

Step 1:

$$\hat{a} = \vec{a}$$
$$\vec{s}_{\parallel} = \text{proj}(\vec{s}, \hat{a}) = \hat{a} (\vec{s} \cdot \hat{a}) = a a^T \vec{s}$$
$$\vec{s}_{\perp} = \vec{s} - \vec{s}_{\parallel}$$
$$\hat{b} = \text{normalize}(\vec{s}_{\perp}) = \frac{\vec{s}_{\perp}}{\|\vec{s}_{\perp}\|}$$

Step 2:

$$\hat{c} = \hat{a} \times \hat{b} = \frac{\hat{a} \times \vec{s}_{\perp}}{\|\vec{s}_{\perp}\|}$$
$$= \frac{\hat{a} \times \vec{s}}{\|\vec{s}_{\perp}\|}$$

Step 3:

$$\vec{s}_{\perp}^{\text{ROT}} = \|\vec{s}_{\perp}\| \cos \theta \hat{b} + \|\vec{s}_{\perp}\| \sin \theta \hat{c}$$