Lecture 6 Convolutional Neural Networks

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- Single artificial neuron to mimic biological neurons
 - each with simple operations
- Logistic Regression and its limitation
- Feedforward neural network (multilayer perceptron)
 - Massive combination of simple units
- Successful example of FFN
 - Deep&Wide model for recommendation system
- Computing Gradient for FFN backpropagation

Feedforward Neural Net (FFN)

- also known as multilayer perceptron (MLP)
- Layers are connected sequentially
- Each layer has full-connection (each unit is connected to all units of next layer)
 - Linear project followed by
 - an element-wise nonlinear activation function
- There is no connection from output to input



Learning FFN: Stochastic Gradient Descent

learning rate eta.

- **1.** set initial parameter $\theta \leftarrow \theta_0$
- 2. for epoch = 1 to maxEpoch or until converge:
- 3. random_shuffle data
- 4. for each data batch (x, y):
- 5. compute error $err(f(x; \theta) y)$ using forward
- 6. compute gradient $g = \frac{\partial \text{err}(\theta)}{\partial \theta}$ using backpropagation
- 7. total_g += g
- 8. update $\theta = \theta$ eta * total_g / batch_size

Forward "Pass"

- Input: *D* dimensional vector $\mathbf{x} = [x_j, j = 1...D]$
- Set:

$$-D_0 = D$$
, is the width of the 0th (input) layer
 $-y_j^{(0)} = x_j, \ j = 1...D; \ y_0^{(k=1...N)} = x_0 = 1$

• For layer
$$k = 1...N$$

- For $j = 1...D_k$ D_k is the size of the kth layer
, $z_j^{(k)} = \sum_{i=0}^{D_{k-1}} w_{i,j}^{(k)} y_i^{(k-1)}$
, $y_j^{(k)} = f_k(z_j^{(k)})$
• Output:

Backward Pass

• Output layer (N) :

For
$$i = 1...D_N$$

 $\frac{\partial \ell}{\partial z_i^{(N)}} = f'_N(z_i^{(N)}) \frac{\partial \ell}{\partial \hat{y}_i^{(N)}}$
 $\frac{\partial \ell}{\partial w_{ij}^{(N)}} = y_i^{(N-1)} \frac{\partial \ell}{\partial z_j^{(N)}}$ for each j

Called "Backpropagation" because the derivative of the loss is propagated "backwards" through the network

• For layer $k = N - 1 \ downto$ Very analogous to the forward pass:

- For
$$i = 1...D_k$$

 $\frac{\partial \ell}{\partial y_i^{(k-1)}} = \sum_j w_{ij}^{(k)} \frac{\partial \ell}{\partial z_j^{(k)}}$
 $\frac{\partial \ell}{\partial z_i^{(k)}} = f'_k(z_i^{(k)}) \frac{\partial \ell}{\partial y_i^{(k)}}$
 $\frac{\partial \ell}{\partial w_{ij}^{(k)}} = y_i^{(k-1)} \frac{\partial \ell}{\partial z_j^{(k)}}$ for each j

Backward weighted combination of next layer

Backward equivalent of activation

Why Learning CNN?

- A fundamental class of models for image recognition
- Vast applications:
 - Autonomous driving vehicle
 - Image search
 - E-commerce recommendation
 - Face identification (iphone faceID)



Visual Search





Answering question about image



Q: what is the color of the bus? A: yellow Q: what are there hanging up? A: umbrellas

Q: What is the color of the cake? A: red ABC-CNN [Chen, Wang et al 2015]

Autonomous Driving in 2015



Convolution

Problem: Classifying Dog and Cat Images

- Use a good camera
- RGB image has 36M elements
- What is the size of a FFN with a single hidden layer (100 hidden units)?
- How to reduce parameter size?











Where is Waldo?





Two Principles

- Translation
 Invariance
- Locality



Full Projection in Tensor Form

- Input image: a matrix with size (h, w)
- Projection weights: a 4-D tensors (h,w) by (h',w')

$$h_{i,j} = \sum_{k,l} w_{i,j,k,l} x_{k,l} = \sum_{a,b} v_{i,j,a,b} x_{i+a,j+b}$$

V is re-indexes W such as that $v_{i,j,a,b} = w_{i,j,i+a,j+b}$ Tensor is a generalization of matrix

Idea #1 - Translation Invariance

$$h_{i,j} = \sum_{a,b} v_{i,j,a,b} x_{i+a,j+b}$$

- A shift in x also leads to a shift in h
- v should not depend on (i,j). Fix via

$$v_{i,j,a,b} = v_{a,b}$$
$$h_{i,j} = \sum_{a,b} v_{a,b} x_{i+a,j+b}$$

Idea #2 - Locality

$$h_{i,j} = \sum_{a,b} v_{a,b} x_{i+a,j+b}$$

- We shouldn't look very far from x(i,j) in order to assess what's going on at h(i,j)
- Outside range $|a|, |b| > \Delta$ parameters vanish $v_{a,b} = 0$

$$h_{i,j} = \sum_{a=-\Delta}^{\Delta} \sum_{b=-\Delta}^{\Delta} v_{a,b} x_{i+a,j+b}$$

2-D Convolution Layer

- input matrix $\mathbf{X} : n_h \times n_w$
- kernel matrix $\mathbf{W}: k_h \times k_w$
- b: scalar bias
- output matrix

$$\mathbf{Y} : (n_h - k_h + 1) \times (n_w - k_w + 1)$$
$$\mathbf{Y} = \mathbf{X} \star \mathbf{W} + b$$
$$y_{i,j} = \sum_{a=1}^h \sum_{b=1}^w w_{a,b} x_{i+a,j+b}$$

• W and b are learnable parameters





*



19	25
37	43

Examples

$$\begin{bmatrix} -1 & -1 & -1 \\ -1 & 8 & -1 \\ -1 & -1 & -1 \end{bmatrix}$$



(wikipedia)

$$egin{array}{ccc} 0 & -1 & 0 \ -1 & 5 & -1 \ 0 & -1 & 0 \end{array}
ight]$$

 $\frac{1}{16} \begin{bmatrix} 1 & 2 & 1 \\ 2 & 4 & 2 \\ 1 & 2 & 1 \end{bmatrix}$

$$\begin{bmatrix} 0\\ -1\\ 0 \end{bmatrix}$$



Edge Detection

Sharpen

Gaussian Blur

Examples







(Rob Fergus)



Padding

- Given a 32 x 32 input image
- Apply convolutional layer with 5 x 5 kernel
 - 28 x 28 output with 1 layer
 - 4 x 4 output with 7 layers
- Shape decreases faster with larger kernels
 - Shape reduces from $n_h \times n_w$ to $(n_h - k_h + 1) \times (n_w - k_w + 1)$



Padding

Padding adds rows/columns around input





 $0 \times 0 + 0 \times 1 + 0 \times 2 + 0 \times 3 = 0$

Padding

- Padding p_h rows and p_w columns, output shape will be

$$(n_h - k_h + p_h + 1) \times (n_w - k_w + p_w + 1)$$

- A common choice is $p_h = k_h 1$ and $p_w = k_w 1$
 - Odd k_h : pad $p_h/2$ on both sides
 - Even k_h : pad $[p_h/2]$ on top, $[p_h/2]$ on bottom

Stride

- Padding reduces shape linearly with #layers
 - Given a 224 x 224 input with a 5 x 5 kernel, needs 44 layers to reduce the shape to 4 x 4
 - Requires a large amount of computation





Stride

Stride is the #rows/#column

Strides of 3 and 2 for height and width



 $0 \times 0 + 0 \times 1 + 1 \times 2 + 2 \times 3 = 8$ $0 \times 0 + 6 \times 1 + 0 \times 2 + 0 \times 3 = 6$



Stride

Given stride s_h for the height and stride s_w for the width,
 the output shape is

$$\lfloor (n_h - k_h + p_h + s_h)/s_h \rfloor \times \lfloor (n_w - k_w + p_w + s_w)/s_w \rfloor$$

- With $p_h = k_h 1$ and $p_w = k_w 1$ $\lfloor (n_h + s_h - 1)/s_h \rfloor \times \lfloor (n_w + s_w - 1)/s_w \rfloor$
- If input height/width are divisible by strides $(n_h/s_h) \times (n_w/s_w)$

Multiple Channels

- Color image may have three RGB channels
- Converting to grayscale loses information



- Color image may have three RGB channels
- Converting to grayscale loses information



- Input is a tensor
- Have a kernel for each channel, and then sum results over channels



 $(1 \times 1 + 2 \times 2 + 4 \times 3 + 5 \times 4) + (0 \times 0 + 1 \times 1 + 3 \times 2 + 4 \times 3) = 56$

- $\mathbf{X}: c_i \times n_h \times n_w$ input tensor
- $\mathbf{W}: c_i \times k_h \times k_w$ kernel tensor
- $\mathbf{Y}: m_h \times m_w$ output

$$\mathbf{Y} = \sum_{i=0}^{c_i} \mathbf{X}_{i,:,:} \star \mathbf{W}_{i,:,:}$$

Multiple Output Channels

- No matter how many inputs channels, so far we always get single output channel
- We can have multiple 3-D kernels, each one generates a output channel
- Input $\mathbf{X}: c_i \times n_h \times n_w$
- Kernel W : $c_o \times c_i \times k_h \times k_w$
- Output $\mathbf{Y} : c_o \times m_h \times m_w$

$$Y_{i,:,:} = X \star W_{i,:,:,:}$$

for $i = 1,..., c_o$

Multiple Input/Output Channels

 Each output channel may recognize a particular pattern



 Input channels kernels recognize and combines patterns in inputs

1 x 1 Convolutional Layer

 $k_h = k_w = 1$ is a popular choice. It doesn't recognize spatial patterns, but fuse channels.



Equal to a dense layer with $n_h n_w \times c_i$ input and $c_o \times c_i$ weight.

2-D Convolution Layer Summary

- Input $\mathbf{X} : c_i \times n_h \times n_w$
- Kernel $\mathbf{W}: c_o \times c_i \times k_h \times k_w$
- Bias $\mathbf{B}: c_o$
- Output $\mathbf{Y} : c_o \times m_h \times m_w$

$$\mathbf{Y} = \mathbf{X} \star \mathbf{W} + \mathbf{B}$$

- Complexity (number of floating point operations FLOP) $c_i = c_o = 100$ $k_h = h_w = 5$ $m_h = m_w = 64$ $O(c_i c_o k_h k_w m_h m_w)$ 1GFLOP
- 10 layers, 1M examples: 10PF (CPU: 0.15 TF = 18h, GPU: 12 TF = 14min)
Pooling Layer

Pooling

Convolution is sensitive to position
 Detect vertical edges

0 output with 1

- We need some degree of invariance to translation
 - Lighting, object positions, scales, appearance vary among images

2-D Max Pooling

 Returns the maximal value in the sliding window



 $\max(0,1,3,4) = 4$



2-D Max Pooling

 Returns the maximal value in the sliding window

Vertical edge detectio for voutput 2 x 2 max pooling

[[1. 1. 0. 0. 0.][1. 1. 0. 0. 0.



Padding, Stride, and Multiple Channels

- Pooling layers have similar padding and stride as convolutional layers
- No learnable parameters
- Apply pooling for each input channel to obtain the corresponding output channel

#output channels = #input channels



Average Pooling

- Max pooling: the strongest pattern signal in a window
- Average pooling: replace max with mean in max pooling
 - The average signal strength in a window

Max pooling









 https://edstem.org/us/courses/22801/ lessons/45024/slides/257680

LeNet Architecture



Handwritten Digit Recognition

An instance of optical character recognition (OCR)

Philip Marlowe PORTLAND OR 970 638 Hollywood Blia # 615 Los Angeles, CA 1544 2019 2M21 Dave Fernik vletter, inc 509 Casiade Ave, Suite H Hood River, OR 97031 վիկվել կան հանգանությունը կան հանգերությունը 97091206080 CARROLL O'CONNOR 11725 715 **BUSINESS ACCOUNT** % NANAS, STERN, BIERS AND CO. march 10 19 80 9454 WILSHIRE BLVD., STE. 405 273-2501 **BVERLY HILLS, CALIF. 90212** 5000 DOLLARS theausa deposit 1980 Chev. pickup for UGO "°0000500000"'

MNIST

- Centered and scaled
- 50,000 training data
- 10,000 test data
- 28 x 28 images
- 10 classes

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Y. LeCun, L. Bottou, Y. Bengio, P. Haffner, 1998 Gradientbased learning applied to document recognition

EARCH

Expensive if we have many outputs



LeNet-5

Layer	#channels	kernel	stride	activation	feature
		size			map size
Input					32 x 32 x 1
Conv 1	6	5 x 5	1	tanh	28 x 28 x 6
Avg Pooling 1		2 x 2	2		14 x 14 x 6
Conv 2	16	5 x 5	1	tanh	10 x 10 x 16
Avg Pooling 2		2 x 2	2		5 x 5 x 16
Conv 3	120	5 x 5	1	tanh	120
FC 1					84
FC 2					10

LeNet in Pytorch

```
class LeNet(nn.Module):
```

```
def init (self):
    super(LeNet, self).__init__()
    self.model = nn.Sequential(
      nn.Conv2d(in_channels = 1, out_channels = 6, kernel_size = 5, stride = 1,
padding = 0),
      nn.Tanh(),
      nn.AvgPool2d(kernel size = 2, stride = 2),
      nn.Conv2d(in channels = 6, out channels = 16, kernel size = 5, stride = 1,
padding = 0),
      nn.Tanh(),
      nn.AvgPool2d(kernel_size = 2, stride = 2),
      nn.Conv2d(in_channels = 16, out_channels = 120, kernel_size = 5, stride =
1, padding = 0),
      nn.Flatten(),
      nn.Linear(120, 84),
      nn.Tanh().
      nn.Linear(84, 10))
  def forward(self, x):
    y = self.model(x)
    return v
                                                                            50
```

Recap

- Convolutional layer
 - Reduced model capacity compared to dense layer
 - Efficient at detecting spatial pattens
 - High computation complexity
 - Control output shape via padding, strides and channels
- Max/Average Pooling layer
 - Provides some degree of invariance to translation

AlexNet



ImageNet (2010)



Images	Color images	Gray image for
	with nature	hand-written
	objects	digits
Size	469 x 387	28 x 28
#	1.2 M	60 K
examples		
# classes	1,000	10

AlexNet

- AlexNet won ImageNet competition in 2012
- Deeper and bigger LeNet
- Key modifications
 - Dropout (regularization)
 - ReLu (training)
 - MaxPooling
- Paradigm shift for computer vision



AlexNet Architecture



and more output

channels.

AlexNet Architecture



AlexNet Architecture



More Tricks

- Change activation function from sigmoid to ReLu (no more vanishing gradient)
- Add a dropout layer after two hidden FFN layers (better robustness / regularization)
- Data augmentation

Data Augmentation

 Create additional training data with existing data



ReLU Activation

ReLU: rectified linear unit

 $\operatorname{ReLU}(x) = \max(x,0)$



Dropout Layer

• For every input x_i , Dropout produces $x'_i = \begin{cases} 0 & \text{with probablity } p \\ \frac{x_i}{1-p} & \text{otherise} \end{cases}$

AlexNet



Complexity

					Dense (1000)
	#parameters		FLOP		Dense (4096)
	AlexNet	LeNet	AlexNet	LeNet	f Dense (4096)
Conv1	35K	150	101M	1.2M	Max Pooling
Conv2	614K	2.4K	415M	2.4M	1 3x3 Conv (384)
Conv3-5	3M		445M		1 2x2 Copy (284)
Dense1	26M	0.48M	26M	0.48M	
Dense2	16M	0.1M	16M	0.1M	3x3 Conv (384)
Total	46M	0.6M	1G	4M	Max Pooling
Increase	11x	1x	250x	1x	5x5 Conv (256)
					Max Pooling

11x11 Conv (96), stride 4

image (224x224)

ImageNet Results: ILSVRC Winners







VGG

- AlexNet is deeper and bigger than LeNet to get performance
- Go even bigger & deeper?
- Options
 - More dense layers (too expensive)
 - More convolutions
 - Group into blocks



VGG Blocks

- Deeper vs. wider?
 - 5x5 convolutions
 - 3x3 convolutions (more)
 - Deep & narrow better
- VGG block
 - 3x3 convolutions (pad 1)
 (n layers, m channels)
 - 2x2 max-pooling (stride 2)

VGG Architecture

- Multiple VGG blocks followed by dense layers
- Vary the repeating number to get different architectures, such as VGG-16, VGG-19, ...



Going Deeper

- LeNet (1995)
 - 2 convolution + pooling layers
 - 2 hidden dense layers
- AlexNet
 - Bigger and deeper LeNet
 - ReLu, Dropout, preprocessing
- VGG
 - Bigger and deeper AlexNet (repeated VGG blocks)

Residual Networks

Best paper CVPR 2016

Does adding layers improve accuracy?



generic function classes



nested function classes

Residual Networks

- Adding a layer
 changes function
 class
- We want to add to the function class
- 'Taylor expansion' style f(x) = x + g(x) parametrization



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Kaiming He, Xiangyu Zhang, Shaoqing Ren, Jian Sun, Deep Residual Learning for Image Recognition. 2016
ResNet Block in detail



Code



Batch Normalization

- Loss occurs at last layer
 - Last layers learn quickly
- Data is inserted at first layer
 - Input layers change everything changes
 - Last layers need to relearn many times
 - Slow convergence
- This is like covariate shift
 - The distribution of each layer shift across over training process



Batch Normalization

OSS

data

• For each layer, compute mean and variance

$$\mu_B = \frac{1}{|B|} \sum_{i \in B} x_i \text{ and } \sigma_B^2 = \frac{1}{|B|} \sum_{i \in B} (x_i - \mu_B)^2 + \epsilon$$

and adjust it separately

$$x_{i+1} = \gamma \frac{x_i - \mu_B}{\sigma_B} + \beta$$

• γ and β are learnable parameters

Sergey loffe, Christian Szegedy. Batch Normalization: Accelerating Deep Network Training by Reducing Internal Covariate Shift. 2015

This was the original motivation ...

What Batch Norms really do

- Doesn't really reduce covariate shift (Lipton et al., 2018)
- Regularization by noise injection



- Random shift per minibatch
- Random scale per minibatch
- No need to mix with dropout (both are capacity control)
- Ideal minibatch size of 64 to 256



torch.nn.BatchNorm1d(num_features)

```
torch.nn.BatchNorm2d(num_features)
>>> m = nn.BatchNorm2d(100)
>>> input = torch.randn(20, 100, 32, 32)
>>> output = m(input)
```

ResNet Module

- Downsample per module (stride=2)
- Enforce some nontrivial nonlinearity per module (via 1x1 convolution)
- Stack up in blocks



Putting it all together

- Same block structure as e.g. VGG or GoogleNet
- Residual connection to add to expressiveness
- Pooling/stride for dimensionality reduction
- Batch Normalization for capacity control





ResNet in Pytorch

https://medium.datadriveninvestor.com/cnnarchitectures-from-scratch-c04d66ac20c2

Deeper is better



ImageNet Results: ILSVRC Winners



Notes

- ResNet won the champion for ILSVRC 2015
- The ResNet paper won the best paper award from CVPR 2016 (one of the leading CV conferences)
- Kaimin He won multiple best papers.

Papers of Kaimin He

- Exploring Simple Siamese Representation Learning. CVPR Best Paper Honorable Mention, 2021
- Group Normalization. ECCV Best Paper Honorable Mention, 2018



- Mask R-CNN. ICCV Best Paper Award (Marr Prize), 2017
- Focal Loss for Dense Object Detection. ICCV Best Student Paper Award, 2017
- Deep Residual Learning for Image Recognition. CVPR Best Paper Award, 2016
- Single Image Haze Removal using Dark
 Channel Prior. CVPR Best Paper Award, 2009

The first publication from Kaimin He

Discussion

- Your manager assigns a task for you: build a system to automatically select the cover photo for a short video on Tiktok
- Please discuss in groups how you plan to build the system

Summary

- Building blocks
 - Convolution
 - Stride
 - Padding
 - Channel
 - Pooling
 - Dropout
 - Batch Norm
 - Residual connection
- Data Augmentation
- Deeper is better but still efficient