1. Given the conditional “The visiting team wins whenever it is sunny,” write in English its
   (a) negation
   (b) converse
   (c) contrapositive
   (d) inverse.

2. (a) Consider the English sentence “You cannot ride the roller coaster if you are under 4 feet tall unless you are older than 16 years old.” Let \( q, r \) and \( s \) represent “You can ride the roller coaster,” “You are under 4 feet tall,” and “You are older than 16 years old,” respectively. Express the given sentence as a logical expression using \( q, r \) and \( s \).

   (b) Let \( A \) be the proposition “All jokers are mean” and \( B \) be the proposition “Arthur Fleck is a joker.” Restate “A necessary condition for \( B \) is \( A \)” in the form of a conditional proposition (i.e. of the form “If ..., then ...”) in English.

3. Let \( S(x, y) \) be the predicate \((x-y)^2 \geq (x+y)^2\). The domain of any quantified variables below is the set of all natural numbers. What are the truth values of the following statements? Give a brief reason for each answer.
   (a) \( S(0,0) \)
   (b) \( S(2,1) \)
   (c) \( S(1,1) \)
   (d) \( \forall x S(x, x) \)
   (e) \( \exists x S(0, x - 0.5) \)
   (f) \( \exists x \forall y S(x, y) \)
   (g) \( \forall x \exists y S(x + 4, y) \)
   (h) \( \exists x \exists y S(x, 3y) \)

4. Compose a compound proposition involving the propositional variables \( p, q, \) and \( r \) that is true when at least two of the variables are true, and is false otherwise.

5. Suppose that the domain of the propositional function \( P(x) \) is \( \{-1, 0, 1, 2\} \). Write out each of the following propositions using disjunctions, conjunctions and negations only:
   (a) \( \exists x P(x) \)
   (b) \( \forall x P(x) \)
   (c) \( \neg \exists x P(x) \)
   (d) \( \exists x ((x \geq 0) \land P(x)) \)

6. Let \( A = \{1, 2, 3, 4, 5\} \) and \( B = \{0, 3, 6\} \). Then
   (a) \( A \cup B = \)
   (b) \( A \cap B = \)
   (c) \( A \setminus B = \)
   (d) \( B \setminus A = \)
(e) \( A \oplus B = \)

(f) \( \{x, y\} \times B = \)

(g) The power set \( \mathcal{P}(B) = \)

7. Prove that \( \neg(r \lor (q \land (\neg r \rightarrow \neg p))) \equiv \neg r \land (p \lor \neg q) \) by using a series of logical equivalences. Justify each step by stating what rule is applied.

8. Suppose \( \mathcal{I} = (0, 1) \) is the interval of real numbers strictly between 0 and 1. State whether each of these statements is true or false:

(a) \( x \in \{x\} \)
(b) \( \{x\} \in \{x\} \)
(c) \( x \subseteq \{x\} \)
(d) \( \{x\} \subseteq \{x\} \)
(e) \( \emptyset \in \{x\} \)
(f) \( \emptyset \subseteq \{x\} \)
(g) \( \emptyset \in \{\} \)
(h) There is a bijection between \( \mathcal{I} \) and the real numbers \( \mathbb{R} \).
(i) There is a bijection between \( \mathcal{I} \) and \( \mathcal{I} \times \mathcal{I} \).
(j) There is a bijection between \( \mathcal{I} \) and the set of all Java programs.

9. Provide the negation of the following statements, in English, but do not start with a negation (i.e., “It is not the case that . . .”). For example, the negation of “Fido is a dog” should be “Fido is not a dog,” rather than “It is not the case that Fido is a dog.”

(a) If the Clippers win the NBA championship, then I will dye my hair purple.
(b) Clippers win the NBA championship only if I will dye my hair purple.
(c) Every CS graduate has taken discrete math.
(d) Some rich people have taken discrete math.
(e) At every NBA game, someone sings the national anthem.

10. Assume \( n \) is an integer. Prove that \( n \) is odd if and only if \( n^2 + 1 \) is even.