Introduction to Computer Graphics
GAMES101, Lingqi Yan, UC Santa Barbara

Lecture 17:
Materials and Appearances

Announcements

• Homework 6: 82 submissions so far (note: 1.5 weeks for it)

• New assignment on path tracing has been worked out!
  - Followed the pseudocode in the last lecture as much as possible
  - Will be released this Friday

• Final project ideas: to be released soon

• From today: the lectures will be much easier!
The Appearance of Natural Materials

[Courtesy of Prof. Henrik Wann Jensen, UCSD]
What is Material in Computer Graphics?

3D coffee mug model

Rendered

Rendered

[From TurboSquid, created by artist 3dror]
Material == BRDF
What is this material?
Diffuse / Lambertian Material (BRDF)

Uniform colored diffuse BRDF

Textured diffuse BRDF

[Mitsuba renderer, Wenzel Jakob, 2010]
Diffuse / Lambertian Material

Light is equally reflected in each output direction

Suppose the incident lighting is uniform:

\[ L_o(\omega_o) = \int_{H^2} f_r L_i(\omega_i) \cos \theta_i \, d\omega_i \]

\[ = f_r L_i \int_{H^2} (\omega_i) \cos \theta_i \, d\omega_i \]

\[ = \pi f_r L_i \]

\[ f_r = \frac{\rho}{\pi} \quad \text{— albedo (color)} \]
What is this material?
Glossy material (BRDF)

[Copper]

[Copper]

[Aluminum]

[Aluminum]

[Mitsuba renderer, Wenzel Jakob, 2010]
What is this material?
Ideal reflective / refractive material (BSDF*)

Air <-> water interface

Air <-> glass interface (with absorption)
Perfect Specular Reflection

[Zátonyi Sándor]
Perfect Specular Reflection

\[ \theta = \theta_o = \theta_i \]

\[ \omega_o + \omega_i = 2 \cos \theta \vec{n} = 2(\omega_i \cdot \vec{n})\vec{n} \]

\[ \omega_o = -\omega_i + 2(\omega_i \cdot \vec{n})\vec{n} \]

Top-down view
(looking down on surface)

\[ \phi_o = (\phi_i + \pi) \mod 2\pi \]
Perfect Specular Reflection BRDF
Specular Refraction

In addition to reflecting off surface, light may be transmitted through surface.

Light refracts when it enters a new medium.
Snell’s Law

Transmitted angle depends on
index of refraction (IOR) for incident ray
index of refraction (IOR) for exiting ray

\[ \eta_i \sin \theta_i = \eta_t \sin \theta_t \]

<table>
<thead>
<tr>
<th>Medium</th>
<th>( \eta^* )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Vacuum</td>
<td>1.0</td>
</tr>
<tr>
<td>Air (sea level)</td>
<td>1.00029</td>
</tr>
<tr>
<td>Water (20°C)</td>
<td>1.333</td>
</tr>
<tr>
<td>Glass</td>
<td>1.5-1.6</td>
</tr>
<tr>
<td>Diamond</td>
<td>2.42</td>
</tr>
</tbody>
</table>

* index of refraction is wavelength dependent (these are averages)
Law of Refraction

\[ \eta_i \sin \theta_i = \eta_t \sin \theta_t \]

\[ \cos \theta_t = \sqrt{1 - \sin^2 \theta_t} \]

\[ = \sqrt{1 - \left( \frac{\eta_i}{\eta_t} \right)^2 \sin^2 \theta_i} \]

\[ = \sqrt{1 - \left( \frac{\eta_i}{\eta_t} \right)^2 (1 - \cos^2 \theta_i)} \]

Total internal reflection:

When light is moving from a more optically dense medium to a less optically dense medium: \( \frac{\eta_i}{\eta_t} > 1 \)

Light incident on boundary from large enough angle will not exit medium.
Snell’s Window / Circle

Total internal reflection

[Livingston and Lynch]
Fresnel Reflection / Term

Reflectance depends on incident angle (and polarization of light)

This example: reflectance increases with grazing angle

[Lafortune et al. 1997]
Fresnel Term (Dielectric, $\eta=1.5$)
Fresnel Term (Conductor)
Fresnel Term — Formulae

Accurate: need to consider polarization

\[
R_s = \left| \frac{n_1 \cos \theta_i - n_2 \cos \theta_t}{n_1 \cos \theta_i + n_2 \cos \theta_t} \right|^2 = \frac{\left( n_1 \cos \theta_i - n_2 \sqrt{1 - \left( \frac{n_1}{n_2} \sin \theta_i \right)^2} \right)^2}{n_1 \cos \theta_i + n_2 \sqrt{1 - \left( \frac{n_1}{n_2} \sin \theta_i \right)^2}},
\]

\[
R_p = \left| \frac{n_1 \cos \theta_t - n_2 \cos \theta_i}{n_1 \cos \theta_t + n_2 \cos \theta_i} \right|^2 = \frac{\left( n_1 \sqrt{1 - \left( \frac{n_1}{n_2} \sin \theta_i \right)^2} - n_2 \cos \theta_i \right)^2}{n_1 \sqrt{1 - \left( \frac{n_1}{n_2} \sin \theta_i \right)^2} + n_2 \cos \theta_i}.
\]

\[
R_{\text{eff}} = \frac{1}{2} \left( R_s + R_p \right).
\]

Approximate: Schlick’s approximation

\[
R(\theta) = R_0 + (1 - R_0)(1 - \cos \theta)^5
\]

\[
R_0 = \left( \frac{n_1 - n_2}{n_1 + n_2} \right)^2
\]
Microfacet Material
Microfacet Material: Motivation

https://twitter.com/Cmdr_Hadfield/status/318986491063828480/photo/1
Microfacet Theory

Rough surface
- Macroscale: flat & rough
- Microscale: bumpy & specular

Individual elements of surface act like mirrors
- Known as Microfacets
- Each microfacet has its own normal
Microfacet BRDF

• Key: the **distribution** of microfacets’ normals
  
  • Concentrated \(\iff\) glossy

  • Spread \(\iff\) diffuse
Microfacet BRDF

- What kind of microfacets reflect \( \mathbf{w}_i \) to \( \mathbf{w}_o \)? (hint: microfacets are mirrors)

\[
f(\mathbf{i}, \mathbf{o}) = \frac{F(i, h) G(i, o, h) D(h)}{4(n, i)(n, o)}
\]
Microfacet BRDF: Examples

[Autodesk Fusion 360]
Isotropic / Anisotropic Materials (BRDFs)

Inside an elevator

Slide courtesy of Prof. Ren Ng, UC Berkeley
Isotropic / Anisotropic Materials (BRDFs)

- Key: directionality of underlying surface

**Isotropic**

- Surface (normals)
- BRDF (fix $w_i$, vary $w_o$)

**Anisotropic**

- Surface (normals)
- BRDF (fix $w_i$, vary $w_o$)
Anisotropic BRDFs

Reflection depends on azimuthal angle $\phi$

$$f_r(\theta_i, \phi_i; \theta_r, \phi_r) \neq f_r(\theta_i, \theta_r, \phi_r - \phi_i)$$

Results from oriented microstructure of surface, e.g., brushed metal
Anisotropic BRDF: Brushed Metal

- How is the pan brushed?
Anisotropic BRDF: Nylon

[Westin et al. 1992]
Anisotropic BRDF: Velvet

[Westin et al. 1992]
Anisotropic BRDF: Velvet

[https://www.youtube.com/watch?v=2hjoW8TYTd4]
Properties of BRDFs

- Non-negativity

\[ f_r(\omega_i \rightarrow \omega_r) \geq 0 \]

- Linearity

\[
L_r(p, \omega_r) = \int_{H^2} f_r(p, \omega_i \rightarrow \omega_r) L_i(p, \omega_i) \cos \theta_i \, d\omega_i
\]

[Sillion et al. 1990]
Properties of BRDFs

- Reciprocity principle

\[ f_r(\omega_r \rightarrow \omega_i) = f_r(\omega_i \rightarrow \omega_r) \]

- Energy conservation

\[ \forall \omega_r \int_{H^2} f_r(\omega_i \rightarrow \omega_r) \cos \theta_i \, d\omega_i \leq 1 \]
Properties of BRDFs

• Isotropic vs. anisotropic

• If isotropic, \( f_r(\theta_i, \phi_i; \theta_r, \phi_r) = f_r(\theta_i, \theta_r, \phi_r - \phi_i) \)

• Then, from reciprocity,

\[
f_r(\theta_i, \theta_r, \phi_r - \phi_i) = f_r(\theta_r, \theta_i, \phi_i - \phi_r) = f_r(\theta_i, \theta_r, |\phi_r - \phi_i|)
\]
Measuring BRDFs
Measuring BRDFs: Motivation

Avoid need to develop / derive models

• Automatically includes all of the scattering effects present
Can accurately render with real-world materials

• Useful for product design, special effects, ...

Theory vs. practice:

[Bagher et al. 2012]
Image-Based BRDF Measurement

[Marschner et al. 1999]
Measuring BRDFs: gonioreflectometer

Spherical gantry at UCSD
Measuring BRDFs

General approach:

```plaintext
foreach outgoing direction wo
    move light to illuminate surface with a thin beam from wo
for each incoming direction wi
    move sensor to be at direction wi from surface
measure incident radiance
```

Improving efficiency:

- Isotropic surfaces reduce dimensionality from 4D to 3D
- Reciprocity reduces # of measurements by half
- Clever optical systems...
Challenges in Measuring BRDFs

• Accurate measurements at grazing angles
  • Important due to Fresnel effects
• Measuring with dense enough sampling to capture high frequency specularities
• Retro-reflection
• Spatially-varying reflectance, ...
Representing Measured BRDFs

Desirable qualities

• Compact representation
• Accurate representation of measured data
• Efficient evaluation for arbitrary pairs of directions
• Good distributions available for importance sampling
Tabular Representation

Store regularly-spaced samples in \((\theta_i, \theta_o, |\phi_i - \phi_o|)\)
- Better: reparameterize angles to better match specularities

Generally need to resample measured values to table

Very high storage requirements

MERL BRDF Database
[Matusik et al. 2004]
90 * 90 * 180 measurements
Thank you!

(And thank Prof. Ravi Ramamoorthi and Prof. Ren Ng for many of the slides!)