1. (20 pts) Suppose $A$ and $B$ are two sorted arrays, each with $n$ numbers. Design an $O(\log n)$ time algorithm to find the median of the set $A \cup B$. (You can assume that there are no duplicates in the set $A \cup B$.)

2. (20 pts)
   - Solve the following recurrence both using the Master Method and the basic recurrence expansion method.
     \[ T(n) = \sqrt{n}T(\sqrt{n}) + n \]
     Why do you think you get different answers? Which answer is the correct one?
   - Find the exact solution for the following recurrence, with precise constants.
     \[ T(n) = 2T(n/2) + 2 \]

3. (20 pts) The linear-time **Selection** algorithm described in class divides the input elements into groups of 5, and then recursively solves the problem.
   - Re-analyze the algorithm when we divide the elements into groups of 3. Specifically, determine the sizes of the subproblems on which recursive calls will be made. Write down the final recurrence. Does the algorithm still run in $O(n)$ time? Support your answer with an argument.
   - Re-analyze the algorithm if the elements are divided into groups of 7. Again, determine the sizes of the recursive subproblems, and derive the final recurrence. Does the algorithm still run in $O(n)$ time? Support your answer with an argument.

4. (20 pts) In a population of $n$ people, at most $k$ have been infected by a deadly virus. Our task is to identify the infected people and treat them. The infection can be detected by testing a patient’s blood. Unfortunately the test is very expensive, and so we want to minimize the number of tests to carry out.

   Fortunately, one can mix the blood samples of multiple people and test at once. In such a **group** test, if at least one of the patients is infected, the test result is positive; otherwise, it is negative. Specifically, for a subset $S$ of people, $test(S)$ carries out one test and returns a YES if at least one person in $S$ is infected, and NO otherwise.

   Describe an algorithm that identifies all the infected people using only $O(k \log n)$ tests.

**Submission Instructions.**

- Typeset your solution.
- After signing up at Gradescope, select **Homework 2** and just follow the instructions. You can either submit one image per question, or upload one pdf and identify the pages for each question.