CS-235
Computational Geometry

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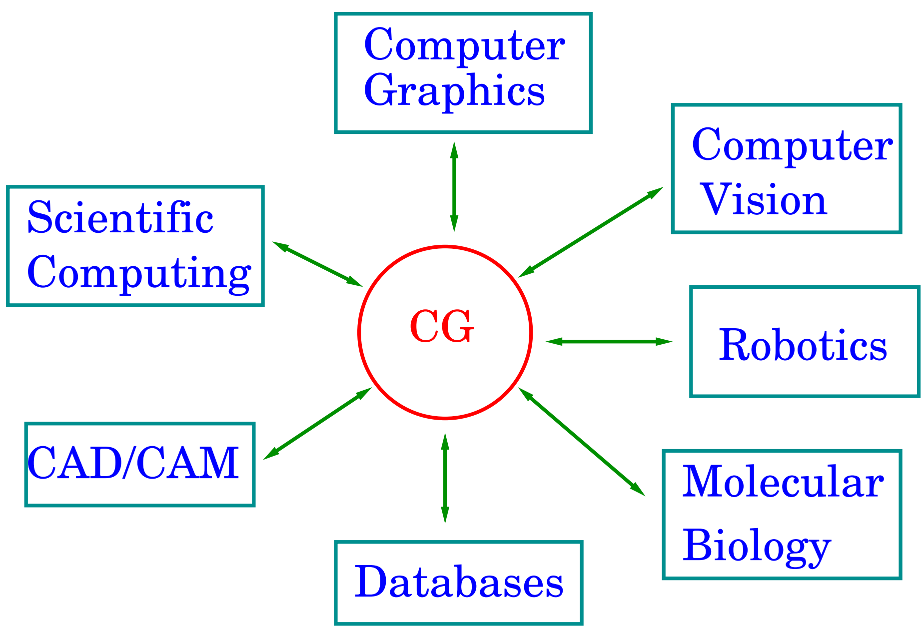
Computational Geometry

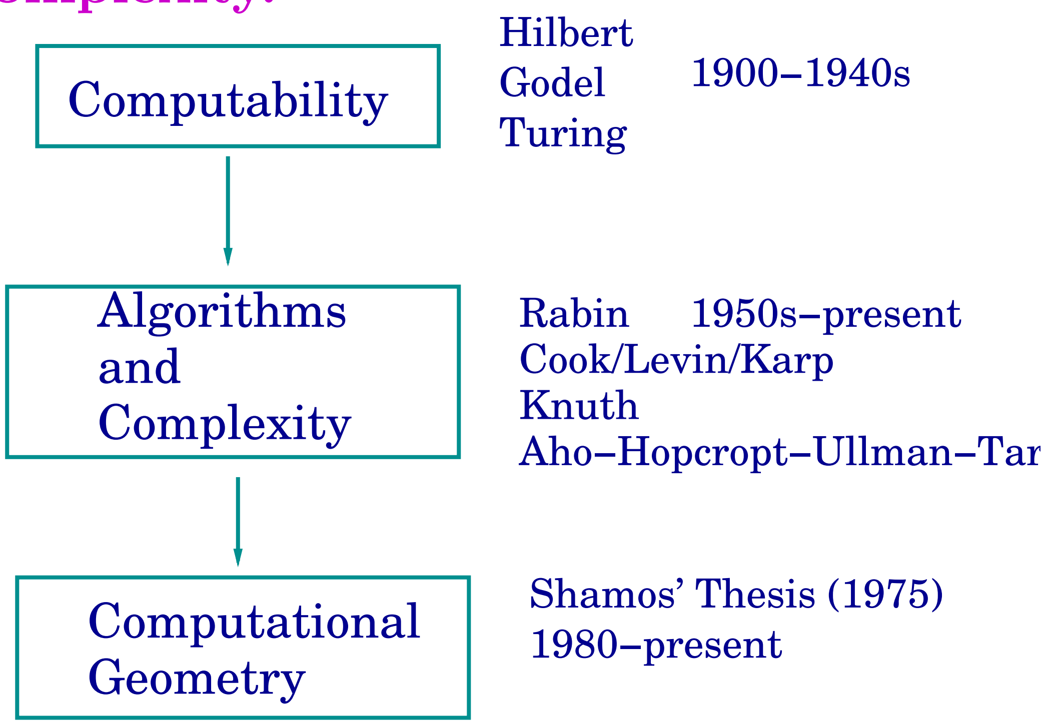
- **Study of algorithms for geometric problems.**
- **Deals with discrete shapes: points, lines, polyhedra, polygonal meshes.**
- **Abstraction of problems in different applied areas.**



- **Occlusion, visibility, augmented reality, collision detection, motion or assembly planning, drug design, databases, GIS, layout, fluid dynamics, etc.**

CG and Computer Science

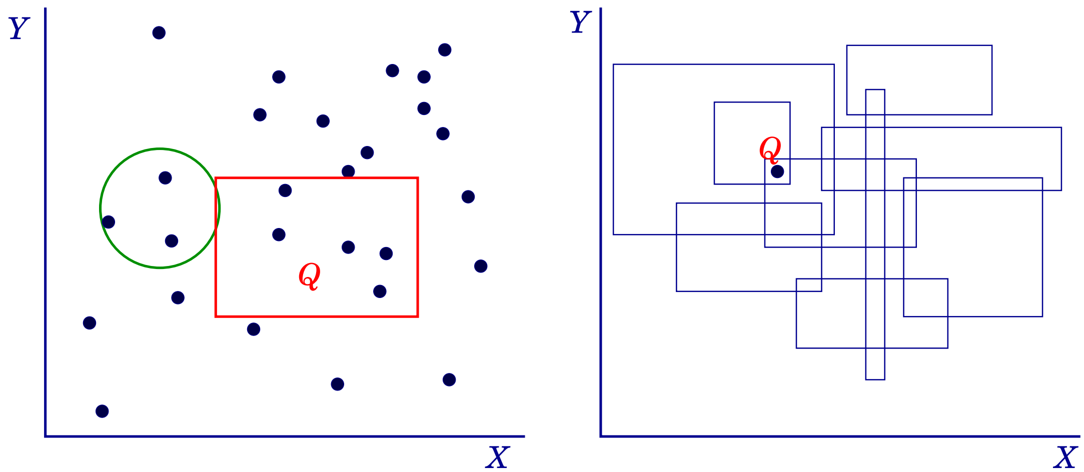
- **CG is a sub-discipline of algorithms and complexity.**



- **Develops fundamental techniques and tools for geometric problems.**
- **Motivated by applications in other CS fields.**
- **Significant interaction with discrete mathematics.**

Some Examples

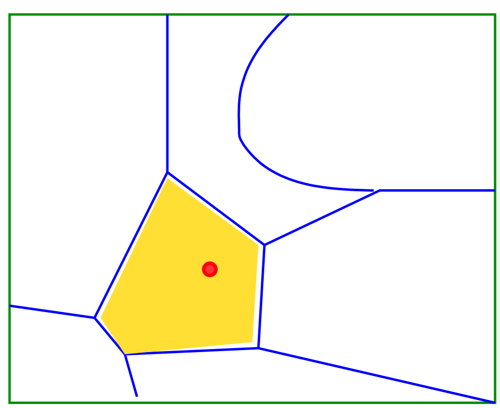
- **Range Searching Data Structures.**



Point objects — Rectangular objects

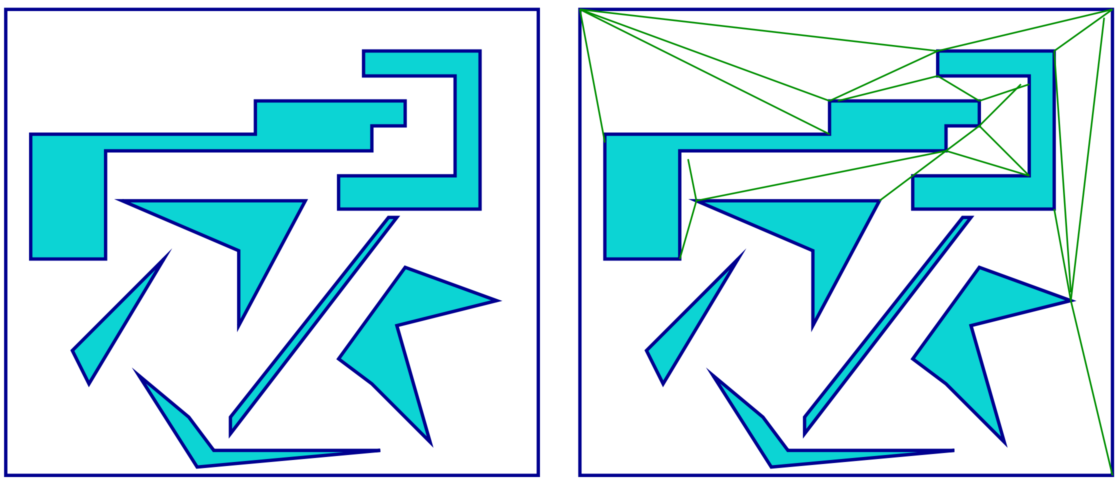
- **Location Queries.**

p = (18, 11)



Some Examples

- **Decomposition.**

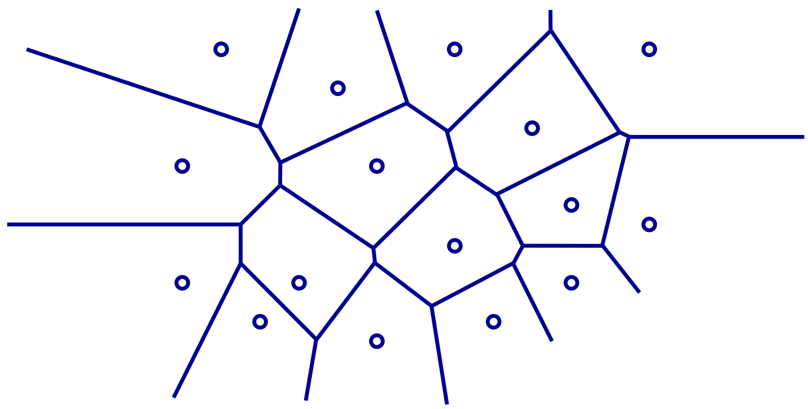


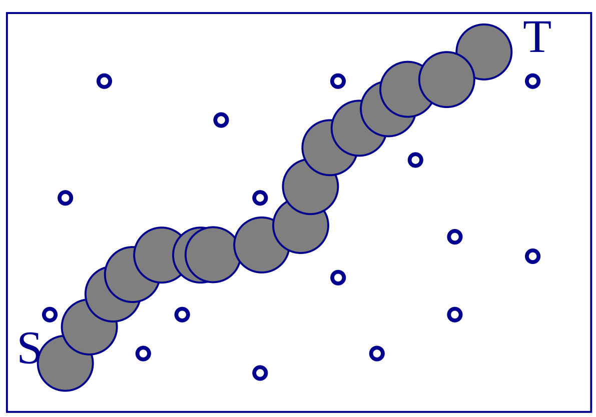
Geometric Scene   Partial Triangulation

- **Is this always possible?**
- **In three dimension?**
- **Other examples: Shortest paths, geometric structures, visibility, pattern matching.**

Some Examples

- **Spatial Data Structures.**





- **Voronoi diagram, Delaunay triangulation.**

- **Robot motion planning.**

Taste of Comb. Geometry

- **Helly's Theorem:** Let $C_1, \ldots, C_n$ be a family of convex sets in the plane. If every triple intersects, then $\bigcap C_i$ is non-empty.

- **Center Points:** Given points $p_1, p_2, \ldots, p_n$ in the plane, a point x is called center point if any line through x contains at least $n/3$ points on each side.

- **Ham Sandwich Theorem:** Take n red points and n blue points in the plane. There is a line simultaneously bisecting both red and blue points.

- **Crossing Number Theorem:** If G is a graph with n nodes and m edges, then every drawing of G in the plane contains at least $c\left(\frac{m^3}{n^2}\right) - n$ crossings.

Taste of Comb. Geometry

- **Among any 5 points in the plane in general position, we can find 4 forming a convex polygon.**
- **Erdös-Szekeres Theorem: For every positive integer k, there exists a number F_k, such that every set of F_k points in the plane contains k that form a convex k-gon.**
- **Empty k-gon: How large must the set be to guarantee that we can find k points forming a convex polygon, which does not include any other point inside?**
- **Empty k-gon: Known values:**
  $G_3 = 3$.
  $G_4 = 5$.
  $G_5 = 10$.
  $G_6 = ???$.
  $G_7 = \infty$!

Spirit of CG

1. **CG is a product of marriage between classical geometry and computer science.**
2. **Emphasis on design of efficient algorithms and data structures.**
3. **In classical approach, reducing the complexity to a finite number of choices was enough.**
4. **Alas!** 10^{100} **is mathematically finite but computationally infinite.**
5. **Point of Reference: 1 Year** $\approx 3.15 \times 10^7$ **seconds.**
6. **1 Century** $\approx \pi \times 10^9$ **seconds.**
7. **A 1-Giga flop computer does only** 10^{20} **ops in a century!**

Model of Computation

1. **Assume an abstract programming language model.**
2. **Traditionally, real-number extension of Random Access Machine (RAM).**
3. **Each memory cell can hold one int or real geometric coordinate. We will discuss later numerical precision issues.**
4. **Standard repertoire of operators:**
   - **Arithmetic:** $+, -, \div, *, \sqrt{}$.
   - **Trigonometry:** $\sin, \cos, \tan, \exp, \log$.
   - **Comparators:** $\leq, \geq, =$.
   - **Array indexing, pointers.**

Elementary Objects

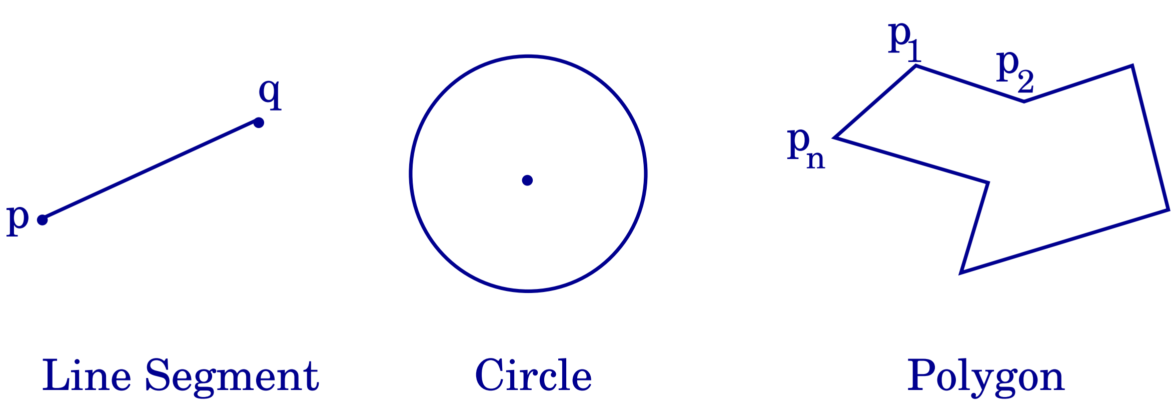
1. **Point** $p = (x, y)$, **where** x, y **reals.**

2. **Line** $\ell := \; ax + by = 1$.

3. **Line segment** $s \; = \; [p, q]$.

4. **Circle** $C \; = \; (p, r)$. **(Center, radius)**

5. **Polygon** $< p_1, p_2, \ldots, p_n >$.



Elementary Operations

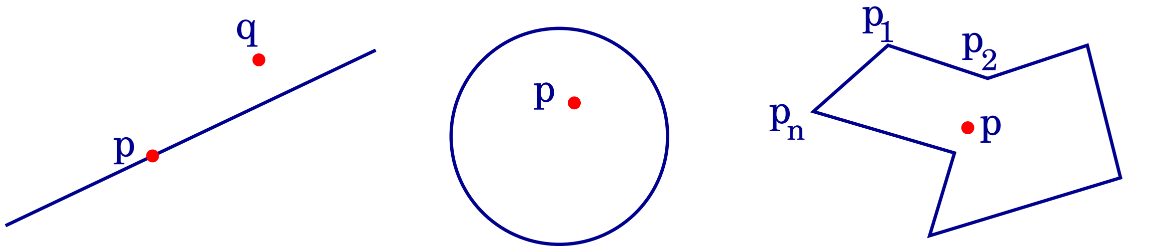
1. **Algorithm receives algebraic input. It must perform computation to see the underlying geometric relationships.**

2. **Is point p on line ℓ?**

3. **Is point p inside or outside circle C?**

4. **Do segments s_1 and s_2 intersect?**

5. **Is point p inside or outside polygon P?**



Point on Line   Point in Circle   Point in Polygon

Overview of the Course

1. **Convex Hulls.**
2. **Intersection Detection and Reporting.**
3. **Triangulation.**
4. **Range Searching.**
5. **Point Location.**
6. **Delaunay Triangulation.**
7. **Voronoi Diagrams.**
8. **Arrangements.**
9. **Binary Space Partitions.**
10. **Epsilon Net and VC Dimension.**
11. **Volume and paradoxes in higher dimensions.**
12. **Misc.**