florida)

R. W. Lucky[4], ``[Algebraic]Coding is Dead,"
IEEE Spectrum, pp. 243-246, 1991.



[4] Electrical Engineer at Bell Labsand Telcordia; specialist in communication theory.

Achievingthe Shannon Coding Theorem Rate

A.Probabilistic Coding (Algebraic Block Codes Versus Probabilistic Coding)

P. Elias, "Coding for Noisy Channels," *IRE Convention Record*, pt. 4, pp. 37-46, March 1955.

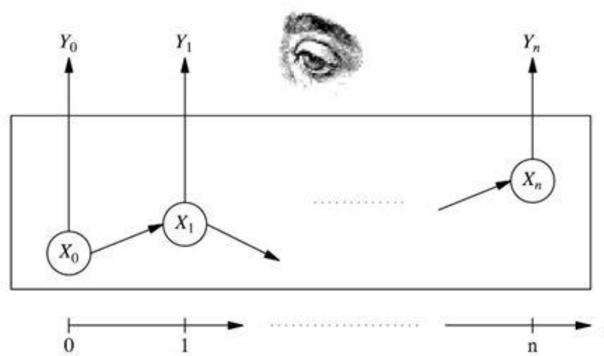
J. M. Wozencraft and B. Reiffen, ``Sequential Decoding," MIT Press, 1961.

B.Soft Versus Hard Decoding(0/1 versus 000, 001, …, 111)

C.Trellis Decoding Algorithm

A. J. Viterbi, `` Convolutional Codes and Their Performance in Communication Systems," *IEEE Transactions on Communications Technology*, **COM-19**(5), pp. 751-772, October 1971.

D. Decoding Using Hidden Markov Model (HMM)



Lalit Bahl, John Cocke, Fred Jelinek, and Joseph Raviv, "Optimum Decoding of Linear Codes for Minimizing Symbol Error Rate," *IEEE Transactions on Information Theory*, **IT-20**, pp. 284-287, March 1974

Berrou et al ⇒Achieving Shannon Coding Theorem Limit[5]

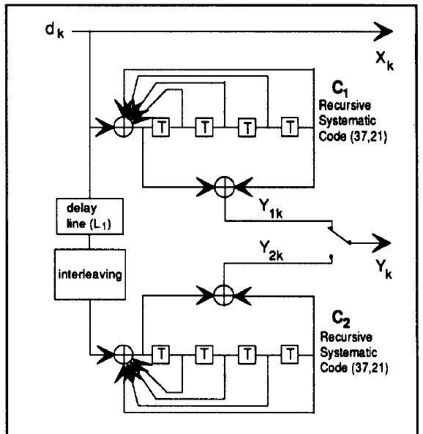


Fig. 2 Recursive Systematic codes with parallel concatenation.

What Claude Shannon Achieved!

He started the investigation of the relationship between the error and transmission rates in communication theoryleading to today's technology!

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[5]C. Berrou, A. Glavieux, and P. Thitimajshima, ``Near Shannon limit Error-correcting Coding and Decoding: Turbo codes", *Proceedings 1993 International Conference on Communications*, Geneva, Switzerland,May 1993, pp. 1064–1070. (Ecole Nationale Supérieure des Télécommunications de Bretagne, France)

(Turbo Coding)U.S. Patent US5446747 A (Berrou), filed 1991.