Table ADT and Sorting

Algorithm topics continuing (or reviewing?) CS 24 curriculum

A table ADT (a.k.a. Dictionary, Map)

`Table` public interface:

// Put information in the table, and a *unique* key to identify it:

```
bool put(KeyType key, ItemType info);
```

// Get information from the table, according to the key value:

```
ItemType get(KeyType key);
```

// Update information that is already in the table:

```
bool update(KeyType key, ItemType newInfo);
```

// Remove information (and associated key) from the table:

```
bool remove(KeyType key);
```

// Above methods return false if unsuccessful (except `get` returns null)

// Print all information in table, in the order of the keys:

```
void printAll();
```

Table implementation options

- Many possibilities – depends on application
  - And how much trouble efficiency is worth
- Option 1: use a BST
  - To put: insertTree using key for ordering
  - To update: deleteTree, then insertTree
  - To printAll: use in-order traversal
- Option 2: use sorted array with binary searching
- Option 3: implement as a “hash table”

Hashing ideas and concepts

- Idea: *transform arbitrary key* domain (e.g., strings) into "dense integer range"
  - Then *use result as index* to table
  - `int index = hash(key);` // transform key to int
- Collisions: `hash(k1)==hash(k2), k1 != k2`
  - Usually impossible to avoid ("perfect hashing" rare)
  - Therefore, must have a way to handle collisions
    - e.g., if using "open addressing" techniques - `while (`*occupied*`(index)) index = probe(key);`
- Concept: insertion/searching is quick – but only until the table starts to get filled up
  - Then collisions start happening too often!

Implementing a hash table

- Constructor allocates memory for array of items, and initializes all items to "empty" key
  - `size` is size of array
  - `n` is the number of items in the table
  - Load factor is `n / size`
- `put` method uses `hash(key)` (and `probe(key)` if open address hashing) to find empty slot for new item
  - May be necessary to *resize* array
    - If so, also necessary to *rehash* existing items
    - If open address hashing, resize when load factor > 50%

Open address hashing

- `get` & `update` methods use `hash(key)` and `probe(key)` in *exact same sequence* as `put`
  - To find existing info where it was put
- `remove` is more complicated
  - Cannot just remove an item – future probes for get and update might terminate prematurely at empty slot
    - Common trick is to have "deleted" key
      - Problem with that is table can seem full prematurely
    - Inefficient alternative rehashes all items when any removed
- Note: to `printAll` in key order – must sort first
  - So `O(n log n)` at best!

Hash functions

- Goal: uniform distribution of keys
  - Means each index of table is equally likely
  - Important for reducing collisions
- Common approach is a *restricted transformation*
  - Step 1 – transform key to *large* integer
  - Step 2 – restrict integer to 0…size-1
    - Usually done with modulus operator - %
- Lots of variations – partly depends on key type
  - General observation: hard to find a good hash function
  - Note: should be "cheap" to compute too – e.g., division is slower on most CPUs than addition

Resolving collisions

- Simplest open address approach is linear probing
  - If (`index = hash(key)`) is not empty, try `index+1`, then `index+2`, …, until empty slot
    - Note: searching for first "open address"
  - Leads to "primary clusters" – collisions bunch up
- Quadratic probing – vary probe, like 1, 3, 6, …
  - Leads to "secondary clusters" but not as quickly
- Double hashing – `probe(key)` varies by key
  - Best open addressing approach for avoiding clusters
- Or completely different approach – "chaining"

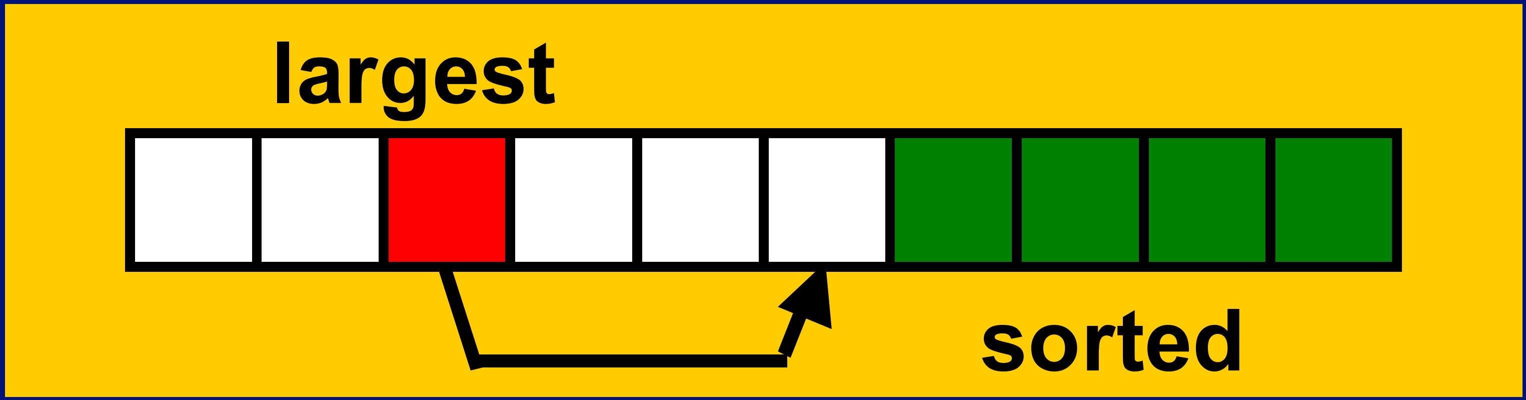
Chaining

- Constructor allocates memory for array of Lists, and creates an empty list for each element of the array
- `put` method uses `hash(key)` and appends to end of list at that index of array
  - Still should resize when load factor approaches 80%
    - Clustering is not a problem, but long lists slow performance
- `remove` method is easier now – just delete from list
- But lots more overhead than open addressing
  - Must store node links as well as key and info
  - Use list method calls instead of direct array access

Sorting

- Probably *the* most expensive common operation
- Problem: arrange `a[0..n-1]` by some ordering
  - e.g., in ascending order: `a[i-1]<=a[i], 0<i<n`
- Two general types of strategies
  - Comparison-based sorting – includes most strategies
    - Apply to any comparable data – (key, info) pairs
    - Lots of simple, inefficient algorithms
    - Some not-so-simple, but more efficient algorithms
  - Address calculation sorting – rarely used in practice
    - Must be tailored to fit the data – not all data are suitable

Selection sort



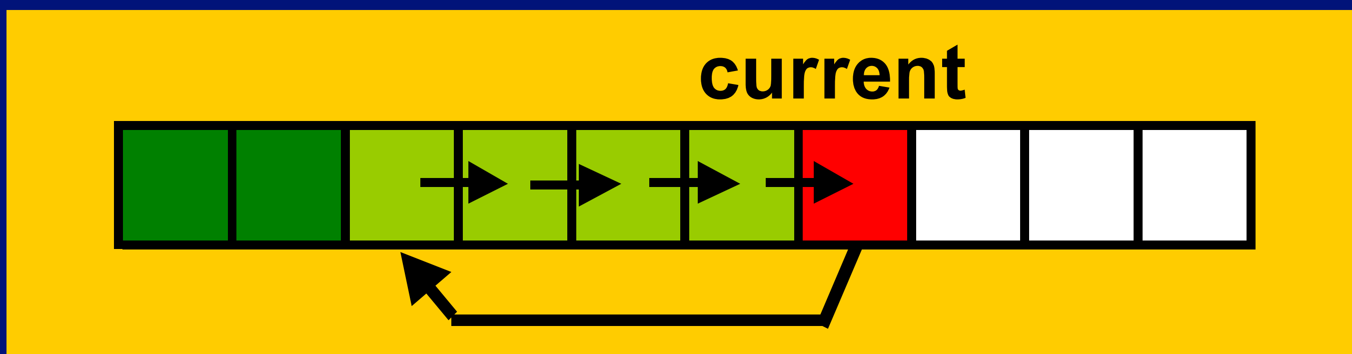
- Idea: build sorted sequence at end of array
- At each step:
  - *Find largest* value in not-yet-sorted portion
  - *Exchange* this value with the one at end of unsorted portion (now beginning of sorted portion)
- Complexity is `O(n`^{2}`)` – but simple to program
- Also best way to find kth largest, or top k values

Heap sort

- Another priority queue sorting algorithm
  - Note about selection sort: unsorted part of array is like a priority queue – remove greatest value at each step
  - Also recall that heaps make faster priority queues
- Idea: create heap out of unsorted portion, then remove one at a time and put in sorted portion
- Complexity is `O(n log n)`
  - `O(n)` to create heap + `O(n log n)` to remove/reheapify
- Note proof: `O(n log n)` is the fastest possible class of any *comparison-based* sorting algorithm
  - But constants do matter – so some are faster than others in practice

Insertion sort

- Generally "better" than other simple algorithms
- Inserts one element into sorted part of array
  - Must move other elements to make room for it



- Complexity is `O(n`^{2}`)` (code)
  - But runs faster than selection sort and others in its class
  - Really quick on *nearly sorted* array
- Often used to supplement more sophisticated sorts

Divide & conquer strategies

- Idea: (1) divide array in two; (2) sort each part; (3) combine two parts to overall solution
- e.g., mergeSort

```
if (array is big enough to continue splitting) →
   divide array into left half and right half;
   mergeSort(left half);
   mergeSort(right half);
   merge(left half and right half together);
else → sort small array in a simpler way
```

  – Cost each time to merge two halves is `O(n)`, and overall complexity is `O(n log n)`
  – But notice it also uses `2n` space
  – Commonly used to sort large files (i.e., when there are too many records to load all of them into memory at once)

Quick sort

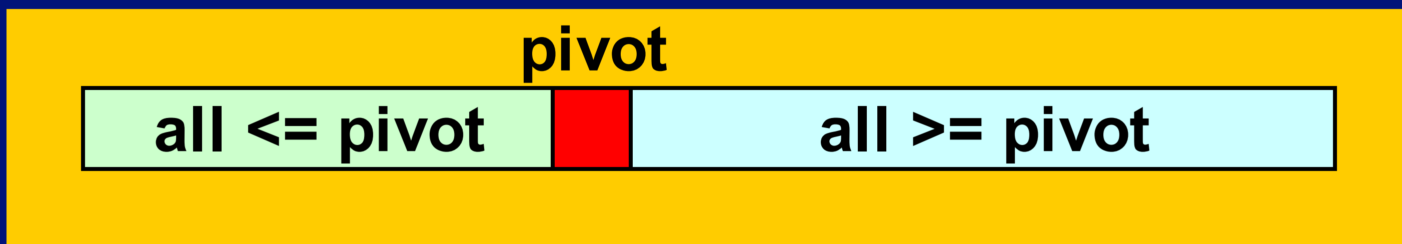
- Invented in 1960 by C.A.R. Hoare
  - Studied extensively by many people since
  - Probably used more than any other sorting algorithm
- Basic (recursive) quicksort algorithm:

```
if (there is something to sort)
{    partition array;
     sort left part;
     sort right part;  }
```

  - All the work is done by partition (a.k.a. split) function
  - And there is no need to merge anything at the end

Partitioning (for quickSort)

- Arrange so elements in the two sub-arrays are on correct side of a pivot element
  - Also means pivot element ends up in its final position



- Done by performing two series of “scans”

```
scan from (i = left) until a[i] >= pivot;
scan from (j = right) until a[j] <= pivot;
swap a[i] and a[j], and continue both scans;
stop scanning when i >= j;
```

(code)

Quick sort (cont.)

- Complexity is `O(n log n)` on average
  - Fastest comparison-based sorting algorithm
  - But overkill, and not-so-fast with small arrays
    - Um … what about a small partition?!
  - One optimization applies insertion sort for partitions smaller than than 7 elements
- And worst case is $O(n^2)$!
  - Depends on initial ordering and choice of pivot
- Btw: C library `qsort`, and C++ STL `sort`

Compare 3 table implementations

Table operation	Hash table	BST	Sorted array
create (new table)	`O(n)`	`O(1)`	`O(n)`
get, update	`O(1)`	`O(log n)`	`O(log n)`
put	`O(1)`	`O(log n)`	`O(n)`
remove	`O(1)`	`O(log n)`	`O(n)`
printAll	`O(n log n)`	`O(n)`	`O(n)`

- Conclusion: choice depends on table purpose and size of `n`
- Q. Ever want to use a sorted array?
  - A. It *depends*!