Using a heap as a priority queue

- To remove highest priority item from heap:

  `remove root;` /* `O(1)` complexity */

  Move last item to root, then …

  `heapify in reverse;` /* `O(log n)` complexity */

  - So overall complexity is `O(log n)`
- Also `O(log n)` for insert function
- Compare to other priority queue strategies
  - Sorted list: insert – `O(n)`; remove – `O(1)`
  - Unsorted array: insert – `O(1)`; remove – `O(n)`
- Choose heap strategy if `n` is expected to be large

A table ADT

- Declare Table type (define in implementation)
```
typedef struct TableTag Table;
```
  - Also define a `KeyType`, and maybe a `DataType` (or just use `void *`)
- Let user define initial size of table
```
Table *createTable(int startingSize);
```
- Can put/get/update/remove info associated with unique key
```
int put(KeyType key, void *info, Table *table);
void* get(KeyType key, Table *table);
int update(KeyType key, void *info, Table *table);
int remove(KeyType key, Table *table);
```
  - Functions return false if unsuccessful (except `get` returns `NULL`)
- Can print all info, usually in key order
```
void printAll(Table *table);
```

Table implementation options

- Many possibilities – depends on application
  - And how much trouble efficiency is worth
- Option 1: use a BST
  - To put: insertTree using key for ordering
  - To update: deleteTree, then insertTree
  - To getAll: use in-order traversal
- Option 2: sorted array with binary searching (later)
- Option 3: implement as a “hash table”
  - Hashing – general technique works great with tables

Hashing ideas and concepts

- Idea: transform arbitrary key domain (e.g., strings) into "dense integer range" – then use result as index to table
  - `index = hash(key);` /* function returns int */
- Collisions: `hash(k1)==hash(k2), k1 != k2`
  - Usually impossible to avoid ("perfect hashing"), so must have a way to handle collisions
  - e.g., probe for empty slot if using "open addressing" - `while (!empty(index)) index = probe(key);`
- Concept: insertion/searching is quick – but only until the table starts to get filled up
  - Then collisions start happening too often!

Open address hashing

– & implementing basic table ADT

- Define structs for table items and whole table of items

```
typedef struct
   { KeyType key; void *info; } TableItem;
typedef struct
   { int size; int n; TableItem *items; } Table;
```

  - `size` is size of array; `n` is the number of items in the table
  - Constructor allocates memory for array of items, and initializes all items to “empty” key
- The `put` function uses `hash(key)` and `probe(key)` to find empty slot for new item
  - *Resizes* array (and *rehashes* existing items) whenever table “load factor” reaches 50 percent (rule of thumb for open addressing)

Open address hashing (cont.)

- `get` & `update` functions use `hash(key)` and `probe(key)` in *exact same sequence* as `put` – to find existing info
- `remove` is more complicated
  - Cannot just remove an item – future probes for get and update might terminate prematurely at empty slot
  - Inefficient technique rehashes all items
  - Alternative technique uses “deleted” key markers
    - But problem with that is table fills up prematurely
- `printAll` in key order – must sort first!
  - So `O(n log n)` at best!

Resolving collisions

- Simplest open address approach is linear probing
  - If (`index = hash(key)`) is not empty, try `index+1`, then `index+2`, …, until empty slot
  - In other words, searching for first "open address"
  - Biggest problem: it leads to "primary clusters"
- Quadratic probing – varies probe, like 1, 3, 6, …
  - Leads to "secondary clusters" but not as quickly
- Double hashing – `probe(key)` varies by key
  - Best open addressing approach for avoiding clusters
- Or a completely different approach: "chaining"

Chaining

- Table is an array of pointers to lists:

```
typedef struct TableTag
        { int size; int n;
          ListPointer *lists; };
```

- Constructor allocates memory for array, and creates an empty list for each element of the array
- `put` function uses `hash(key)` and appends to end of list
  - Clustering not a problem, but long lists can be, so rule of thumb is resize when load factor approaches 80%
- `remove` function is easier now – just delete from list
- But lots more overhead than open addressing
  - Must store node pointers as well as key and info
  - Use list function calls instead of direct array access

Recursive binary searching

- Start with a sorted array: `a[0..n-1]`
  - Useful item in `a` is `struct{key, info} ItemType`;
- Binary searching algorithm is naturally recursive:

```
int bsearch(KeyType key, ItemType a[],
            int left, int right) {
   /* first call is for left=0, and right=n-1 */
   int middle = (left + right) / 2;
   if (key == a[middle].key) return middle; /* success */
   if (left > right) return -1; /* unsuccessful */
   if (key > a[middle].key) /* search one half or the other */
      return bsearch(key, a, middle+1, right);
   else return bsearch(key, a, left, middle-1);
}
```

- Iterative version is a little trickier (but not too hard)

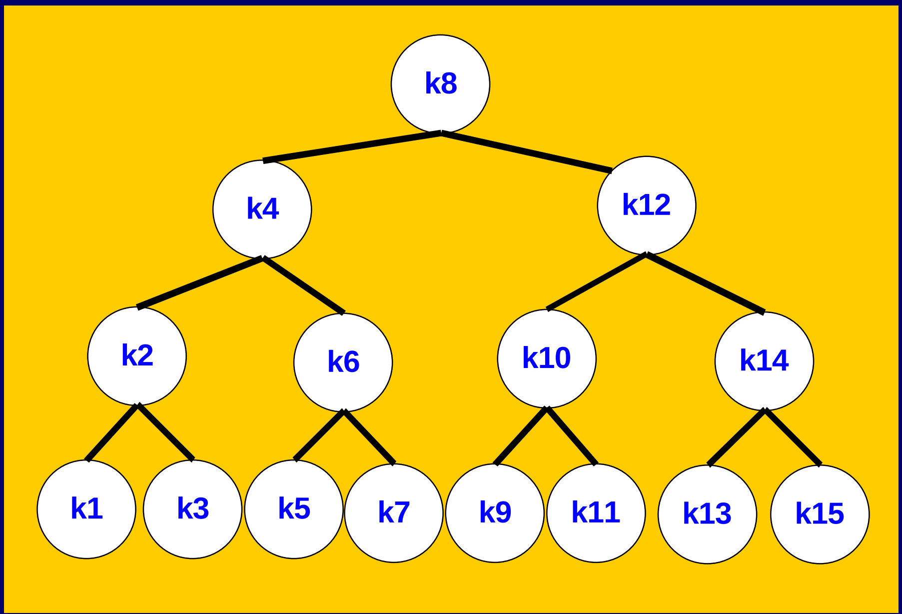
Iterative binary searching

```
int bsearch(KeyType key, ItemType a[], int n) {
   int low = 0, high = n-1, middle;
   while (low <= high) {
      middle = (low + high) / 2;
      if (key == a[middle].key)
         return middle; /* success */
      if (key > a[middle].key) low = middle + 1;
      else high = middle – 1;
   }
   return -1; /* unsuccessful */
}
```

- Both versions are same complexity class
  - But recursive version has more overhead, so actually runs a bit slower than iterative version
  - Interpolation search, by the way, is in a faster class

Complexity of binary search

- Say array has 15 elements, $k_1 .. k_{15}$: `a[0..14]`
  - If key is at k_8 (`a[7]`) then found by 1 comparison
  - If key is at k_4 or k_{12}, takes 3 comparisons …
- i.e., it's just like searching a BST



- Problem size is halved at each step
  - So complexity class is `O(log n)`
- Interpolation search reduces more quickly
  - Class is `O(log log n)`

Compare 3 table implementations

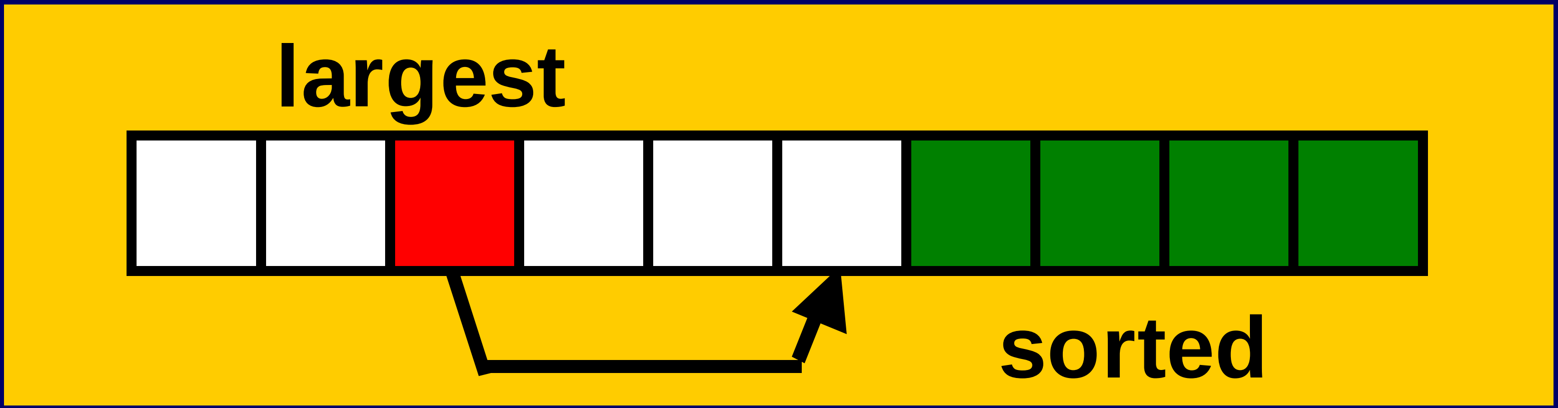
Table operation	Hash table	BST	Sorted array
create	O(n)	O(1)	O(n)
find, get, update	O(1)	O(log n)	O(log n)
put	O(1)	O(log n)	O(n)
remove	O(1)	O(log n)	O(n)
printAll	O(n log n)	O(n)	O(n)

- Conclusion – depends on table purpose & n size
  - Hash table wins for most applications if n is large
  - BST wins if expect to printAll frequently
  - Sorted array might win for small n – to minimize overhead/work

Sorting

- Probably *the* most expensive common operation
- Problem: arrange `a[0..n-1]` by some ordering
  - e.g., in ascending order: `a[i-1]<=a[i], 0<i<n`
- Two general types of strategies
  - Comparison-based sorting – includes most strategies
    - Apply to any comparable data – (key, info) pairs
    - Lots of simple, inefficient algorithms
    - Some not-so-simple, but more efficient algorithms
  - Address calculation sorting – rarely used in practice
    - Must be tailored to fit the data – not all data are suitable
    - Won't cover in CS 12 – see proxmap and radix sorts in sec. 13.6

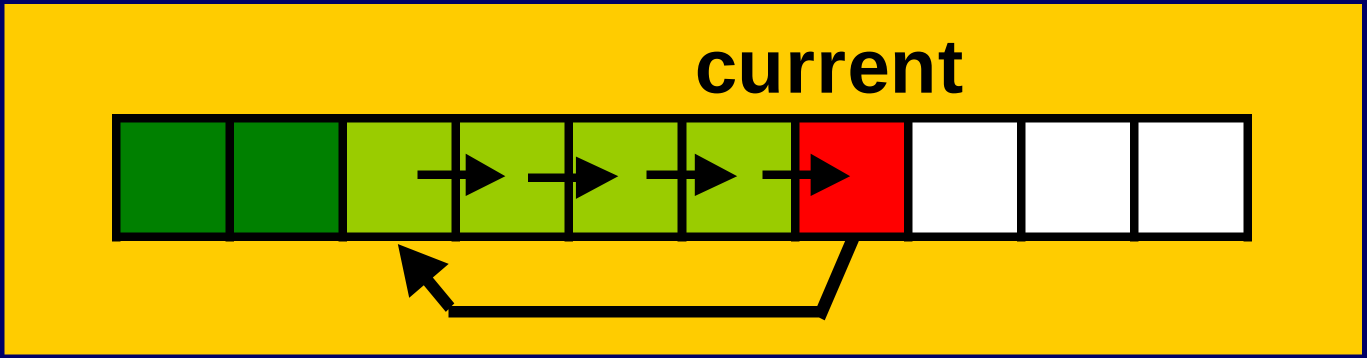
Selection sort



- Idea: build sorted sequence at end of array
- At each step:
  - *Find largest* value in not-yet-sorted portion
  - *Exchange* this value with the one at end of unsorted portion (now beginning of sorted portion)
- Complexity is $O(n^2)$– but simple to program
  - Also – best way to find k^{th} largest, or top k values

Insertion sort

- Generally "better" than other simple algorithms
- Inserts one element into sorted part of array
  - Must move other elements to make room for it



- Complexity is O(n^2) (code)
  - But runs faster than selection sort and others in class
  - Really quick on *nearly sorted* array
- Often used to supplement more sophisticated sorts

Divide & conquer strategies

- Idea: (1) divide array in two; (2) sort each part; (3) combine two parts to overall solution
- e.g., mergeSort

```
if (more than one item in array):
   divide array into left half and right half;
   mergeSort(left half); mergeSort(right half);
   merge(left half and right half together);
```

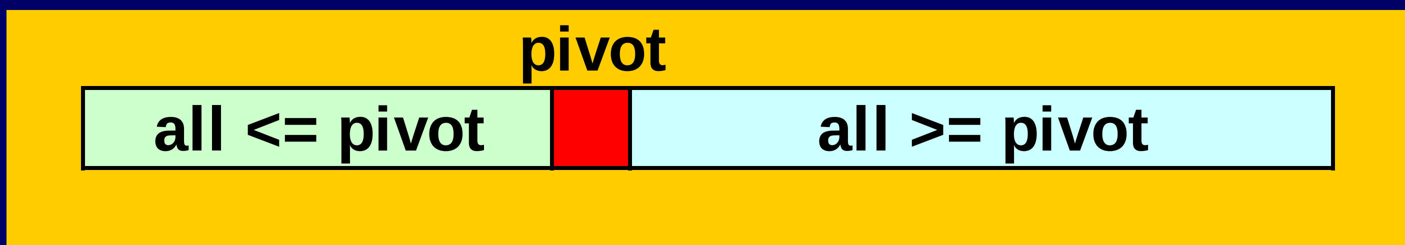
  – Requires helper method to merge two halves
  – Complexity is `O(n log n)`
  – The best sort for large files (too big for memory)
- But for most problems, quickSort is a better divide & conquer strategy

Quick sort

- Basic quicksort algorithm is recursive

```
if (there is something to sort)
{    partition array elements;
     sort left part; sort right part;
} /* It's the utility partition function that does all the work! */
```

- Partition idea: arrange elements around an arbitrary pivot




```
scan from (i = left) until a[i] >= pivot;
scan from (j = right) until a[j] <= pivot;
swap a[i], a[j];
continue both scans until i > j;
```

(code)

Quick sort (cont.)

- Complexity is `O(n log n)` on average
  - Fastest comparison-based sorting algorithm
  - But overkill, and not-so-fast with small arrays
    - One frequently-used optimization applies insertion sort for partitions smaller than than 10 or so
- But worst case is $O(n^2)$!
  - Just like BST worst case – sorted order can be bad
    - Especially if first or last is chosen as pivot – middle is better
- By the way – see `qsort` in `<stdlib.h>` (code)
  - Also by the way – see `O(n)` address calculation sorts if really fast sorting is required for an application