Supplementary: A Realistic Multi-scale Surface-based Cloth Appearance Model

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1 TERMS OF OUR BSDF MODEL

Our complete model is defined as:

$$\begin{aligned} f(x,\mathbf{i},\mathbf{o}) = & f^{sheen}(x,\mathbf{i},\mathbf{o}) \\ & + T^{sheen} I(x)\left(f^{r,s}(x,\mathbf{i},\mathbf{o}) + f^{r,d}(x,\mathbf{i},\mathbf{o})\right) \\ & + T^{sheen} I(x')\left(f^{t,s}(x,\mathbf{i},\mathbf{o}) + f^{t,d}(x,\mathbf{i},\mathbf{o})\right) \\ & + T^{sheen}(1 - I(x)) f^{\delta}(\mathbf{i},\mathbf{o}). \end{aligned} \tag{1}$$

Surface Reflection. The specular reflection term $f^{r,s}$ is defined following the SpongeCake model, as

$$f^{r,s}(x,\mathbf{i},\mathbf{o}) = \frac{k^{r,s}(x) D(\mathbf{h},x) G^r(\mathbf{i},\mathbf{o},x)}{4(\mathbf{i},\mathbf{n_s})(\mathbf{o},\mathbf{n_s})}. \tag{2}$$

Where D is the SGGX distribution [Heitz et al. 2015] centered in the fiber tangent $\mathbf{t}(x)$ distribution, $k^{r,s}$ is the albedo, and G^r is the attenuation defined as

$$G^r(x,\mathbf{i},\mathbf{o}) = \frac{1 - e^{-T(\Lambda(\mathbf{i},x)+\Lambda(\mathbf{o},x))}}{\Lambda(\mathbf{i},x) + \Lambda(\mathbf{o},x)}, \tag{3}$$

with Λ the Smith shadowing/masking function.

The diffuse reflection term $f^{r,d}$, on the other hand, approximates multiple scattering inside yarns, and is defined as

$$f^{r,d}(x,\mathbf{i},\mathbf{o}) = \frac{k^{r,d}(x)}{\pi}, \tag{4}$$

with $k^{r,d}(x)$ the diffuse albedo, $\mathbf{n_s}$ is the surface normal.

Surface Transmission. The specular tranmission term $f^{t,s}$ is defined following the SpongeCake model, as

$$f^{t,s}(x,\mathbf{i},\mathbf{o}) = \frac{k^{t,s}(x) D(\mathbf{h},x) G^t(\mathbf{i},\mathbf{o},x)}{4(\mathbf{i},\mathbf{n_s})(\mathbf{o},\mathbf{n_s})}, \tag{5}$$

where G^t is the transmission attenuation defined as

$$G^t(\mathbf{i},\mathbf{o},x) = \frac{1 - e^{-T(\Lambda(\mathbf{i},x)+\Lambda(\mathbf{o},x))}}{\Lambda(\mathbf{i},x) + \Lambda(\mathbf{o},x)} e^{T\Lambda(\mathbf{o},x)}. \tag{6}$$

The diffuse transmission $f^{t,d}$term is derived analogously to its reflection counterpart as

$$f^{t,d}(\mathbf{i},\mathbf{o},x) = \frac{k^{t,d}(x)}{\pi}. \tag{7}$$

The delta transmission term f^{δ} is defined as:

$$f^{\delta}(x,\mathbf{i},\mathbf{o}) = \frac{\delta(\mathbf{i}+\mathbf{o})}{\langle \mathbf{i}\cdot\mathbf{n_s}\rangle}. \tag{8}$$

2 MULTI-SCALE SHADING DETAILS

Here we provide detailed formula for the base and residual term for our specular reflection and diffuse reflection lobes. The rest of the lobes can be derived in a similar fashion. Note that for brevity we omit the directional dependence on each term.

2.1 Specular Reflection

The multi-scaled integration formula for specular reflection:

$$F^{r,s} = \int_{\mathcal{P}} T^{sheen} I(p) f^{r,s}(p) V(p) \langle \mathbf{n_s}\cdot\mathbf{i}\rangle \, \mathrm{d}p \tag{9}$$

$$= T^{sheen} \frac{G_r}{4\langle \mathbf{n_s}\cdot\mathbf{o}\rangle} \bar{I}\cdot\bar{k} \int_{\mathcal{P}} D(p) V(p) \mathrm{d}p, \tag{10}$$

with $\bar{I} = \int_{\mathcal{P}} I(p)dp$ and $\bar{k} = \int_{\mathcal{P}} k^{r,s}(p)dp$. As stated in the paper, here we assume that albedo $k^{r,s}$ is smoothly varying and has little correlation with the other terms and can be integrated separately via mip-mapping. Inspired by the observation made by Zhu et al. [2022], we applied the same assumption to the gap indicator function I. This means that our focus is on understanding the statistical likelihood of light passing through the gap, rather than the specific location where it passes through. $\bar{I}$ is calculated with similar SAT-based approach introduced in Sec. 4.3 of the paper for constant time lookup. The remainder of the term $\int_{\mathcal{P}} D(p)V(p)\mathrm{d}p$ is then solved with our multi-scale shading scheme. While we already introduced the equations in the paper, for completeness, fitting into the multi-scale scheme via Clustered Control Variates in Eq.8 of the paper:

$$\int_{\mathcal{P}} D(p)V(p)\mathrm{d}p = \sum_{c\in 1,\ldots,N_c} \bar{v}_c F_c + \int_{\mathcal{P}_c} f \cdot (v - \bar{v}_c)\,\mathrm{d}p \tag{11}$$

$$= \sum_{c\in 1,\ldots,N_c} \bar{v}_c \int_{\mathcal{P}} D(p)\mathrm{d}p + \int_{\mathcal{P}} D(p)\cdot(V(p) - \bar{v}_c)\mathrm{d}p \tag{12}$$

2.2 Diffuse Reflection

The multi-scaled integration formula for diffuse reflection:

$$F^{r,d} = \int_{\mathcal{P}} T^{sheen} I(p) f^{r,d}(p) V(p) \langle \mathbf{n_s} \cdot \mathbf{i} \rangle \,\mathrm{d}p \tag{13}$$

$$= T^{sheen} \frac{1}{\pi} \langle \mathbf{n_s} \cdot \mathbf{i} \rangle \,\bar{k} \cdot \bar{I} \cdot \overline{V} \cdot \mathrm{d}p. \tag{14}$$

Following the assumptions we made for the specular reflection, we assume albedo, gap indicator function and the visibility can all be integrated separately. Here $\overline{V} = \int_{\mathcal{P}} V(p)\mathrm{d}p$ is again calculated with the SAT-based approach. By leveraging the visibility clusters that we have, we construct an SAT table where each texel indicates which clusters it belongs to. This allows us to query for the histogram and compute a weighted sum of our total 8 clusters to obtain $\overline{V}$.

3 PSEUDOCODE FOR CORE ALGORITHMS

In this section, we provide the pseudocode for our core algorithm to enhance understanding and facilitate reimplementation.

3.1 Summed Area Table Creation and Query

Algorithm 1 Create SAT

Input: tm as tangent map, dr as directional resolution, D as the cloth bsdf
Output: summed area table sat

function cDirSAT(tm, dr, D)
  $height \leftarrow tm.height$
  $width \leftarrow tm.width$
  declare $sat[dr][dr][height][width]$
  for $i \leftarrow 0$ **to** $dr - 1$ **do**
    for $j \leftarrow 0$ **to** $dr - 1$ **do**
      for $p \leftarrow 0$ **to** $height - 1$ **do**
        for $q \leftarrow 0$ **to** $width - 1$ **do**
          $qd \leftarrow Vectorize(i, j)$
          $pv \leftarrow D(tm[p][q])$
          $left \leftarrow sat[i][j][p][q-1]$ **if** $q > 0$ **else** 0
          $above \leftarrow sat[i][j][p-1][q]$ **if** $p > 0$ **else** 0
          $aboveLeft \leftarrow sat[i][j][p-1][q-1]$ **if** $p > 0$ **and** $q > 0$ **else** 0
          $sat[i][j][p][q] \leftarrow pv + left + above - aboveLeft$
        end for
      end for
    end for
  end for
  return sat
end function

Algorithm 2 Query SAT

Input: sat as precomputed summed area tables, h as half vector, $x1, y1, x2, y2$ as the four corner of the query pixel footprint
Output: radiance integral over the query area $areaSum$

function qAreaSum($sat, h, x1, y1, x2, y2$)
  $(dt, dp) \leftarrow Spherize(h)$
  $total \leftarrow sat[dt][dp][y2][x2]$
  $left \leftarrow sat[dt][dp][y2][x1-1]$ **if** $x1 > 0$ **else** 0
  $above \leftarrow sat[dt][dp][y1-1][x2]$ **if** $y1 > 0$ **else** 0
  $aboveLeft \leftarrow sat[dt][dp][y1-1][x1-1]$ **if** $x1 > 0$ **and** $y1 > 0$ **else** 0
  $areaSum \leftarrow total - left - above + aboveLeft$
  return $areaSum$
end function

3.2 Offset Precomputation

Algorithm 3 Precompute offset based on position map and viewing angles

Input: $posMap$ is the 2D map of 3D positions, ω_o is the viewing direction, x, y are the current intersected positions on the surface corresponding to u_{ori}
Output: The offset length to the primary intersected position $offset$

function OFFSET($posMap, \omega_o, x, y$)
  $maxH \leftarrow -\infty$
  for each pos in $posMap$ **do**
    if $posMap[pos] > maxH$ **then**
      $maxH \leftarrow posMap[pos]$
    end if
  end for
  $\theta, \phi \leftarrow Spherize(\omega_o)$
  $offset \leftarrow 0$
  for t from 0 to $texCount$ **do**
    $currH \leftarrow t \cdot \cos(\theta)$
    $nextH \leftarrow (t+1) \cdot \cos(\theta)$
    $currX \leftarrow x + t \cdot \sin(\theta) \cdot \cos(\phi)$
    $currY \leftarrow y + t \cdot \sin(\theta) \cdot \sin(\phi)$
    $nextX \leftarrow x + (t+1) \cdot \sin(\theta) \cdot \cos(\phi)$
    $nextY \leftarrow y + (t+1) \cdot \sin(\theta) \cdot \sin(\phi)$
    if $posMap[\lfloor currX \rfloor, \lfloor currY \rfloor] < maxH - currH$ **and** $posMap[\lfloor nextX \rfloor, \lfloor nextY \rfloor] > maxH - nextH$ **then**
      $offset \leftarrow t$
      break
    end if
  end for
  return $offset$
end function

3.3 Clustered Control Variates

Algorithm 4 Clustered Control Variates

Input: ω_i, ω_o, are the incident and outgoing direction, A is the pixel footprint covering area, D is the cloth bsdf, p is the sampling position

Output: corrected estimate of radiance over the pixel footprint Val

```
function CCV(ω_i, ω_o, A, D, p)
    C_ID ← FINDCLUSTER(p)
    h ← (ω_i + ω_o)/2
    x1, y1, x2, y2 ← QUERY(A)
    BASE ← QuerySAT(sat, h, x1, y1, x2, y2)
    Residual ← D(h)
    v̄ ← ClusterVis(C_ID)
    v ← Vis(p)
    Val ← BASE · v̄ + Residual · (v − v̄)
    return Val
end function
```

REFERENCES

Eric Heitz, Jonathan Dupuy, Cyril Crassin, and Carsten Dachsbacher. 2015. The SGGX microflake distribution. *ACM Trans. Graph.* 34, 4 (2015), 1–11.

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