Lecture 5:
Rasterization 1 (Triangles)
Announcements

• Homework 0 — 188 submissions
  - No worries if you did not submit

• Homework 1 will be released today
  - Containing basic and advanced requirements (graded separately)
  - Pass or not pass depends on basic requirements only

• Asking on BBS
  - Please try to describe your question more clearly

• Today’s lecture is pretty easy
Last Lecture

- Viewing (观测) transformation
  - View (视图) / Camera transformation
  - Projection (投影) transformation
    - Orthographic (正交) projection
    - Perspective (透视) projection
Today

• Finishing up Viewing
  - Viewport transformation

• Rasterization
  - Different raster displays
  - Rasterizing a triangle

• Occlusions and Visibility
Perspective Projection

- What’s near plane’s l, r, b, t then?
  - If explicitly specified, good
  - Sometimes people prefer:
    vertical **field-of-view** (fovY) and **aspect ratio**
    (assume symmetry i.e. l = -r, b = -t)

Vertical Field of View (fovY)

Aspect ratio = width / height
Perspective Projection

• How to convert from fovY and aspect to l, r, b, t?
  - Trivial

$$\tan \frac{\text{fovY}}{2} = \frac{t}{|n|}$$

$$\text{aspect} = \frac{r}{t}$$
What’s after MVP?

• **Model** transformation (placing objects)

• **View** transformation (placing camera)

• **Projection** transformation
  - Orthographic projection (cuboid to “canonical” cube $[-1, 1]^3$)
  - Perspective projection (frustum to “canonical” cube)

• Canonical cube to ?
Canonical Cube to Screen

• What is a screen?
  - An array of pixels
  - Size of the array: resolution
  - A typical kind of raster display

• Raster == screen in German
  - Rasterize == drawing onto the screen

• Pixel (FYI, short for “picture element”)
  - For now: A pixel is a little square with uniform color
  - Color is a mixture of (red, green, blue)
Canonical Cube to Screen

• Defining the screen space
  - Slightly different from the “tiger book”

Pixels’ indices are in the form of (x, y), where both x and y are integers

Pixels’ indices are from (0, 0) to (width - 1, height - 1)

Pixel (x, y) is centered at (x + 0.5, y + 0.5)

The screen covers range (0, 0) to (width, height)
Canonical Cube to Screen

- Irrelevant to z
- Transform in xy plane: $[-1, 1]^2$ to $[0, \text{width}] \times [0, \text{height}]$
Canonical Cube to Screen

- Irrelevant to \( z \)
- Transform in xy plane: \([-1, 1]^2\) to \([0, \text{width}] \times [0, \text{height}]\)
- Viewport transform matrix:

\[
M_{\text{viewport}} = \begin{pmatrix}
\frac{\text{width}}{2} & 0 & 0 & \frac{\text{width}}{2} \\
0 & \frac{\text{height}}{2} & 0 & \frac{\text{height}}{2} \\
0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1
\end{pmatrix}
\]
Next: Rasterizing Triangles into Pixels
Drawing Machines
CNC Sharpie Drawing Machine

Aaron Panone with Matt W. Moore

Laser Cutters
Different Raster Displays
Oscilloscope
Oscilloscope Art

Jerobeam Fenderson

https://www.youtube.com/watch?v=rtR63-ecUNo
Cathode Ray Tube
Television - Raster Display CRT

Cathode Ray Tube

Raster Scan Pattern of Interlaced Display

Raster Scan
(modulate intensity)
Frame Buffer: Memory for a Raster Display

Image = 2D array of colors
Flat Panel Displays

Low-Res LCD Display

Color LCD, OLED, …
LCD (Liquid Crystal Display) Pixel

Principle: block or transmit light by twisting polarization

Illumination from backlight (e.g. fluorescent or LED)

Intermediate intensity levels by partial twist
LED Array Display

Light emitting diode array
Electrophoretic (Electronic Ink) Display

Greenland or right-whale, he is the best existing authority. But Scoresby knew nothing and says nothing of the great sperm whale, compared with which the Greenland whale is almost unworthymentioning. And here be it said, that the Greenland whale is an usurper upon the throne of the seas. He is not even by any means the largest of the whales. Yet, owing to the long priority of his claims, and the profound ignorance which, till some seventy years back, invested the then fabulous or utterly unknown sperm-whale, and which ignorance to this present day still reigns in all but some few scientific retreats and whale-ports, this usurpation has been every way complete. Reference to nearly all the leviathanic allusions in the great ports of past days, will satisfy you that the Greenland whale, without one rival, was to them the monarch of the seas. But the time has at last come for a new proclamation. This is Charing Cross; hear ye! good people all,—the Greenland whale is deposed,—the great sperm whale now reigneth!

There are only two books in being which at all pretend to put the living sperm whale before you, and at the same time, in the remotest degree succeed in the attempt. Those books are Beale’s and Bennett’s; both in their time surpasse to English South-Sea whale-ships, and both honest and reliable men. The original matter touching the sperm whale to be found in their volumes is necessarily small; but so far as it goes, it is of excellent quality, though
Rasterization:
Drawing to Raster Displays
Polygon Meshes
Triangle Meshes
Triangle Meshes
Triangles - Fundamental Shape Primitives

Why triangles?

• Most basic polygon
  • Break up other polygons

• Unique properties
  • Guaranteed to be planar
  • Well-defined interior
  • Well-defined method for interpolating values at vertices over triangle (barycentric interpolation)
What Pixel Values Approximate a Triangle?

Input: position of triangle vertices projected on screen

Output: set of pixel values approximating triangle

(2.2, 1.3)
(15.3, 8.6)
(4.4, 11.0)
A Simple Approach: Sampling
Sampling a Function

Evaluating a function at a point is sampling.
We can **discretize** a function by sampling.

```c
for (int x = 0; x < xmax; ++x)
    output[x] = f(x);
```

Sampling is a core idea in graphics.
We sample time (1D), area (2D), direction (2D), volume (3D) …
Rasterization As 2D Sampling
Sample If Each Pixel Center Is Inside Triangle
Sample If Each Pixel Center Is Inside Triangle
Define Binary Function: $\text{inside}(\text{tri}, x, y)$

$x, y$: not necessarily integers

$\text{inside}(t, x, y) = \begin{cases} 1 & \text{Point } (x, y) \text{ in triangle } t \\ 0 & \text{otherwise} \end{cases}$
Rasterization = Sampling A 2D Indicator Function

for (int x = 0; x < xmax; ++x)
    for (int y = 0; y < ymax; ++y)
        image[x][y] = inside(tri,
            x + 0.5,
            y + 0.5);
Recall: Sample Locations

Sample location for pixel \((x, y)\)

\((0,0)\) to \((w,0)\)

\((0,h)\) to \((w,h)\)

\((x+0.5, y+0.5)\)
Evaluating $\text{inside}(\text{tri}, x, y)$
Inside? Recall: Three Cross Products!
Edge Cases (Literally)

Is this sample point covered by triangle 1, triangle 2, or both?
Checking All Pixels on the Screen?

Use a **Bounding Box!**
Incremental Triangle Traversal (Faster?)

suitable for thin and rotated triangles
Rasterization on Real Displays
Real LCD Screen Pixels (Closeup)

iPhone 6S

Galaxy S5

Notice R,G,B pixel geometry! But in this class, we will assume a colored square full-color pixel.
Aside: What About Other Display Methods?

Color print: observe half-tone pattern
Assume Display Pixels Emit Square of Light

* LCD pixels do not actually emit light in a square of uniform color, but this approximation suffices for our current discussion
So, If We Send the Display the Sampled Signal
The Display Physically Emits This Signal
Compare: The Continuous Triangle Function
What’s Wrong With This Picture?

Jaggies!
Aliasing (Jaggies)

Is this the best we can do?
Thank you!

(And thank Prof. Ravi Ramamoorthi and Prof. Ren Ng for many of the slides!)